

Useful identities and formulas

$$\sin^2 x + \cos^2 x = 1, \quad \sin(2x) = 2 \sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x = 2 \cos^2 x - 1$$

$$\sin^2 x = \frac{1 - \cos(2x)}{2}, \quad \cos^2 x = \frac{1 + \cos(2x)}{2}$$

$$\sin x \sin y = \frac{\cos(x - y) - \cos(x + y)}{2}$$

$$\cos x \cos y = \frac{\cos(x - y) + \cos(x + y)}{2}$$

$$\sin x \cos y = \frac{\sin(x + y) + \sin(x - y)}{2}$$

$$e^{i\phi} = \cos \phi + i \sin \phi, \quad |e^{i\phi}| = 1 \quad (\phi \text{ real})$$

$$|Z_1 Z_2| = |Z_1| \cdot |Z_2|, \quad \left| \frac{Z_1}{Z_2} \right| = \frac{|Z_1|}{|Z_2|}$$

$$|Z_1 + Z_2| \neq |Z_1| + |Z_2|, \quad |Z_1 \pm Z_2| \leq |Z_1| + |Z_2|$$

Let $\psi' \equiv \frac{d\psi}{dx}$, $\psi'' \equiv \frac{d^2\psi}{dx^2}$, then:

$$\frac{\partial(\psi^*\psi)}{\partial t} = \frac{i\hbar}{2m}(\psi''\psi^* - \psi^{*\prime\prime}\psi)$$

$$\psi''\psi^* - \psi^{*\prime\prime}\psi = [\psi'\psi^* - \psi^{*\prime}\psi]' \equiv \frac{d}{dx}[\psi'\psi^* - \psi^{*\prime}\psi]$$

$$\int [\psi''\psi^* - \psi^{*\prime\prime}\psi] dx = \int d[\psi'\psi^* - \psi^{*\prime}\psi] = [\psi'\psi^* - \psi^{*\prime}\psi] + C$$

$$\int f(x)[\psi''\psi^* - \psi^{*\prime\prime}\psi] dx = \int f(x)d[\psi'\psi^* - \psi^{*\prime}\psi] = f(x)[\psi'\psi^* - \psi^{*\prime}\psi] - \int [\psi'\psi^* - \psi^{*\prime}\psi] f' dx$$

Integrals with trigs

Let $c_{\pm} = a \pm b$, then:

$$\int \sin(ax) \sin(bx) x dx = \frac{1}{2} \left\{ x \frac{\sin(c_- x)}{c_-} - x \frac{\sin(c_+ x)}{c_+} + \frac{\cos(c_- x)}{c_-^2} - \frac{\cos(c_+ x)}{c_+^2} \right\}$$

$$\int \sin(ax) \cos(bx) x dx = \frac{1}{2} \left\{ \frac{\sin(c_- x)}{c_-^2} + \frac{\sin(c_+ x)}{c_+^2} - x \frac{\cos(c_- x)}{c_-} - x \frac{\cos(c_+ x)}{c_+} \right\}$$

$$\int \cos(ax) \cos(bx) x dx = \frac{1}{2} \left\{ x \frac{\sin(c_- x)}{c_-} + x \frac{\sin(c_+ x)}{c_+} + \frac{\cos(c_- x)}{c_-^2} + \frac{\cos(c_+ x)}{c_+^2} \right\}$$

Mathematical Formulas

Trigonometry:

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

Law of cosines:

$$c^2 = a^2 + b^2 - 2ab \cos \theta$$

Integrals:

$$\int x \sin(ax) dx = \frac{1}{a^2} \sin(ax) - \frac{x}{a} \cos(ax)$$

$$\int x \cos(ax) dx = \frac{1}{a^2} \cos(ax) + \frac{x}{a} \sin(ax)$$

Exponential integrals:

$$\int_0^{\infty} x^n e^{-x/a} dx = n! a^{n+1}$$

Gaussian integrals:

$$\int_0^{\infty} x^{2n} e^{-x^2/a^2} dx = \sqrt{\pi} \frac{(2n)!}{n!} \left(\frac{a}{2}\right)^{2n+1}$$

$$\int_0^{\infty} x^{2n+1} e^{-x^2/a^2} dx = \frac{n!}{2} a^{2n+2}$$

Integration by parts:

$$\int_a^b f \frac{dg}{dx} dx = - \int_a^b \frac{df}{dx} g dx + fg \Big|_a^b$$