

PHY 342 HW Ch.6b, 7a

Do problems 6.16, 7.3 (refer to Example 7.3), 7.7, and bonus 6.5*, 6.19*, and 7.13*, plus the following.

q6.4

The virial theorem for the Coulomb potential is $\langle T \rangle = -\frac{1}{2}\langle V \rangle$. Use this result to show that

$$\langle \psi_{nlm} | \frac{1}{r} | \psi_{nlm} \rangle = \frac{1}{n^2 a}.$$

q6.5

Consider the strong Zeeman splitting of $n = 2$ levels of hydrogen. Find the energy of each split level in terms of $\mu_B B_0$ and sketch them. How many distinct levels are there?

q7.1

Consider the linear potential $V = \alpha|x|$. Use a Gaussian $\psi = \exp(-\beta x^2)$ as the trial wave function, and calculate the ground state energy with the variational principle. Determine the parameter β which minimizes the energy, and find E_{min} . Express $E_{min} = f \times (\hbar^2 \alpha^2 / 2m)^{1/3}$, and give the numerical value of the factor f . This is the upper bound of the true ground state energy, E_{gs} .

Compare E_{min} with the exact result, $E_{gs} = 1.019 (\hbar^2 \alpha^2 / 2m)^{1/3}$, and find the relative error between them. Comment on the result.