QM2 Concept test 3.1

Choose all of the following statements that are correct about bosons.

(1) The spin of a boson is an integer.

(2) The overall wavefunction of identical bosons can be anti-symmetric.

(3) Two bosons cannot occupy the same quantum state.

A. 1 only B. 2 only C. 1 and 2 only D. 1 and 3 only
E. All of the above

1

QM2 Concept test 3.2

Choose all of the following statements that are correct about the Pauli exclusion principle.

(1) All identical spin-1/2 particles satisfy the Pauli exclusion principle.

(2) An up quark (u) and a $down$ quark (d) cannot occupy the same quantum state simultaneously.

(3) Two electrons in an atom cannot occupy the same quantum state simultaneously.

A. 1 only B. 3 only C. 1 and 2 only D. 1 and 3 only
E. all of the above

2

QM2 Concept test 3.3

Choose all of the following statements that are correct about the exchange operator $\hat{P}$.

(1) $\hat{P}f(x_1, x_2) = f(x_2, x_1)$, where $f(x_1, x_2)$ is a function of x_1 and x_2.

(2) The eigenvalues of the operator are $\hat{P}$ are ± 1.

(3) $[\hat{P}, \hat{H}] = 0$ if $\hat{H}$ is the Hamiltonian operator of a system of identical particles.

A. 1 only B. 2 only C. 1 and 2 only D. 1 and 3 only
E. All of the above

3

QM2 Concept Test 3.4

Choose all of the following statements that are true about the symmetrization requirement for identical particles:

1) We can find a complete set of functions that are simultaneous eigenstates of the exchange operator and the Hamiltonian operator.
2) All linear superpositions of the stationary states of a system of identical particles must follow the symmetrization requirement.
3) If a system of identical particles starts out in a symmetric or antisymmetric state, it will remain in that state at future times.

A. 1 only B. 1 and 2 only C. 1 and 3 only D. 2 and 3 only
E. All of the above.

4

QM2 Concept test 3.5

Choose all of the following statements that are correct for a system of two-identical fermions or bosons. Ignore spin. $\vec{r}_1$ and $\vec{r}_2$ are real space coordinates.

(1) The stationary state wavefunction for identical bosons should always be written in symmetric form $\psi(\vec{r}_1, \vec{r}_2) = \psi(\vec{r}_2, \vec{r}_1)$ and for identical fermions in anti-symmetric form $\psi(\vec{r}_1, \vec{r}_2) = -\psi(\vec{r}_2, \vec{r}_1)$

(2) Any possible wavefunction for identical bosons should always be written in symmetric form $\psi(\vec{r}_1, \vec{r}_2) = \psi(\vec{r}_2, \vec{r}_1)$ or for identical fermions in anti-symmetric form $\psi(\vec{r}_1, \vec{r}_2) = -\psi(\vec{r}_2, \vec{r}_1)$

(3) The potential energy of this system must be symmetric with respect to the exchange of $\vec{r}_1$ and $\vec{r}_2$, i.e., $V(\vec{r}_1, \vec{r}_2) = V(\vec{r}_2, \vec{r}_1)$.

A. 1 only B. 3 only C. 1 and 2 only D. 1 and 3 only
E. All of the above

5

QM2 Concept test 3.6

The wavefunction for two particles in a one dimensional infinite square well ($V(x) = 0$ for $0 \le x \le a$) is $A \sin\left(\frac{\pi x_1}{a}\right) \sin\left(\frac{\pi x_2}{a}\right)$.Choose all of the following statements that are correct. Ignore spin. A is a normalization constant.

(1) The two particles can be distinguishable.
(2) The two particles can be identical bosons
(3) The two particles can be identical fermions.

A. 1 only B. 2 only C. 1 and 2 only D. 1 and 3 only
E. All of the above

6

QM2 Concept Test 3.7

The stationary states for a particle in an infinite square well ($V(x) = 0$ for $0 \le x \le a$) are $A_n \sin\left(\frac{n\pi x}{a}\right)$ where $n = 1,2,3, \dots$. Choose all of the following statements that are correct about a two particle system in an infinite square well of width a. Ignore spin. A is a normalization constant.

(1) If the two particles are identical bosons, the <u>first excited state</u> of the system is $A[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right) + \sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right)]$

(2) If the two particles are identical bosons, the <u>first excited state</u> of the system is $A\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right)$

(3) If the two particles are identical <u>fermions</u>, the <u>first excited state</u> of the system is $A[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right) - \sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)]$

A. 1 only B. 2 only C. 3 only D. 1 and 3 only E. none of the above

7

QM2 Concept test 3.8

Choose all of the following wavefunctions that are possible for <u>two fermions</u> in a one-dimensional infinite square well with width a. Ignore spin. A is a suitable normalization constant.

(1) $A\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)$

(2) $A[\sin\left(\frac{\pi x_1}{a}\right) - \sin\left(\frac{\pi x_2}{a}\right)]$

(3) $A[\sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{3\pi x_2}{a}\right) - \sin\left(\frac{3\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right)]$

A. only 1 B. only 2 C. 3 only D. 1 and 3 only
E. 2 and 3 only

8

QM2 Concept test 3.9

For one particle in a one dimensional infinite square well ($V(x) = 0$ for $0 \leq x \leq a$) , the energy is $E_n = n^2 K$, where $K = \frac{\pi^2 \hbar^2}{2ma^2}$.
Choose all of the following statements that are correct for two non-interacting particles in the same potential energy well. Ignore spin and assume all particles have the same mass.
(1) If the particles are distinguishable, the ground state energy is $2K$.
(2) If the particles are identical bosons, the ground state energy is $2K$.
(3) If the particles are identical fermions, the ground state energy is $2K$.

A. 1 only B. 2 only C. 1 and 2 only D. 1 and 3 only
E. All of the above

9

QM2 Concept test 3.10

Suppose at time $t = 0$, $\psi_a(x)$ is the wavefunction for particle 1 in a potential energy well and $\psi_b(x)$ is the wavefunction for particle 2 in the same well. Particles 1 and 2 are non-interacting. Choose all of the following statements that are correct for this two-particle system. Ignore spin. A is a normalization constant.
(1) $\Psi(x_1, x_2) = A[\psi_a(x_1)\,\psi_b(x_2) + \psi_b(x_1)\,\psi_a(x_2)]$ is a possible wavefunction for this system if particles 1 and 2 are identical bosons.
(2) If particles 1 and 2 are identical fermions, the wavefunction must be anti-symmetric at all times.
(3) $\Psi(x_1, x_2) = A\psi_a(x_1)\,\psi_b(x_2)$ can be a possible wavefunction for this system if particle 1 is a boson and particle 2 is a fermion.

A. 1 only B. 2 only C. 1 and 2 D. 1 and 3 E. all of the above

10