Useful identities and formulas

$$\sin^2 x + \cos^2 x = 1, \quad \sin(2x) = 2\sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

$$\sin^2 x = \frac{1-\cos(2x)}{2}, \quad \cos^2 x = \frac{1+\cos(2x)}{2}$$

$$\sin x \sin y = \frac{\cos(x-y) - \cos(x+y)}{2}$$

$$\cos x \cos y = \frac{\cos(x-y) + \cos(x+y)}{2}$$

$$\sin x \cos y = \frac{\sin(x+y) + \sin(x-y)}{2}$$

$$e^{i\phi} = \cos\phi + i\sin\phi$$

$$|e^{i\phi}| = 1 \quad (\phi \text{ real})$$

$$|Z_1 Z_2| = |Z_1| \cdot |Z_2|, \quad \left|\frac{Z_1}{Z_2}\right| = \frac{|Z_1|}{|Z_2|}$$

$$|Z_1 + Z_2| \neq |Z_1| + |Z_2|, \quad |Z_1 \pm Z_2| \leq |Z_1| + |Z_2|$$

Integrals with trigs

Let $c_\pm = a \pm b$, then:

$$\int \sin(ax)\sin(bx)x\,dx = \frac{1}{2}\left\{x\frac{\sin(c_- x)}{c_-} - x\frac{\sin(c_+ x)}{c_+} + \frac{\cos(c_- x)}{c_-^2} - \frac{\cos(c_+ x)}{c_+^2}\right\}$$

$$\int \sin(ax)\cos(bx)x\,dx = \frac{1}{2}\left\{\frac{\sin(c_- x)}{c_-^2} + \frac{\sin(c_+ x)}{c_+^2} - x\frac{\cos(c_- x)}{c_-} - x\frac{\cos(c_+ x)}{c_+}\right\}$$

$$\int \cos(ax)\cos(bx)x\,dx = \frac{1}{2}\left\{x\frac{\sin(c_- x)}{c_-} + x\frac{\sin(c_+ x)}{c_+} + \frac{\cos(c_- x)}{c_-^2} + \frac{\cos(c_+ x)}{c_+^2}\right\}$$

Mathematical Formulas

Trigonometry:

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

Law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

Integrals:

$$\int x \sin(ax)\, dx = \frac{1}{a^2}\sin(ax) - \frac{x}{a}\cos(ax)$$

$$\int x \cos(ax)\, dx = \frac{1}{a^2}\cos(ax) + \frac{x}{a}\sin(ax)$$

Exponential integrals:

$$\int_0^\infty x^n e^{-x/a}\, dx = n!\, a^{n+1}$$

Gaussian integrals:

$$\int_0^\infty x^{2n} e^{-x^2/a^2}\, dx = \sqrt{\pi}\,\frac{(2n)!}{n!}\left(\frac{a}{2}\right)^{2n+1}$$

$$\int_0^\infty x^{2n+1} e^{-x^2/a^2}\, dx = \frac{n!}{2}a^{2n+2}$$

Integration by parts:

$$\int_a^b f\frac{dg}{dx}\, dx = -\int_a^b \frac{df}{dx} g\, dx + fg\Big|_a^b$$