$$\langle p\rangle = \int \psi^* \left(-i\hbar \frac{\partial}{\partial x}\right)\psi \, dx \qquad \left(\frac{\partial}{\partial x}\right)\psi f \equiv \frac{\partial(\psi f)}{\partial x}$$

$$\frac{d\langle p\rangle}{dt} = \frac{d}{dt} \qquad = -i\hbar \int \frac{\partial}{\partial t}\left(\psi^* \frac{\partial}{\partial x}\psi\right) dx \qquad \psi' = \frac{\partial \psi}{\partial x}$$

$$= -i\hbar \int \left[\frac{\partial \psi^*}{\partial t}\psi' + \psi^* \frac{\partial \psi'}{\partial t}\right] dx$$

$$= -i\hbar \int \left[\frac{\partial \psi^*}{\partial t}\psi' + \psi^* \frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial t}\right)\right] dx \qquad \frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial x}\right) = \frac{\partial^2 \psi}{\partial t \partial x} = \frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial t}\right) = \left(\frac{\partial \psi}{\partial t}\right)'$$

$$\frac{\partial \psi}{\partial t} = \frac{\gamma}{i\hbar}\psi'' + \frac{V}{i\hbar}\psi \qquad \gamma = -\frac{\hbar^2}{2m}$$

$$\psi' \frac{\partial \psi^*}{\partial t} = \left[\frac{\gamma}{-i\hbar}\psi^{*\prime\prime} - \frac{V}{i\hbar}\psi^*\right]\psi'$$

$$= -i\hbar\left[\frac{1}{-i\hbar}\left(\gamma\psi^{*''}\psi' + V\psi^{*}\psi'\right)\right.$$

$$\left. + \frac{1}{-i\hbar}\frac{\partial}{\partial x}\left(\gamma\psi'' + V\psi\right)\cdot\psi^{*}\right]$$

$$\langle f\rangle = \int \psi^{*} f \psi \, dx$$

$$\left\langle -\frac{\partial V}{\partial x}\right\rangle = \int \psi^{*}\left(-\frac{\partial V}{\partial x}\right)\psi \, dx$$

$$= \gamma\psi^{*''}\psi' + \underline{V\psi^{*}\psi'} - \psi^{*}\left[\gamma\psi''' + V'\psi + \underline{V\psi'}\right]$$

$$-\gamma\psi^{*}\psi''' - \psi^{*}V'\psi$$

$$-\psi^{*}V\psi'$$

$$= \gamma\left[\psi^{*''}\psi' - \psi^{*}\psi'''\right] - \psi^{*}V'\psi$$

$$\frac{d\langle p\rangle}{dt} = \gamma\underbrace{\int\left[\psi^{*''}\psi' - \psi^{*}\psi'''\right]dx}_{I} - \int\psi^{*}V'\psi\,dx$$

$$+\left\langle -\frac{\partial V}{\partial x}\right\rangle$$

$$I = \int [\psi^{*''}\psi' - \psi^{*}\psi'''] dx \qquad [\psi^{*'}\psi' - \psi^{*}\psi'']'$$

$$\psi^{*''}\psi' + \psi^{*'}\psi'' - \psi^{*'}\psi'' - \psi^{*}\psi'''$$

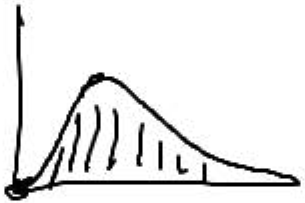
$$I = \int_{-\infty}^{\infty} \frac{d}{\cancel{dx}} [\psi^{*'}\psi' - \psi^{*}\psi''] \cancel{dx} \qquad \psi(\pm\infty)$$

$$[\qquad\qquad] \Big|_{-\infty}^{\infty} = 0 \qquad = \psi'(\pm\infty) = \psi'(\pm\infty)$$

$$\frac{d\langle P\rangle}{dt} = \left\langle -\frac{\partial V}{\partial x} \right\rangle \qquad F = \frac{dp}{dt} = ma = m\frac{dv}{dt} = 0$$

2nd Law.

$\Psi = A e^{-\lambda|x|} e^{i\omega t}$ $\quad |\Psi|^2 = A^2 e^{-2\lambda|x|}$



$\langle x \rangle = \int |\Psi|^2 x\,dx = A^2 \int_{-\infty}^{\infty} x \underbrace{e^{-2\lambda|x|}}_{\text{odd}} dx = 0$

$\langle x^2 \rangle = A^2 \int_{-\infty}^{\infty} \underbrace{x^2 e^{-2\lambda|x|}}_{\text{even}} dx = A^2 \int_{-\infty}^{0} x^2 e^{2\lambda x} dx + A^2 \int_{0}^{\infty} x^2 e^{-2\lambda x} dx$

$x \to -y$

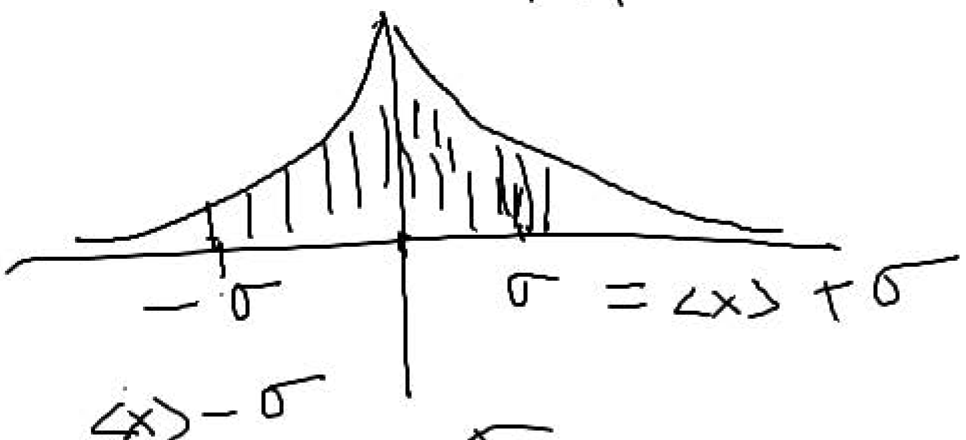
$A^2 \int_{\infty}^{0} y^2 e^{-2\lambda y} (-dy) = A^2 \int_{0}^{\infty} y^2 e^{-2\lambda y} dy$

$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$

$= 2A^2 \int_0^\infty x^2 e^{-2\lambda x} dx = 2 \cdot A^2 \frac{2!}{(2\lambda)^3} = \boxed{\frac{1}{2\lambda^2}}$

$\frac{1}{4\lambda^2} - \frac{1}{4\lambda^2}$

$$\sqrt{\langle x^2\rangle - \langle x\rangle^2} = \sigma$$



$$\text{Outside} = 1 - \int_{-\sigma}^{\sigma} |\psi|^2\,dx$$

$$|\psi|^2 = A^2 e^{-2\lambda|x|}$$

$$A^2\int_0^\sigma e^{-2\lambda x}\,dx = A^2 \cdot \frac{e^{-2\lambda x}}{-2\lambda}\Big|_0^\sigma \qquad = 1 - 2\int_0^\sigma |\psi|^2\,dx$$

$$= A^2\,\frac{1-e^{-2\lambda\sigma}}{2\lambda} = 0.7$$