

## PHY 341 HW Ch.3a

Do problems 3.4; plus the following (\* = optional bonus):

### q3-1

For each of the following wave functions, sketch it and determine whether it is a Hilbert space wave function ( $a > 0$ ,  $|x| < \infty$ , and  $x > 0$  when square roots are involved.):

(a)  $A \exp(-ax)$ ; (b)  $A \sin(kx)$ ; (c)  $A \exp(-ax^2)$ ; (d)  $A/\sqrt{x}$ ; (e)  $A \exp(-a|x|^2)/x^{1/4}$ .

### q3-2

Which of the following operators are Hermitian, and why?

$d^2/dx^2$ ,  $ix$ ,  $a_-a_+$ ,  $a_+a_-$ ,  $i\partial/\partial t$ .

### q3-3

Let  $\psi_n$  be the stationary states of a particle in the box. Determine if  $\psi_n$  are eigenfunctions of the Hamiltonian  $H$ , momentum  $p$ , position  $x$ , and momentum squared  $p^2$ .

### q3-4

Calculate the following inner product:

(a)  $\langle a|b \rangle$  and  $\langle b|a \rangle$ , where  $a = [1, 3, 2i, -2]^T$ ,  $b = [-i, -1, i, 1]^T$ .

(b)  $\langle f|g \rangle$  where  $f = e^{ikx-x} \sin 2x$ ,  $g = e^{ikx-x}$ ,  $0 \leq x < \infty$ , and  $k$  is real.

**q3-5** Earlier we proved  $[f(x), p] = i\hbar f'$  in **q2-4**. Now prove this explicitly in momentum space. Hint: First show that  $(p\Phi)^{(n)} = p\Phi^{(n)} + n\Phi^{(n-1)}$ , then expand  $f(x)$  as a Taylor series, and calculate the commutator of the  $n$ -th term with  $p$ . Here  $\Phi = \Phi(p)$  is a test wave function in momentum space, and  $\Phi^{(n)} \equiv \frac{\partial^n \Phi}{\partial p^n}$ .