MTH150 Project 3

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1 Chapter 3 Polynomial and Rational Functions

1.1 3.1 Power Functions and Polynomial Functions

Question 1, Page 166
Find the long run behavior of each function as $x goes to \infty$ and $x goes to -\infty$
$f(x) = x^4$
a) $f(x)$ will approach positive ∞ as x approaches positive ∞
b) $f(x)$ will approach positive ∞ as x approached $-\infty$ because x is being raised by a positive exponent.

Question 15, Page 166
Find the degree and leading coefficient of each polynomial.
$(2x + 3)(x - 4)(3x + 1)$
Simplify: $(2x^2 - 5x - 12)(3x + 1)$
$6x^3 - 13x^2 - 41x - 12$
The leading coefficient is 6 and the degree is 3.

Question 17, Page 166
Find the long run behavior of each function as $x goes to \infty$ and $x goes to -\infty$.
$-2x^4 - 3x^2 + x - 1$
a) The leading coefficient is negative so as x approaches positive ∞ the function will be $-\infty$.
b) The leading coefficient is negative and the polynomial has an even degree, so as x approaches $-\infty$ the function will be $-\infty$.

Question 21 Page 166
What is the maximum number of x-intercepts and turning points for a polynomial of degree 5?
a) The function will have 5 x-intercepts.
b) The maximum turning points of a polynomial of degree n is $n - 1$ Because the degree is 5 in this problem, the maximum number of turning points will be 4.

Question 31, Page 166
Find the vertical and horizontal intercepts of each function.

$f(t) = 2(t-1)(t+2)(t-3)$
To find our vertical intercept, plug in 0 for t. We then get: $f(0) = 2((0)-1)((0)+2)((0)-3)$
Multiply $(-2)(2)(-3)$ to then get -12
The y-intercept of $f(t)$ is $(0, 12)$.
To find the x-intercepts, find the products when $f(t)$=0.
$(0-1) = 0$=1
$(0+2) = 0$=-2
$(0-3) = 0$=3
our products are -2,1, and 3, meaning that the x-intercepts are, $(-2, 0), (1, 0)$ and $(3, 0)$.

At the beginning of this section, 3.1, Power Functions and Polynomial Functions, I was somewhat confused. I had jumped straight to the questions instead of watching any of the videos beforehand, thinking that I would understand what to do (that wasn't the case). After I went back and watched the linked videos for Chapter 3, section 1 I remembered how too solve for these questions since I took a pre-calc class while I was in high school. I simply needed a refresher. After watching all of the videos I seemed to understand how to solve all the questions easily, besides question 17. I understood I had to find the leading coefficient and the degree, however because the exponent was positive, I thought that when x was approaching ∞ or $-\infty$ the function would be approaching ∞. I went back and looked in the solution manual to make sure that I was right, however I saw that the function would be approaching $-\infty$. That confused me, however I think that I now understand what it was looking for. Because we are working with Polynomials, when the leading coefficient is negative, it will remain negative, regardless of the degree. Besides question 17, I understood how to solve for the problems once watching the videos that you left for us to watch for this section.

1.2 3.2 Quadratic Functions

Question 7, Page 177
For each of the follow quadratic functions, find a) the vertex, b) the vertical intercept, and c) the horizontal intercepts.
$y(x) = 2x^2 + 10x + 12$
7a) find the vertex.
To find the vertex (h, k), begin with the equation of line of symmetry, which is $\frac{-b}{2a}$. But before we are able to substitute the values into that equation, we need to know what the standard form is for this equation. $y(x) = ax^2 + bx + c$. $a = 2$, $b = 10$, $c = 12$. Substitute the known values into the line of symmetry equation.
$\frac{-(10)}{2(2)}$= $\frac{-10}{4}$= $-2\frac{1}{2}$
We now know that the x cordinate for the vertex is $-2\frac{1}{2}$, however, in the vertex,

this point is known as h. Now, we need to find the y cordinate of the vertex, which is known as k.
First, begin subsitituting $-2\frac{1}{2}$ for all x values in the equation.
$y(-2\frac{1}{2}) = 2(-2\frac{1}{2})^2 + 10(-2\frac{1}{2}) + 12 = -\frac{1}{2}$
The k value for the vertex is $-\frac{1}{2}$
Therefore, the vertex of the quadratic equation is $(-\frac{10}{4}, -\frac{1}{2})$
7b) Find the vertical intercept of the equation.
The y-intercept of the equation is 12.
7c) Find the horizontal intercepts of the equation.
Divide the equation by 2. $\frac{2x^2+10x+12}{2} = x^2+5x+6$ After this, break the equation down into a binomial to find the horizontal intercepts. $(x+2)(x+3) = x^2+5x+6$
Set them to equal to zero, and you end up getting:
$x + 2 = 0 = -2$
$x + 3 = -3$
The horizontal intercepts would be $(-2, 0)$ and $(-3, 0)$.

Question 13, Page 177
Rewrite the quadratic function into vertex form.
$f(x) = x^2 - 12x + 32$
Vertex form is $f(x) = a(x-h)^2 + k$, when the equation is currently in standard form, which looks like $f(x) = ax^2 + bx + c$. The points h and k symbolize were the vertex is, hence the name "Vertex Form". To begin, to find x, divide $-b$ over $2a$. For this problem, that would look like $\frac{12}{2} = 6$. 6 is equivalent to k. Now that k is known, substitute 6 for all x values.
$f(6) = (6)^2 - 12(6) + 32 = 36 - 72 + 32$ Simplify: $f(6) = -4$
Now, both the k and h values are known. The vertex is $(6, -4)$, and substituting these values into the vertex form will give us the vertex form of our equation.
$f(x) = 1(x-6)^2 - 4$

Question 19, Page 178
x-intercepts (-3,0) and (1,0) and y-intercept (0,2).
To begin, we can set up the equation $f(x) = a(x - r1)(x - r2)$ and insert our x-intercepts into the equation. That will look like, $f(x) = a(x+3)(x-1)$. Once we have that, we can subsititue our y-intercept, $(0, 2)$ into the equation as well. Set the x values to 0, and then the equation will also be equal to 2.
$2 = a(3)(-1) = f(0) = 2 = -3a = -\frac{2}{3}$
We now know our a value as well, so we can substitute all known values into the equation. $f(x) = -\frac{2}{3}(x+3)(x-1)$
Standard form:$f(x) = -\frac{2}{3}x^2 - \frac{4}{3}x - 2$

Question 27, Page 178
A rocket is launched in the air. Its height, in meters above sea level, as a function of time, in seconds, is given by $h(t) = -4.9t^2 + 229t + 234$
27a) From what height was the rocket launched?
234 feet.

27b) How high above sea level does the rocket reach its peak?
You can find this by $\frac{-b}{2a}$. This equation gives the axis of symmetry of the parabola, which will also be the highest or lowest point. That looks like $\frac{-(229)}{2(4.9)}$=23.3673 for this equation. After, plug in 23.3673 for all t values in the equation.
$f(23.3673) = 4.9(23.3673)^2 + 229(23.3673) + 234$= 2909.56
2909.56 feet.
27c) Assuming the rocket will splash down in the ocean, at what time does the splashdown occur?
To find the time it took for the rocket to reach water, use the equation $\frac{-b}{2a}$ and multiply by 2. We can do that because $\frac{-b}{2a}$ is the equation to find the axis of symmetry which is also the highest point that the rocket will reach. If we multiply it by two, that will allow us to find the total time the rocket was in the air. We already know that that is equal to 23.3673 and now multiply by 2.
46.73 seconds.

Question 33, Page 179
A farmer wishes to enclose two pens with fencing, as shown. If the farmer has 500 feet of fencing to work with, what dimensions will maximize the area enclosed?
We know that there are two enclosures, and that there needs to be enough fencing for the two. There are 3 shorter sections (y-axis) of the enclosures, and two longer sides (x-axis). The equation would be $\frac{500-3x}{2}$=$250-\frac{3x}{2}$ which would then look like $250-\frac{3x}{2}^2$ to find the area of the two enclosures.
To begin, the equation $\frac{-b}{2a}$ will give us the shorter side legnths of the enclosure.
$\frac{-250}{2(\frac{3}{2}}$=$83\frac{1}{3}$ ft.
After that, plug in $83\frac{1}{3}$ for x in the equation to find the two longer side lengths of the enclosures.
$250-\frac{3(83\frac{1}{3}}{2}$= 125 ft.
To check our work, $83\frac{1}{3}(3) + 125(2) = 500$

For this section, 3.2, Quadratic Functions, I struggled tremendously at the beginning. I was never good with quadratic functions, so this section took me a very long time to complete. What was hard for me was understanding how to find the vertex of a parabola, but also how to write a quadratic function with the given intercepts. I watched the videos that you linked on your website, however I was still left confused. I then had to try and find more videos on youtube to help to show me more examples on the topic. After some time, I finally got the hang of how to solve for the equations, and I'm hoping that as this chapter progresses, I will have a better understanding on quadratics. Now that I have finished this section, I feel more comfortable solving questions like this.

1.3 3.3 Graphs of Polynomial Functions

Question 19, Page 191
Solve Each Inequality
$(x-3)(x-2)^2 > 0$
First, set the equations to equal zero.
$x - 3 = 0 = x = 3$
$x - 2 = 0 = x = 2$
After this, find it so that the Inequality value is true. Substitute 3 in for each x value.
$(3-3)(3-2)^2 > 0 = 1 > 0$, which is true. This inequality is true when $x > 3$

Question 31, Page 192
Write and equation for a polynomial with the given features.
Degree 3 zeros at $x = -2$, $x = 1$, and $x = 3$ vertical intercept $(0, 4)$

To start, we can use our intercepts to find $f(x)$. We know that our vertical intercept is equal to y, and that y is equal to $f(x)$, so therefore we can plug 4 into $f(x)$, and our x values in the equation will be equal to 0. Therefore, the function will look like, $-4 = (0+2)(0-3)(0-1)$.
This can be simplified so that it would look like $-4 = (2)(-3)(-1)$. We can multiply all of our values together so that we get 6. for this problem, our equation would look like $f(x) = -\frac{2}{3}(x+2)(x-3)(x-1)$.

Question 51, Page 193
A rectangle is inscribed with its base on the x axis and its upper corners on the parabola $y = 5 - x^2$ What are the dimensions of such a rectangle that has the greatest possible area?

To begin, to some this may look like a difficult question by the writers of the textbook using the word parabola while talking about rectangles. However, it isn't that scary. The formula to find the area of a rectangle is $A = 2xy$. From the information given, we know that this formula would look like $A = 2x(5-x^2)$= $10x - 2x^3$. In order to see the maximum of $10x - 2x^3$, we must use a graphing calculator. We need to find the maximum and minimum of the x axis to know how long the base of the rectangle is.

Once we graph $y = 5 - x^2$ into our calculator we can see that the Xmax is located at $(2.606, 0)$ and that the Xmin is located at $(-2.393, 0)$. We can now add those two together to find how long the base of the rectangle is. That comes out to 5 units long. Now, we have to solve where exactly the two top points lie on the graph

For this section, I found that for the most part these questions were difficult to solve. I understood how to do question 19, however after that I was having some difficulties. This section was the last one that I finished in the whole project, but mostly because I was so stuck on question 51 I didn't know what to do. I was absent that day from class, so I have rewatched that lecture however was still having a lot of difficulty solving for the equation. I still am not exactly sure if what I have is correct, but I did put a lot of time into this one question, and I'm hoping that it managed to come out all right.

1.4 3.4 Factor Theorem and Remainder Theorem

Question 21, Page 202
Use the techniques in this section to find the rest of the real zeros and factor the polynomial.
$x^3 - 6x + 11x - 6$ when $c = 1$
To begin, to find other zeros of the function, I started by using a calculator to graph out the function to have a visual of what the function looked like. From the views on the graph, it looks as though 2 and 3 are other possible zeros. To test this out, use synthetic division to determine if they are zeros of the equation.

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 11 & -6 \\ \hline & & 2 & -8 & 6 \\ \hline & 1 & -4 & 3 & 0 \end{array}$$

For the synthetic division above is the work used to show that 2 is a zero of the function $x^3 - 6x + 11x - 6$.

$$\begin{array}{r|rrrr} 3 & 1 & -6 & 11 & -6 \\ \hline & & 3 & -9 & 6 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

Above is the work for the synthetic division to determine is 3 is a zero of the function $x^3 - 6x + 11x - 6$. Since it comes out to equal zero, we can guarantee that 3 is one of the zeros for this function.
$(x - 1)(x - 2)(x - 3)$

For this section, 3.4 Factor Theorem and Remainder Theorem, I understood how to solve for the questions with ease. I remembered how to use long and synthetic division from high school. The only thing that I found remotely confusing was what was meant by $c = 1$ in question 21. This confused me because I wasn't sure what I was supposed to do with that information or what it was equal to. I went back and reread the section for some extra knowledge and finished this section rather quickly once I made it over that small set back.

1.5 3.5 Real Zeros of Polynomials

Question 1, Page 209
For each of the following polynomials, use a) Cauchy's Bound to find an interval

containing all the real zeros, b) then use Rational Roots Theorem to make a list of possible rational zeros.
$f(x) = x^3 - 2x^2 - 5x + 6$
a) To begin, to find an interval containing using all real zeros while using Cauchy's Bound is quite easy. The equation is $(-\frac{M}{a_n} - 1, \frac{M}{a_n} + 1)$.
The value M is the largest coefficient in absolute value, so therefore M equals 5. The value a_n is the leading coefficient, therefore a_n equals 1. Now, plug in the known values into the function to find all real zeros in the equation.
$(-\frac{5}{1} - 1, \frac{5}{1} + 1)$= $[-6, 6]$
All real zeros lay between $(-6, 6)$.
b) To find all possible rational zeros, the equation would be $r = \pm\frac{p}{q}$.
r symbolizes the rational zeros of f. p is the constant term, which is a_0, and q is the leading coefficient a_n. In this equation, p=6, because the constant term is 6. q is equal to 1.
The factors of 6 are -6,-3,-2,-1,1,2,3, and 6.
The factors of 1 are -1, and 1.
Therefore the only possible rational roots would be -6,-3,-2,-1,1,2,3,6.

Question 11, Page 209
Find the real zeros of each polynomial.
$f(x) = x^3 - 2x^2 - 5x + 6$

Luckily, all of our work for this problem was done in question 1. We already know where all the possible zeros lie, and what they could be. Now, all that is needed to be done is solve what they are. To get a good idea of what these possible zeros could be, using a graphing calculator to see the visual of the line would be helpful. From the visual, it looks as though -2,1, and 3 are good possibilities. To test if these are the zeros of $f(x) = x^3 - 2x^2 - 5x + 6$, we can use synthetic division, similar to what was used in the previous section.
Testing if -2 is a zero of the function is shown below.

$$\begin{array}{r|rrrr} -2 & 1 & -2 & -5 & 6 \\ \hline & & -2 & 8 & -6 \\ \hline & 1 & -4 & 3 & 0 \end{array}$$

As we can see, -2 is one of he real zeros of the function.
Below will be the work to test if 1 is a zero of the function. If this was to be factored, it would look like $(x + 2)$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -5 & 6 \\ \hline & & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

As we can see, 1 is a real zero of the function. If this was too be factored, it would look like $(x - 1)$

Lastly, we will be testing if 3 is a real zero of the function.

$$\begin{array}{c|cccc} 3 & 1 & -2 & -5 & 6 \\ \hline & & 3 & 3 & -6 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

As we can see, 3 is a real zero of the function. If this was to be factored it would look like $(x-3)$.
if everything was put together, the factored form of $f(x) = x^3 - 2x^2 - 5x + 6$ would look like $f(x) = (x+2)(x-1)(x-3)$

For section 3.5, Real Zeros of Polynomials, I found the material easy to understand and to cover the the material mentioned in it (Cauchy's Bound and the Rational Root Theorem). I found Cauchy's Bound the easiest covered material in this section, however we did go over one of the questions based on it during one of our lectures. For an example, we can take the first part of question 7 on page 209 into more depth. To begin, the first part of question 7 asks to use Cauchy's Bound to find he interval where all the real zeros would lay between. $f(x) = 17x^2 + 5x^2 + 34x - 10$ is the equation at question. The formula for Cauchy's Bound is $(-\frac{M}{a_n} - 1, \frac{M}{a_n} + 1)$ However, we don't know what M is equal to or what ta_n is equal to either. The value M is the largest coefficient in absolute value. This means that for this equation, M is equal o 34. Secondly, a_n is the leading coefficient, meaning that a_n is equal to 17. Now that we know the material that is necessary for the equation, we are able to plug in the knowns into the equation, $(-\frac{34}{17} - 1, \frac{34}{17} + 1)$. Now we can simplify the equation to find the intervals that the real zeros lie. $(-\frac{34}{17} - 1, \frac{34}{17} + 1) = (-3, 3)$ Therefore, we can guarantee that all real zeros will lay in between $(-3, 3)$.

1.6 3.6 Complex Zeros

Question 5, Page 217
Simplify each expression to a single complex number.
$\frac{2+2\sqrt{-12}}{2}$

To start, lets understand what exactly we're looking at. In the nominator, we have $\sqrt{-12}$, meaning that we will have a complex number as our final answer of the question. We cannot have a negative square root with real numbers, meaning that an imaginary number must be introduced. We can break down the square root to look like $(\sqrt{12})(\sqrt{-1}) = \sqrt{-12}$. However, we can symbolize $\sqrt{-1}$ as ι. Therefore, we have $(\sqrt{12})(\iota)$.

Next, we should simplify $\sqrt{12}$ to a simpler number. We can do this by checking for perfect squares. We know that $4 * 3 = 12$, and that f is a perfect square. Therefore, we can now write $\sqrt{-12}$ as $(\iota)(\sqrt{4})(\sqrt{3})$.
We know that $\sqrt{4} = 2$, meaning that when everything is put together, $\sqrt{-12}$ would now look like $2\sqrt{3}\iota$.

With this information, we can now put the whole equation together, so that it now looks like $\frac{2+2\sqrt{3}\iota}{2}$. We can now approach this question by dividing by 2. After that division, we are left with $1+\sqrt{3}i\iota$ (if we wanted this to be even more simplified we could use 2.732ι. since 1.732^2 is equal to 3).

Question 19, Page 217
Simplify each expression to a single complex number.
$\frac{3+4\iota}{2}$

Unlike question 5, there is not a square root in the question, meaning that we do not have to break that down to a simpler form. Because of that, we would be able to skip right to dividing $3+4\iota$ by 2. After that, we are left with $\frac{3}{2}+\frac{4\iota}{2}$. We can see that we can simplify this even more, by diving 4ι by 2, meaning that that we would then have $1.5+2\iota$

Question 37, Page 217
Find all the zeros of the polynomial then completely factor it over the real numbers and completely factor over the complex numbers.
$f(x) = x^4 + x^3 + 7x^2 + 9x - 18$

We can start off this problem by using Cauchy's Bond in order to find the coordinates in which all possible zeros would lie. We know that the equation to find the coordinates is $(-\frac{M}{a_n} - 1, \frac{M}{a_n} + 1]$. We know that M is equal to the largest coefficient, meaning that $M = 9$. We also know that a_n is equal to the leading coefficient, so that means that $a_n = 1$. Now we are able to substitute our known values into the equation. $(-\frac{9}{1} - 1, \frac{9}{1} + 1)$. From this, we know that all of our possible zeros will have to lie in between $[-10, 10]$.

From here, we are able to use The Rational Root Theorem to find possible zeros of the function $f(x) = x^4+x^3+7x^2+9x-18$. We can do this by using the equation, $r = \pm\frac{p}{q}$. p is equal to the constant term, so in this case, $p = 18$, and q is equal to the leading coefficient, meaning hat for this function $q = 1$. From here we have a large list of possible zeros, however, using a graphing calculator and inserting the function and the coordinates $[-10, 10]$ for the Xmin and Xmax values, will give us a better look at what the zeros could be. After graphing, it looks at though -2, or 1 could be possible zeros of his function. We can see if hey are by using synthetic division.
Below will be the work to see if -2 is a possible zero of $f(x) = x^4 + x^3 + 7x^2 + 9x - 18$

$$\begin{array}{r|rrrrr} -2 & 1 & 1 & 7 & 9 & -18 \\ \hline & & -2 & 2 & -18 & 18 \\ \hline & 1 & -1 & 9 & -9 & 0 \end{array}$$

From this information, we can guarantee that -2 is one of the zeros of the function.
Now that we know that -2 is a zero, lets test 1. newline

1	1	1	7	9	-18
		1	2	9	18
	1	2	9	18	0

We now know that 1 is also a zero of the function $f(x) = x^4 + x^3 + 7x^2 + 9x - 18$.

However, the question states that we need to factor this problem. We already know that $(x+2)(x-1)$ will be part of it, but we are still missing a large portion of the problem. Once we factor $(x+2)(x-1)$ we can see that we will have $x^2 + x - 2$. From here, we can use synthetic division to solve for the other factor that we need. Once we finish synthetic division we get (x^2+9), Therefore, our factored function so far is $f(x) = (x+2)(x-1)(x^2+9)$. We do however need to include our complex numbers. We can do this by using (x^2+9).
$9 + x^2 = 0$
$x = -3\iota$
$x = 3\iota$
From this information, our entire factored function would be $f(x) = (x+2)(x-1)(x^2+9)(x+3\iota)(x-3\iota)$.

For 3.6, Complex zeros, I found that the first two problems I did we easy to solve and I could complete quickly. I felt the same for the beginning of question 37 on page 217, however it asked me to factor the complex numbers along the real numbers, and I was unsure on how to do that. I completely understood finding the coordinates that the real zeros would lie between (I did this using Cauchy's Bound), and I understood how to find the possible zeros of the function (Beginning with The Rational Roots Theorem, and then using Synthetic division to test if the numbers at questions were actually zeros). What confused me when it said to factor the real and complex numbers was that at first glance, I didn't that there were any complex numbers. I had always associated complex numbers with a negative square root (i.e. $\sqrt{-12}$) or with the imaginary number, ι. I wasn't aware that there were imaginary numbers because of one of the factors, $(9+x^2)$. However, now that we had gone over it in our meeting on Friday the 16th, I see how there is a complex number. Just like any other factor of the function, we set $x = 0$ when we want our zero of the function. For this one, x is squared, meaning that we would have to square root 9. When we do that, we have $(x^2 - 9)$, so when we then do $x + \sqrt{-9}$ we have a complex number at hand. $\sqrt{-9} = 3\iota$ and -3ι, meaning that our complex factors would be $(x - 3\iota)$ and $(x + 3\iota)$.

1.7 3.7 Rational Functions

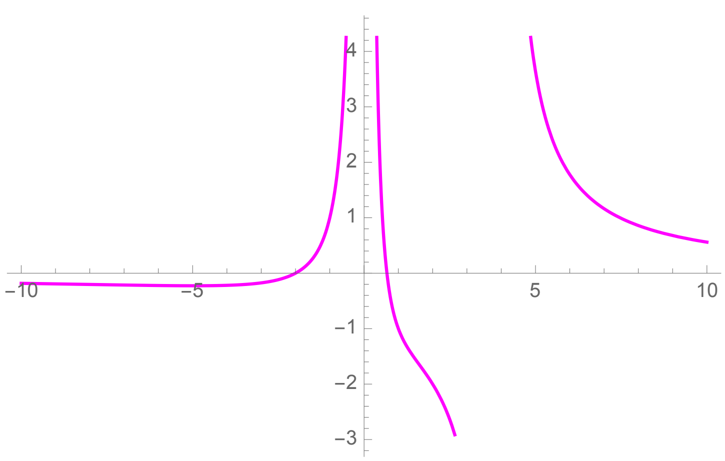
Question 15, Page 234
For each function, find the horizontal intercepts, the vertical intercept, the vertical asymptotes, and the horizontal asymptote. Use that information to sketch

a graph.
$p(x) = \frac{3x^2+4x-4}{x^3-4x^2}$

We can factor the function so that we can find the horizontal intercepts, vertical intercept and the vertical asymptotes. $p(x) = \frac{(3x-2)(x+2)}{(x^2)(x-4)}$
To begin, we will work on finding the horizontal intercepts of the function.
$p(x) = \frac{(3x-2)(x+2)}{(x^2)(x-4)}$
Here we have to set the numerator of the function to equal zero in order to find the horizontal intercepts of the function.
$3x - 2 = 0$ $x = \frac{2}{3}$
$x + 2 = 0$ $x = -2$
From this, we can see that we will have horizontal intercepts at both $(\frac{2}{3}, 0)$ and $(-2, 0)$.
Secondly, we can work on finding the vertical asymptotes by setting the the denominator equal to zero.
$x^2 = 0$ $x = 0$
$x - 4 = 0$ $x = 4$
We can see that we have vertical asymptotes at $(0, 0)$ and $(4, 0)$.

Now, we can solve for the vertical intercept by setting the all the x values equal to zero.
$p(0) = \frac{(3(0)-2)((0)+2)}{((0)^2((0)-4)}$
$p(0) = \frac{(3(0)-2)((0)+2)}{((0)^2((0)-4)} = 0$
We have a vertical intercept at $(0, 0)$

Now that we know what our vertical intercept, horizontal intercept, and vertical asymptote is, we can determine the horizontal asymptote by checking the degree of the denominator and numerator. When the degree of the denominator is larger than the degree of the numerator, we will have a horizontal asymptote at $y = 0$. Since that is the case for this problem, we now know all of our needed information in order to draw a graph of the function.
Below is the graph of the function $f(x) = x^4 + x^3 + 7x^2 + 9x - 18$.



Question 19,Page 235

Write an equation for a rational function with the given characteristics.

Vertical Asymptotes at $x = 5$ and $x = -5$, x intercept at $(2, 0)$ and $(1, 0)$, and y intercept at $(0, 4)$

To start, lets begin by putting what we know together into a factored version of the function at hand. $f(x) = \frac{(x-2)(x-1)}{(x-5)(x+5)}$. We are also aware that there is a vertical intercept at $(0, 4)$. There is an equation that allows the function to be found just from the information above, and it looks like $f(x) = a\frac{(x-x_1)^{p1}(x-x_2)^{p2}}{(x-v_1)^{q1}(x-v_2)^{q2}}$ We can insert our x intercepts into the x values, and our vertical asymptotes into the v values. $f(x) = a\frac{(x-2)(x-1)}{(x-5)(x+5)^2}$. a is equal to the stretch factor of the the function, and we can find it by using a clear point on the graph, for instance the y-intercept at $(0, 4)$. Once we substitute this into the equation we will be able to determine our final function.

$4 = a\frac{(x-2)(x-1)}{(x-5)(x+5)}$

$4 = a\frac{2}{25}$

$a = \frac{8(x-2)(x-1)}{100(x-5)(x+5)}$

Question 40, Page 237

Find the oblique asymptote of each function.

$f(x) = \frac{2x^2+3x-8}{x-1}$

To solve for this question, we need to first describe the long behavior of the function. As we can see, the long behavior is $f(x) \approx \frac{2x^2}{x} = 2x$ meaning that $x \to \pm\infty, f(x) \to \pm\infty$ as well, and that there is no horizontal asymptote. However, if we used long division we would get a better understanding of $x \to \pm\infty$.

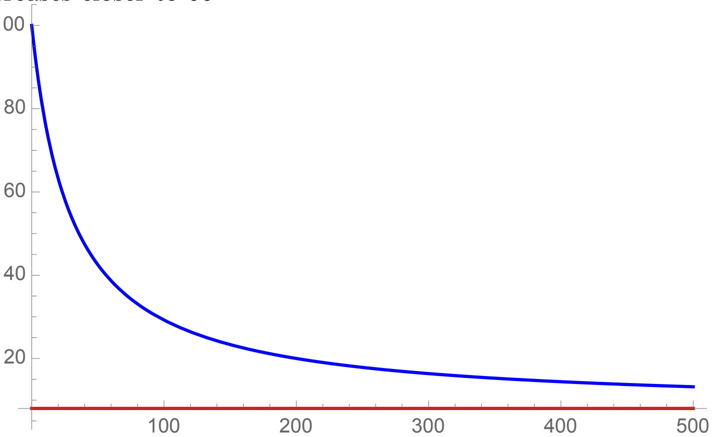
$x - 1$	$2x^2 + 3x - 8$
	$2x$
	$-(2x^2 + 2x)$
	$x - 8$
	1
	$-(x - 1)$
	9

This means that $f(x) = \frac{2x^2+3x-8}{x-1}$ can be written as $f(x) = 2x + 1 + \frac{9}{x-1}$.

As $x \to \pm\infty$, the term $\frac{9}{x-1}$ will become very small and approach zero, becoming insignificant. The remaining $2x + 1$ then describes the long-run behavior of the function: as $x \to \pm\infty$, $f(x) \to 2x + 1$.

$y = 2x + 1$ is the oblique asymptote of the function $f(x) = \frac{2x^2+3x-8}{x-1}$.

Question 46, Page 237
A scientist has a beaker containing 30 mL of a solution containing 3 grams of potassium hydroxide. To this, she mixes a solution containing 8 milligrams per mL of potassium hydroxide.
a) Write an equation for the concentration in the tank after adding n mL of the second solution.
Solution$= 3000 + 8n$
Potassium Hydroxide$= 30 + 1n$
$C(n) = \frac{3000+8n}{30+n}$
b) Find the concentration if 10 mL of the second solution has been added.
$C(10) = \frac{3000+(8(10)}{30+8(10)}$
$C(10) = \frac{3080}{110} = 28$
c) How many mL of water must be added to obtain a 50 mg/mL solution?
We have to find the value of n in orderfor the concentration to be 50.
From this, I used mathematica in order to solve for n, which looked like: Solve[Conc[n]==50,n]
From there, I got that $n = \frac{250}{7} = 35.7143$.
The concentration will be 50 $milligrams/mL$ when the scientists add 35.7 mL of the second solution.
d) What is the behavior as $n \to \infty$, and what is the physical significance of this?
Below is the graph of the equation, $C(n) = \frac{3000+8n}{30+n}$ in the long run, as n increases closer to ∞



$As we can see, as 8 milligram/mLwill eventually dilute the solution.$

For this section, I mostly understood how to solve for everything that was asked of me from the questions. The only question that left me confused was question 46 on page 237. I was confused as to how to solve for part c and d, or if my equation and calculations from part a and b were correct. Luckily, I emailed you and you were able to help walk me through on how to solve for the question. Everything else from this section (calculating asymptotes, and intercepts) I found rather easy to tackle. Besides from this last question, I believe that I have all the other concepts down. My only issue is that I typically check my work by looking through the solution manual, and if I am wrong I would rework my way through the problem with the extra information that the solution manual gave me. Besides that, I found this section one of the least frustrating to complete.

1.8 3.8 Inverse and Rational Functions

Question 1, Page 245
For each function, find a domain on which the function is one-to-one and non-decreasing, then find an inverse of the function on this domain.
$(x-4)^2$
To begin, we can find the inverse of the function by switching x and y and solving for y.
$x=(y-4)^2$
We can now solve for y. We can start by square rooting both sides.
$\sqrt{x}=\sqrt{(y-4)^2}=\sqrt{x}=y-4$
We now will now add 4 to both sides of the function.
$\sqrt{x}+4=y.$
Therefore, our inverse is $\sqrt{x}+4=y$.
$f^-1(x)=\sqrt{x}+4$

Question 7, Page 245
Find the inverse of each function.
$f(x)=9+\sqrt{4x-4}$
Just like the last problem, we will start by switching the x and y values.
$x=9+\sqrt{4y-4}$
Next, we can solve for y. To remove the radical, we will need to square both sides of the equation so that now the equation will look like, $x^2=81+4y+4=$ $x^2=9^2+4y-4$.
$9^2=81$, and we also have -4 We can add them together to have 78.
Subtract 78 from both sides.
$x^2-78=4y$. Divide both sides by 4.
$\frac{x^2-78}{4}=\frac{4y}{4}$. Simplify the equation.
We now have $\frac{x^2}{4}+19.5=y$
$f^-1(x)=\frac{x^2}{4}+19.5$

Question 17, Page 245
Police use the formula $v=\sqrt{20L}$ to estimate the speed of a car, v, in miles per hour, based on the length, L, in feet, of its skid marks when suddenly braking on a dry, asphalt road.
At the scene of an accident, a police officer measures a car's skid marks to be 215 feet long. Approximately how fast was the car traveling?

$x = \sqrt{20(215)}$=$\sqrt{4300}$= 65.57
The car was traveling at approximately 65.57 mph.

Question 21, Page 246
A drainage canal has a cross- section in the shape of a parabola. Suppose that the canal is 10 feet deep and 20 feet wide at the top. If the water depth in the ditch is 5 feet, how wide is the surface of the water in the ditch? $[UW]$.
The equation that we will be using is $y(x) = ax^2$.
$x = 10$
$y = 5$
$5 = a(10)^2$
$a = \frac{5}{100}$ $a = \frac{1}{20}$
Our parabolic cross section has equation $a = \frac{5}{100}$ $a = \frac{1}{20}x^2$.
To solve for y, depth, the width would be given at $2x$, meaning that we need to solve for x.
$y = \frac{1}{20}x^2$
$20y = x^2$
$x = \pm\sqrt{20y}$.
, even though this is true, we are limiting our answers to the positives, meaning that there will be no negative answer for this problem. The equation we will be using is $x(y) = \sqrt{20y}$
Since the width of the canal is 10 feet, the surface area will be $10(2x)$, or in terms of our y, $Area = 20x = 20\sqrt{20y}$.

For this section, I found it also easy to understand. I managed to fly through this section without any real difficulty besides being confused once or twice and having to go back and reread a section of the textbook. Besides that, this section was one of the easiest ones I believe I have done for this chapter.

Overall, this week has been a very stressful week. To start as you are aware, I was having difficulties with my computer and was unable to work on my project as much as I would have liked to. Secondly, my great aunt is about to die, and even though I personally was not that close to her, my mother was. And on top of all of this, I found out I have an exam next week for biology on Monday, and I knew I had my pyschology exam tuesday, and I also have to have a full lab report due Friday. This is not typical of me to talk about in my summary, however I do feel like it is important to include by saying that this was one of my most stressful math projects I have had to do. Like I mentioned, I was unable to charge my macbook for a little bit, and couldn't work on my project. On Friday the 16th, I worked on this project from 9am all the way until 4pm. I'm not sure if this will be some of my best work, but I do want you to know that I tried my hardest for this to look professional and well done, with logical reasoning for the questions I solved. Besides everything else, I do want to say that I do struggle with quadratics, and this chapter defiinitley gave me a run for my money.