

PHY 342 HW Ch.5

Do problem 5.2 (use results of 5.1), 5.16, 5.33, 5.1* & 5.14* [both bonus], plus the following.

q5.1

For a system of two spins, show that:

(a) $\vec{s}_1 \cdot \vec{s}_2 = s_{1z}s_{2z} + \frac{1}{2}(s_{1+}s_{2-} + s_{1-}s_{2+})$ where $s_{1\pm}$ and $s_{2\pm}$ are the raising and lowering spin operators for particle 1 and 2, respectively;

(b) the last two members of the triplet are eigenfunctions of $\hat{S}^2 = \hat{s}_1^2 + \hat{s}_2^2 + 2s_{1z}s_{2z} + s_{1+}s_{2-} + s_{1-}s_{2+}$ with the eigenvalue $2\hbar^2$ and, of S_z with the eigenvalues $0\hbar, -1\hbar$.

q5.2

Let the wave function of a system of two identical particles be

$$\psi_{\pm}(x_1, x_2) = A[\phi_m(x_1)\phi_n(x_2) \pm \phi_n(x_1)\phi_m(x_2)].$$

(a) Verify that $\psi_{\pm}(x_1, x_2) = \pm\psi_{\pm}(x_2, x_1)$.

(b) Given ϕ_n forming an orthonormal basis set, calculate the normalization constant A for $m \neq n$ and for $m = n$. The latter is possible only for symmetric wave functions (bosons), of course.

QM2 Concept test 3.1

Choose all of the following statements that are correct about bosons.

- (1) The spin of a boson is an integer.
- (2) The overall wavefunction of identical bosons can be anti-symmetric.
- (3) Two bosons cannot occupy the same quantum state.

A. 1 only B. 2 only C. 1 and 2 only D. 1 and 3 only
E. All of the above

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QM2 Concept test 3.2

Choose all of the following statements that are correct about the Pauli exclusion principle.

- (1) All identical spin-1/2 particles satisfy the Pauli exclusion principle.
- (2) An *up* quark (*u*) and a *down* quark (*d*) cannot occupy the same quantum state simultaneously.
- (3) Two electrons in an atom cannot occupy the same quantum state simultaneously.

A. 1 only B. 3 only C. 1 and 2 only D. 1 and 3 only
E. all of the above

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QM2 Concept test 3.3

Choose all of the following statements that are correct about the exchange operator \hat{P} .

- (1) $\hat{P}f(x_1, x_2) = f(x_2, x_1)$, where $f(x_1, x_2)$ is a function of x_1 and x_2 .
- (2) The eigenvalues of the operator are \hat{P} are ± 1 .
- (3) $[\hat{P}, \hat{H}] = 0$ if \hat{H} is the Hamiltonian operator of a system of identical particles.

A. 1 only B. 2 only C. 1 and 2 only D. 1 and 3 only
E. All of the above

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QM2 Concept Test 3.4

Choose all of the following statements that are true about the symmetrization requirement for identical particles:

- 1) We can find a complete set of functions that are simultaneous eigenstates of the exchange operator and the Hamiltonian operator.
- 2) All linear superpositions of the stationary states of a system of identical particles must follow the symmetrization requirement.
- 3) If a system of identical particles starts out in a symmetric or antisymmetric state, it will remain in that state at future times.

A. 1 only B. 1 and 2 only C. 1 and 3 only D. 2 and 3 only
E. All of the above.

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QM2 Concept test 3.5

Choose all of the following statements that are correct for a system of two-identical fermions or bosons. Ignore spin. \vec{r}_1 and \vec{r}_2 are real space coordinates.

- (1) The stationary state wavefunction for identical bosons should always be written in symmetric form $\psi(\vec{r}_1, \vec{r}_2) = \psi(\vec{r}_2, \vec{r}_1)$ and for identical fermions in anti-symmetric form $\psi(\vec{r}_1, \vec{r}_2) = -\psi(\vec{r}_2, \vec{r}_1)$
- (2) Any possible wavefunction for identical bosons should always be written in symmetric form $\psi(\vec{r}_1, \vec{r}_2) = \psi(\vec{r}_2, \vec{r}_1)$ or for identical fermions in anti-symmetric form $\psi(\vec{r}_1, \vec{r}_2) = -\psi(\vec{r}_2, \vec{r}_1)$
- (3) The potential energy of this system must be symmetric with respect to the exchange of \vec{r}_1 and \vec{r}_2 , i.e., $V(\vec{r}_1, \vec{r}_2) = V(\vec{r}_2, \vec{r}_1)$.

A. 1 only B. 3 only C. 1 and 2 only D. 1 and 3 only
E. All of the above

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QM2 Concept test 3.6

The wavefunction for two particles in a one dimensional infinite square well ($V(x) = 0$ for $0 \leq x \leq a$) is

$A \sin\left(\frac{\pi x_1}{a}\right) \sin\left(\frac{\pi x_2}{a}\right)$. Choose all of the following statements that are correct. Ignore spin. A is a normalization constant.

- (1) The two particles can be distinguishable.
- (2) The two particles can be identical bosons
- (3) The two particles can be identical fermions.

A. 1 only B. 2 only C. 1 and 2 only D. 1 and 3 only
E. All of the above

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QM2 Concept Test 3.7

The stationary states for a particle in an infinite square well ($V(x) = 0$ for $0 \leq x \leq a$) are $A_n \sin\left(\frac{n\pi x}{a}\right)$ where $n = 1, 2, 3, \dots$. Choose all of the following statements that are correct about a two particle system in an infinite square well of width a . Ignore spin. A is a normalization constant.

- (1) If the two particles are identical bosons, the first excited state of the system is $A[\sin\left(\frac{\pi x_1}{a}\right) \sin\left(\frac{\pi x_2}{a}\right) + \sin\left(\frac{2\pi x_1}{a}\right) \sin\left(\frac{2\pi x_2}{a}\right)]$
- (2) If the two particles are identical bosons, the first excited state of the system is $A \sin\left(\frac{\pi x_1}{a}\right) \sin\left(\frac{2\pi x_2}{a}\right)$
- (3) If the two particles are identical fermions, the first excited state of the system is $A[\sin\left(\frac{\pi x_1}{a}\right) \sin\left(\frac{2\pi x_2}{a}\right) - \sin\left(\frac{2\pi x_1}{a}\right) \sin\left(\frac{\pi x_2}{a}\right)]$

A. 1 only B. 2 only C. 3 only D. 1 and 3 only E. none of the above

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QM2 Concept test 3.8

Choose all of the following wavefunctions that are possible for two fermions in a one-dimensional infinite square well with width a . Ignore spin. A is a suitable normalization constant.

- (1) $A \sin\left(\frac{\pi x_1}{a}\right) \sin\left(\frac{\pi x_2}{a}\right)$
- (2) $A[\sin\left(\frac{\pi x_1}{a}\right) - \sin\left(\frac{\pi x_2}{a}\right)]$
- (3) $A[\sin\left(\frac{2\pi x_1}{a}\right) \sin\left(\frac{3\pi x_2}{a}\right) - \sin\left(\frac{3\pi x_1}{a}\right) \sin\left(\frac{2\pi x_2}{a}\right)]$

A. only 1 B. only 2 C. 3 only D. 1 and 3 only
E. 2 and 3 only

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QM2 Concept test 3.9

For one particle in a one dimensional infinite square well ($V(x) = 0$ for $0 \leq x \leq a$), the energy is $E_n = n^2 K$, where $K = \frac{\pi^2 \hbar^2}{2ma^2}$.

Choose all of the following statements that are correct for two non-interacting particles in the same potential energy well. Ignore spin and assume all particles have the same mass.

- (1) If the particles are distinguishable, the ground state energy is $2K$.
- (2) If the particles are identical bosons, the ground state energy is $2K$.
- (3) If the particles are identical fermions, the ground state energy is $2K$.

A. 1 only B. 2 only C. 1 and 2 only D. 1 and 3 only
E. All of the above

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QM2 Concept test 3.10

Suppose at time $t = 0$, $\psi_a(x)$ is the wavefunction for particle 1 in a potential energy well and $\psi_b(x)$ is the wavefunction for particle 2 in the same well. Particles 1 and 2 are non-interacting. Choose all of the following statements that are correct for this two-particle system. Ignore spin. A is a normalization constant.

- (1) $\Psi(x_1, x_2) = A[\psi_a(x_1) \psi_b(x_2) + \psi_b(x_1) \psi_a(x_2)]$ is a possible wavefunction for this system if particles 1 and 2 are identical bosons.
- (2) If particles 1 and 2 are identical fermions, the wavefunction must be anti-symmetric at all times.
- (3) $\Psi(x_1, x_2) = A\psi_a(x_1) \psi_b(x_2)$ can be a possible wavefunction for this system if particle 1 is a boson and particle 2 is a fermion.

A. 1 only B. 2 only C. 1 and 2 D. 1 and 3 E. all of the above

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