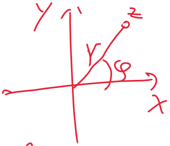
Complex numbers: $z = x + iy$

$= \underset{\text{mag.}}{r} e^{i\varphi}$ φ = phase



$r = \sqrt{x^2+y^2}$

$\tan\varphi = \frac{y}{x}$

$\varphi = \arctan\left(\frac{y}{x}\right)$

$r = |z| = r|e^{i\varphi}| = r$

$|e^{i\varphi}| = 1$ $e^{i\varphi} = \underbrace{\cos\varphi}_{x} + i\underbrace{\sin\varphi}_{y}$

$|e^{i\varphi}| = \sqrt{\cos^2\varphi + \sin^2\varphi} = 1$

$z^* = x - iy$

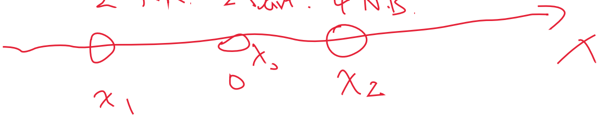
$= r e^{-i\varphi}$

$|z|^2 = z^* z = r e^{-i\varphi} \cdot r e^{i\varphi}$

$= r^2 \underbrace{e^{i\varphi} \cdot e^{-i\varphi}}_{e^0 = 1}$

$|e^{i\varphi}|^2 = \underline{e^{i\varphi} \cdot e^{-i\varphi}} = 1$

2 F.R. 2 Dart. 4 N.B.



$$\bar{x} = \frac{\sum \text{(persons' position)}}{\sum \text{persons}} = \frac{2 * x_1 + 2 * x_0 + 4 x_2}{2 + 2 + 4}$$

$$= \frac{1}{4} x_1 + \frac{1}{4} x_0 + \frac{1}{2} x_2$$

$$p_1 + p_0 + p_2 = 1$$

$$\bar{x} = \sum_{j=0,1,2} p_j x_j \text{ , Weighted avg.}$$

$d_1 = 12$ mi $\quad d_2 = 4$ mi

$d_0 = 0$ mi

$$\bar{d} = \sum_{j=0,1,2} p_j d_j = \frac{1}{4} \cdot 0 + \frac{1}{4} \times 12 + \frac{1}{2} \cdot 4$$

$$= 5 \text{ mi}$$

$p_0 = p_1 = 0$

$p_2 = 1$

$\bar{d} = d_2$

$\overline{d^2} = d_2^2$

$$\overline{d^2} = \sum_{j=0,1,2} p_j d_j^2 = \frac{1}{4} \cdot 0^2 + \frac{1}{4} \times 12^2 + \frac{1}{2} \times 4^2$$

$$0 \quad \frac{144}{4} \quad + 8$$

$$36$$

$\overline{d^2} = (\bar{d})^2$? when?

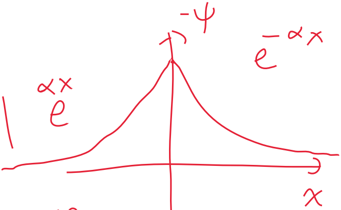
$$= 44 \text{ mi}^2$$

$$\overline{d^2} = 44 \text{ mi}^2 \neq (\bar{d})^2 = 25 \text{ mi}^2$$

$$\sigma = \sqrt{44 - 25} = \sqrt{19} = 4.5 \text{ mi}$$

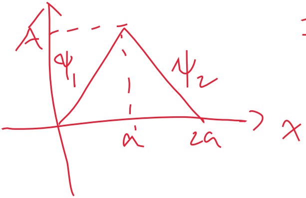
$\exp(x) = e^{x}$

$\psi = A e^{-\alpha|x|}$ $\quad e^{\alpha x}$ $\quad e^{-\alpha x}$



$$\int |\psi|^2 dx = A^2 \int_{-\infty}^{\infty} e^{-2\alpha|x|} dx$$

$$= A^2 \cdot 2 \int_0^{\infty} e^{-2\alpha x} dx$$



$0 < x < a \qquad \frac{Ax}{a} = \psi_1$

$a < x < 2a : \quad \frac{(2a - x)}{a} A = \psi_2$

$$\int_0^{2a} |\psi|^2 dx = \int_0^{a} |\psi_1|^2 dx + \int_a^{2a} |\psi_2|^2 dx$$

Normalization is conserved! $\int_{-\infty}^{\infty} |\psi|^2 dx = 1$

Show $\frac{d}{dt}\left[\int |\psi|^2 dx\right] = 0$

$$I = \frac{d}{dt} \int \psi^*_{(x,t)} \psi_{(x,t)} dx = \int \frac{\partial(\psi^* \psi)}{\partial t} dx$$

$$\frac{\partial(\psi^* \psi)}{\partial t} = \frac{\partial \psi^*}{\partial t} \psi + \psi^* \frac{\partial \psi}{\partial t}$$

$$i\hbar \frac{\partial\psi}{\partial t} = \overset{\gamma}{\left(-\frac{\hbar^2}{2m}\right)} \frac{\partial^2\psi}{\partial x^2} + V\psi$$

$$\frac{\partial\psi}{\partial t} = \underbrace{\frac{\gamma}{i\hbar}\frac{\partial^2\psi}{\partial x^2} + \frac{V\psi}{i\hbar}}$$

$$\psi'' = \frac{\partial^2\psi}{\partial x^2}$$

$$\psi' = \frac{\partial\psi}{\partial x}$$

$$-i\hbar \frac{\partial\psi^*}{\partial t} = \gamma\psi^{*''} + V^*\psi^*$$

$$\frac{\partial\psi^*}{\partial t} = -\frac{\gamma}{i\hbar}\psi^{*''} - \frac{1}{i\hbar}V\psi^*$$

$V =$ real

$V^* = V$

$$\frac{\partial(\psi^*\psi)}{\partial t} = \psi[\quad] + \psi^*[\quad]$$

$$= \psi\left(-\frac{\gamma}{i\hbar}\right)\psi^{*''} - \frac{V\psi\psi^*}{i\hbar} + \psi^*\left(\frac{\gamma}{i\hbar}\right)\psi'' + \frac{V}{i\hbar}\psi^*\psi$$

$$= \frac{\gamma}{i\hbar}\left[\psi^*\psi'' - \psi\psi^{*''}\right]$$

$$= -\frac{\cancel{\hbar^2}}{2m\, i\cancel{\hbar}}[\quad] = \frac{i\hbar}{2m}\left[\psi^*\psi'' - \psi\psi^{*''}\right]$$

$$= \frac{\partial(\psi^*\psi)}{\partial t}$$

$$\frac{d}{dx}\left[\psi^*\psi' - \psi\psi^{*'}\right] = \underline{\psi^{*'}\psi'} + \psi^*\psi'' - \underline{\psi'\psi^{*'}} - \psi\psi^{*''}$$

$$\int_{-\infty}^{\infty} \frac{\partial(\psi^*\psi)}{\partial t}\,dx = \frac{i\hbar}{2m}\int_{-\infty}^{\infty} \frac{d}{\cancel{dx}}\left[\psi^*\psi' - \psi\psi^{*\prime}\right]\cancel{dx}$$

$$= \frac{i\hbar}{2m}\int_{-\infty}^{\infty} d\left[\psi^*\psi' - \psi\psi^{*\prime}\right]$$

$$= \frac{i\hbar}{2m}\left[\psi^*\psi' - \psi\psi^{*\prime}\right]\Big|_{-\infty}^{\infty} = 0$$

Normalizable w.F. must vanish at $\pm\infty$

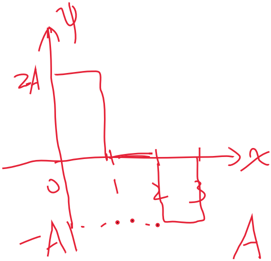
$$\psi(x \to \pm\infty, t) \Rightarrow 0.$$

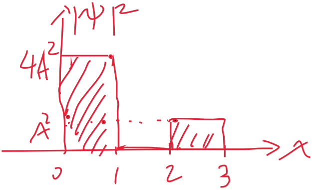
$$\psi^*(\pm\infty) = \psi(\pm\infty) = \psi'(\pm\infty) = \psi^{*\prime}(\pm\infty) = 0$$

Well-behaved w.F.

$$\frac{d}{dt}\left[\int_{-\infty}^{\infty} \psi^*\cdot\psi\,dx\right] = 0$$

$$\psi(x) = \begin{cases} 2A & 0\ x < 1 \\ -A & 2 < x\ 3 \\ 0 & \text{elsewhere} \end{cases}$$





$A = ?$ making W.F. normalized

(A) $\frac{1}{5}$

(B) 1

(C) $\frac{1}{3}$

(D) $\frac{1}{\sqrt{5}}$

$$\int_{-\infty}^{\infty} |\psi|^2 dx = \int_0^1 4A^2 dx + \int_2^3 A^2 dx$$

$$= 5A^2 = 1$$

$$A = \frac{1}{\sqrt{5}}$$