EFFECTIVE STRESS CONCEPT

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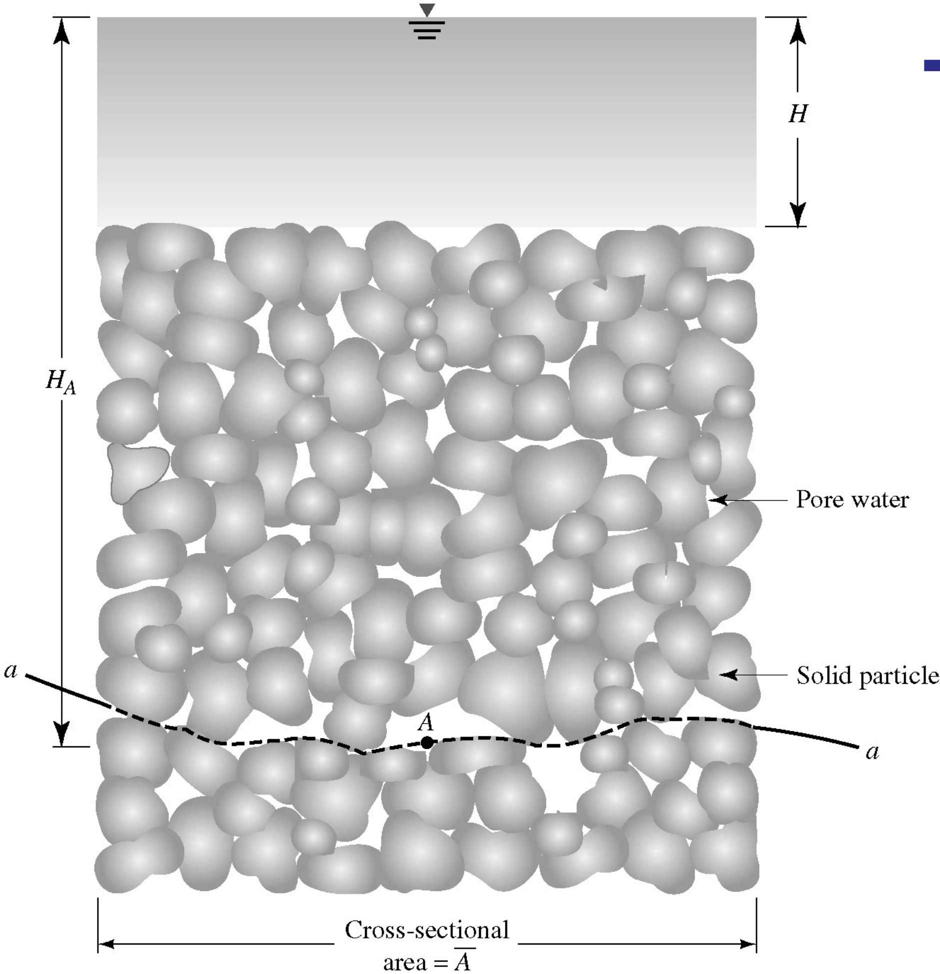

Saturated Soil Column (**Figure 6.1.** Das FGE (2005))

Total Stress (σ) at Point A

$$\sigma = H\gamma_w + (H_A - H)\gamma_{sat}$$

$H\gamma_w$: from Water

$(H_A - H)\gamma_{sat}$: from Soil

Where:

γ_w = Unit Weight of Water

γ_{sat} = Saturated Unit Weight of Soil

H = Height of water above Soil

H_A = Depth of Point A below water table

EFFECTIVE STRESS CONCEPT

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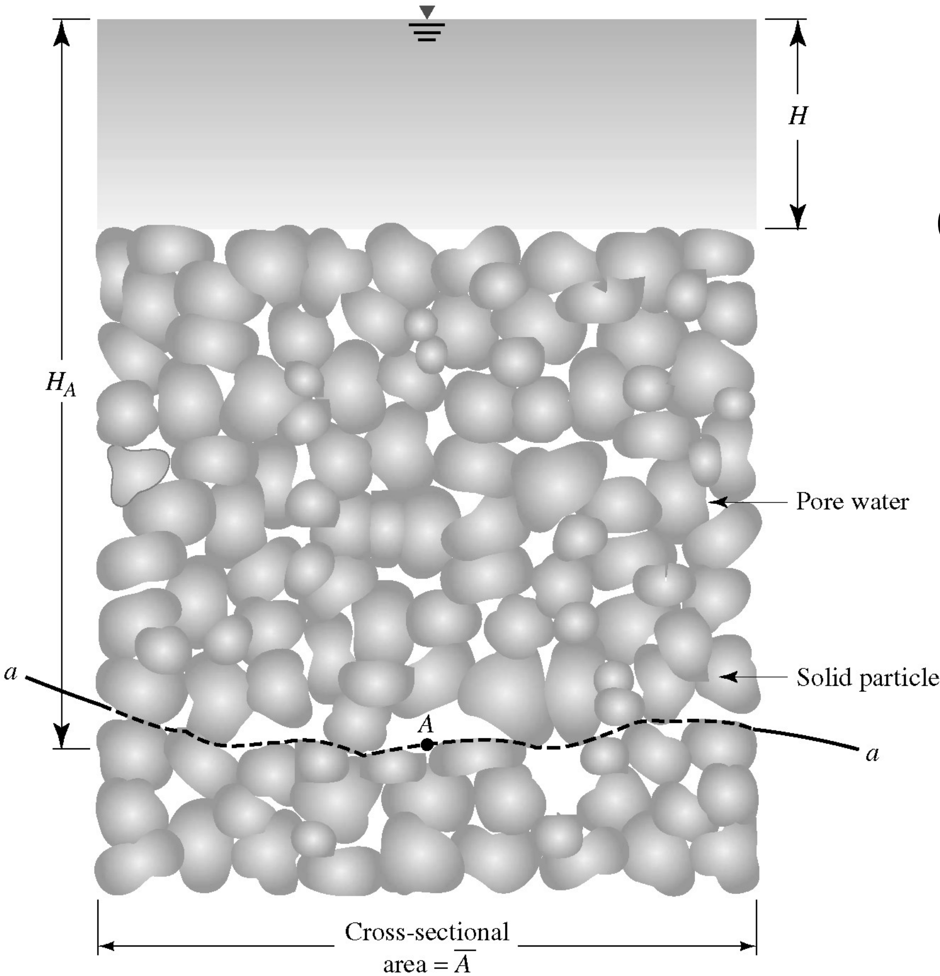


Saturated Soil Column (**Figure 6.1.** Das FGE (2005))

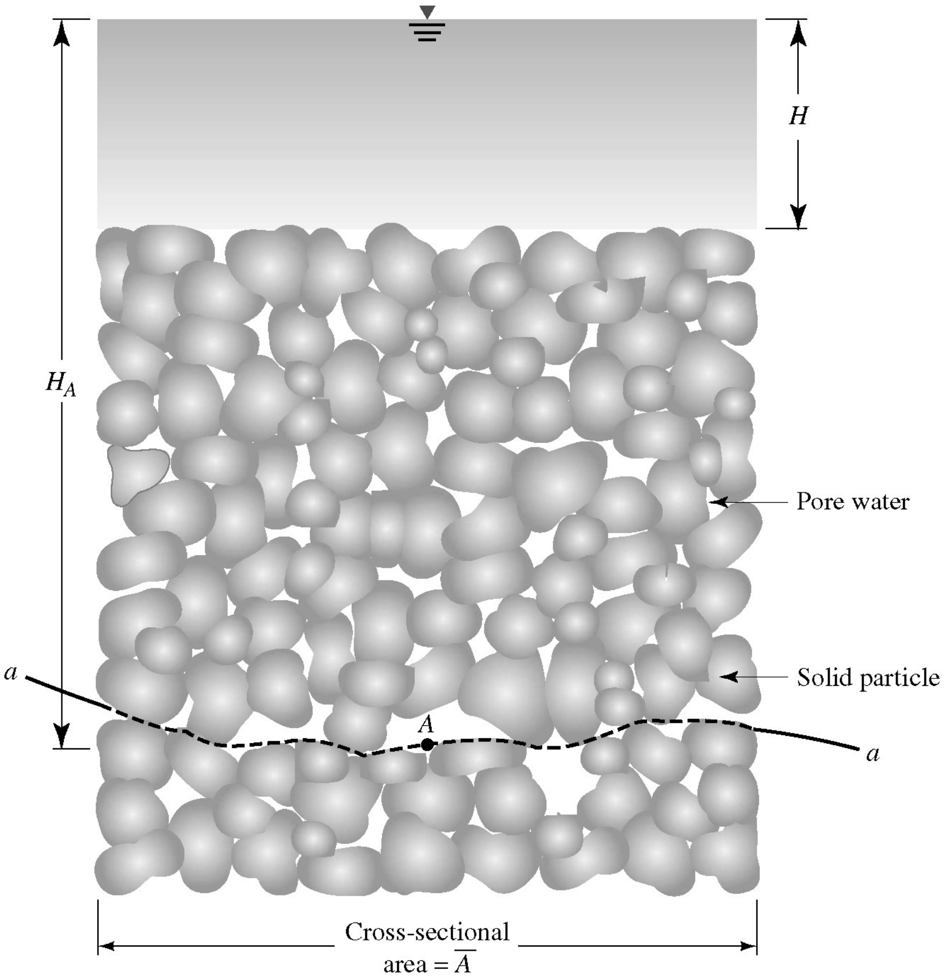
Total Stress (σ)

can be divided into 2 Parts:

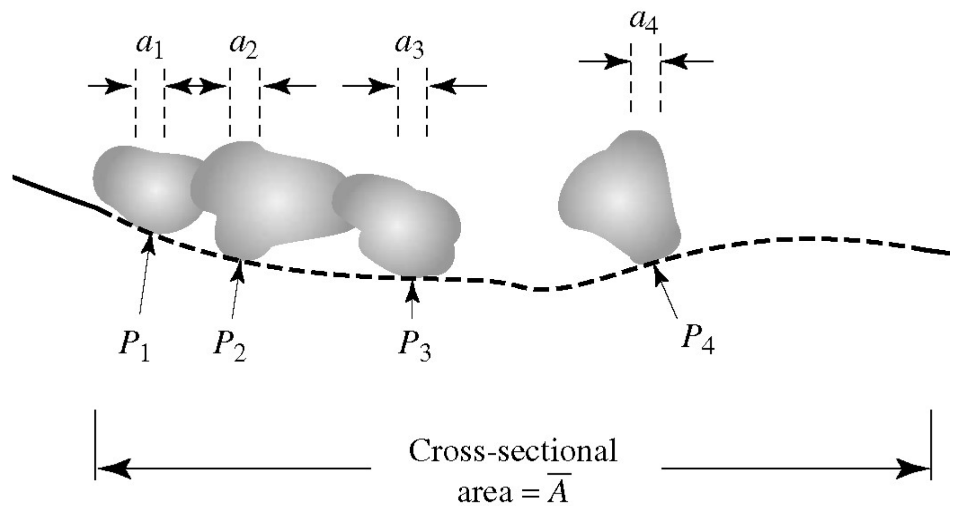
1. Portion carried by water in void spaces. **THIS IS THE PORE PRESSURE (*u*).**
2. Portion carried by soil solids at points of contact. **THIS IS THE EFFECTIVE STRESS (σ´).**

EFFECTIVE STRESS CONCEPT

NO SEEPAGE



Saturated Soil Column (**Figure 6.1.** Das FGE (2005))



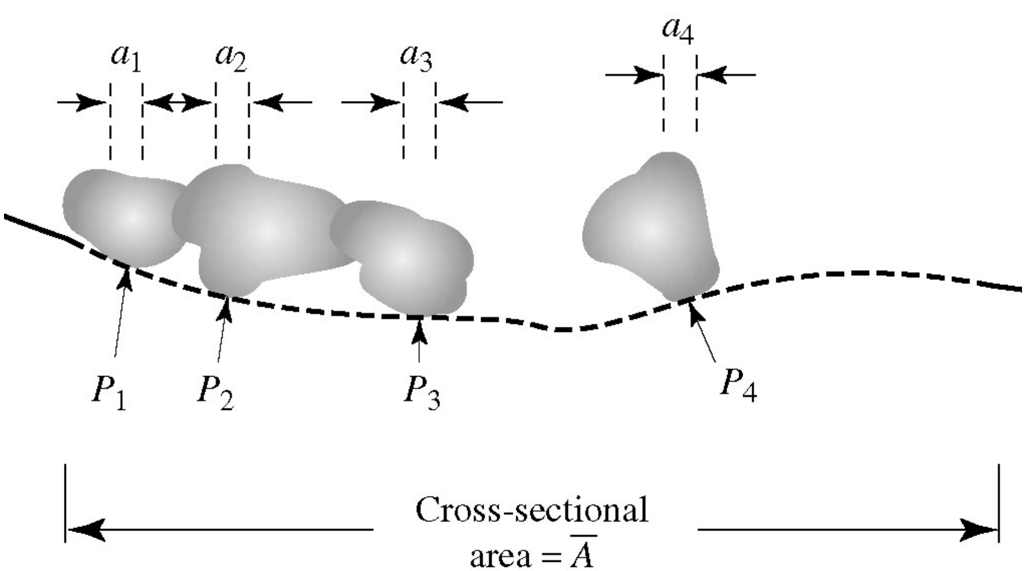
Forces acting at Soil Particle Points of Contact at level of Point A (i.e. along Line *a-a*)
Figure 6.1. Das FGE (2005)

Effective Stress (σ´) along Line *a-a*

$$\sigma' = \frac{P_{1(v)} + P_{2(v)} + P_{3(v)} + \ldots + P_{n(v)}}{\bar{A}}$$

Effective Stress Concept

No Seepage



Forces acting at Soil Particle Points of Contact at level of Point A (i.e. along Line *a-a*)

Figure 6.1. Das FGE (2005)

Effective Stress (σ´) along Line *a-a*

$$\sigma' = \frac{P_{1(v)} + P_{2(v)} + P_{3(v)} + \ldots + P_{n(v)}}{\bar{A}}$$

Where:

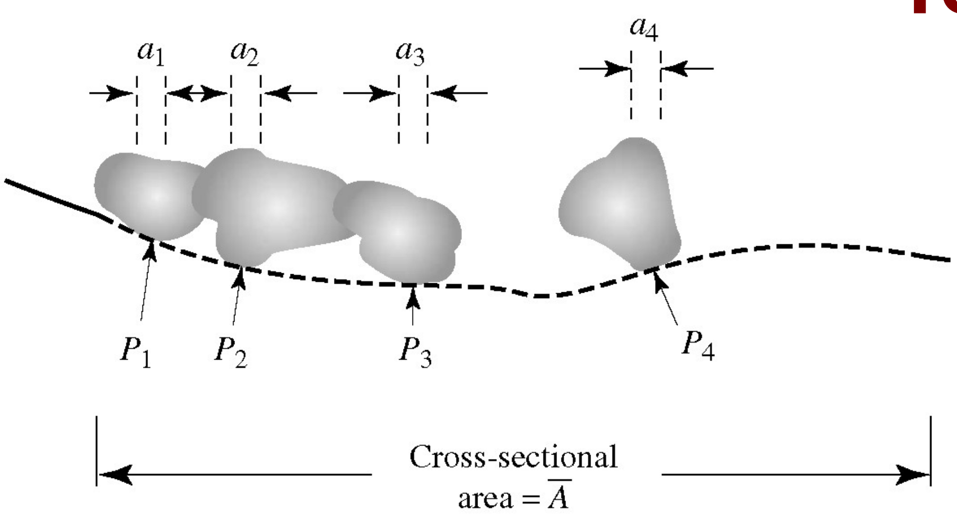
$P_{1(v)}$ = Vertical Component of P_1

$\bar{A}$ = Cross-sectional Area of Soil Mass Under Consideration

EFFECTIVE STRESS CONCEPT

NO SEEPAGE

Total Stress (σ) along Line *a-a*



Forces acting at Soil Particle Points of Contact at level of Point A (i.e. along Line *a-a*)
Figure 6.1. Das FGE (2005).

$$\sigma = \sigma' + \frac{u(\bar{A} - a_s)}{\bar{A}} = \sigma' + u(1 - a'_s)$$

Where:

a_s = Cross-section Area of Soil Contacts = $a_1+a_2+a_3+\ldots+a_n$

$\bar{A}$ = Cross-sectional Area of Soil Mass Under Consideration

$a'_s = a_s/\bar{A}$ = Fraction of unit cross-sectional area of soil mass occupied by solid to solid contacts.

EFFECTIVE STRESS CONCEPT

NO SEEPAGE

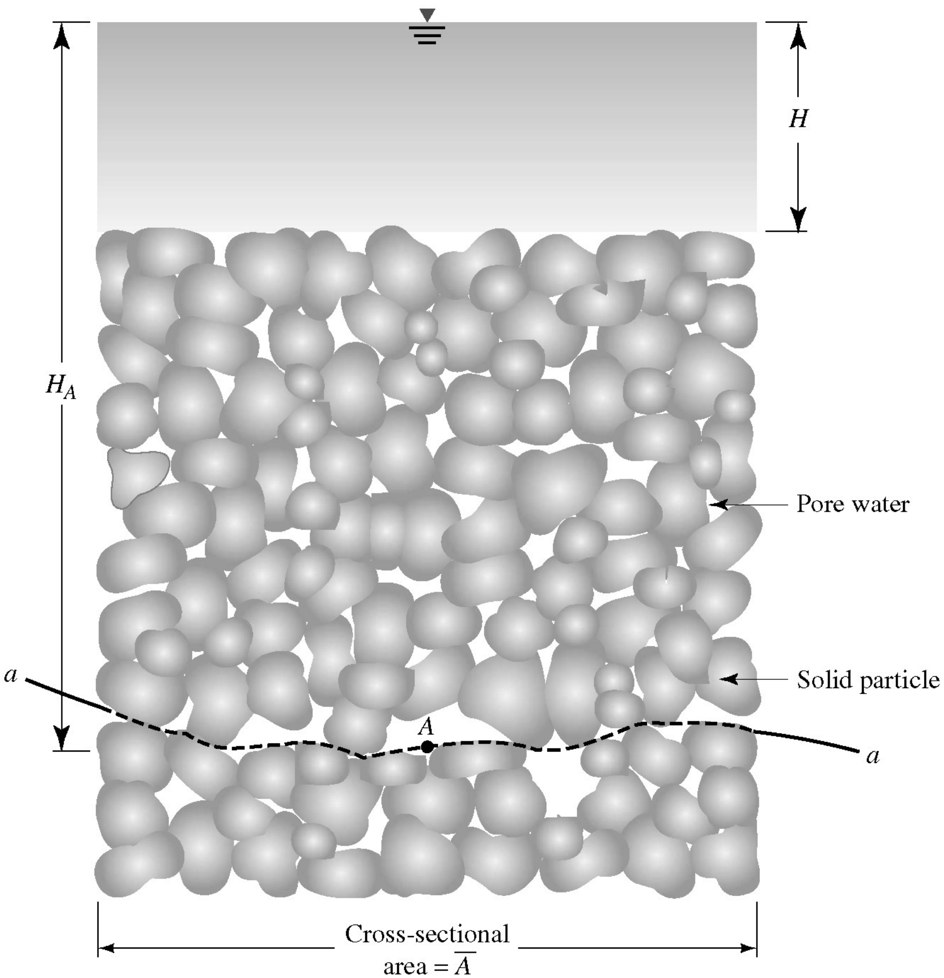
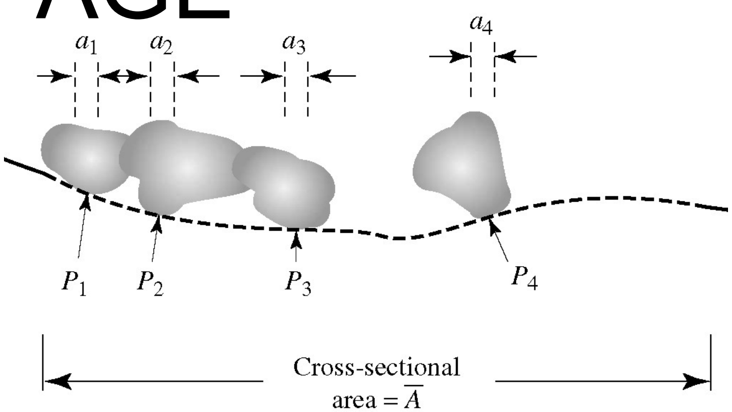


Figure 6.1. Das FGE (2005)



Total Stress (σ) along Line *a-a*

$$\sigma = \sigma' + \frac{u(\bar{A} - a_s)}{\bar{A}} = \sigma' + u(1 - a_s')$$

$a'_s \approx 0$ (i.e. very small), so therefore:

$$= \sigma' + u \quad \text{or} \quad \sigma' = \sigma - u$$

THE EFFECTIVE STRESS EQUATION

EFFECTIVE STRESS CONCEPT

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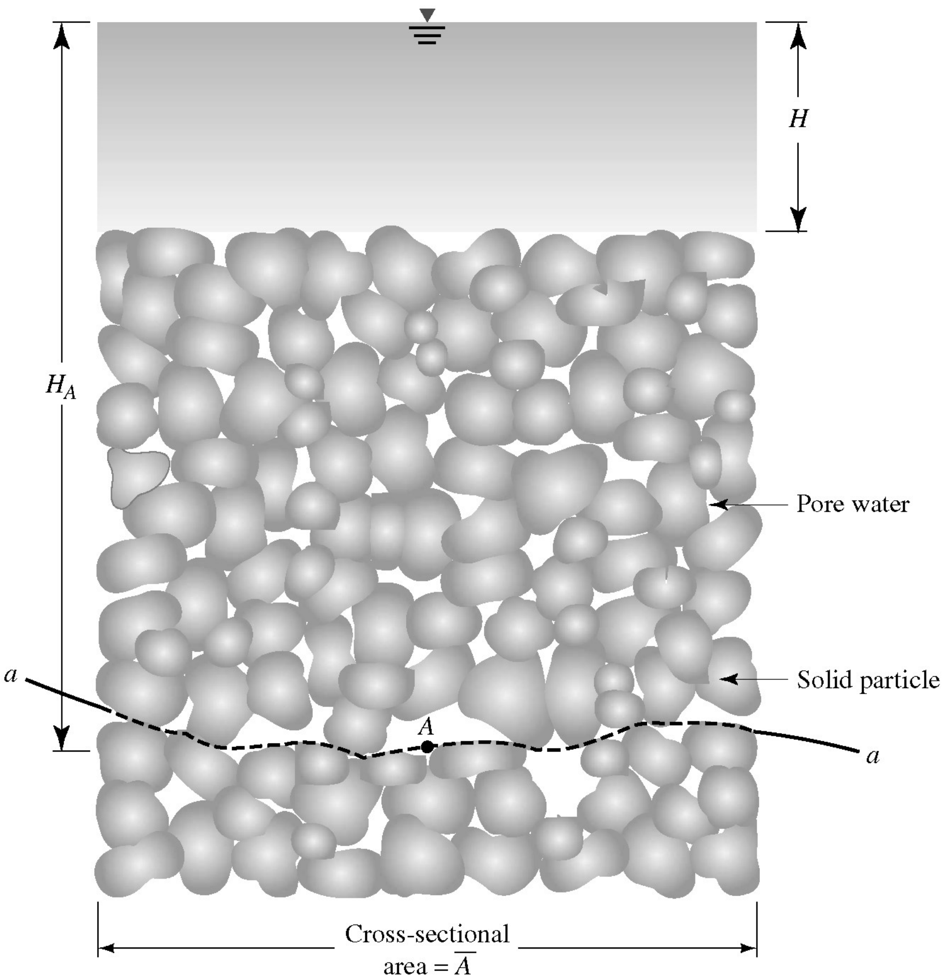


Figure 6.1. Das FGE (2005)

THE EFFECTIVE STRESS EQUATION

$$\sigma' = \sigma - u$$

$$\sigma' = \left[H\gamma_w + (H_A - H)\gamma_{sat}\right] - H_A\gamma_w$$

σ = Total Stress (the bracketed term)

u = Pore Pressure ($H_A\gamma_w$)

$$\sigma' = (H_A - H)(\gamma_{sat} - \gamma_w)$$

$(H_A - H)$: Height of soil column

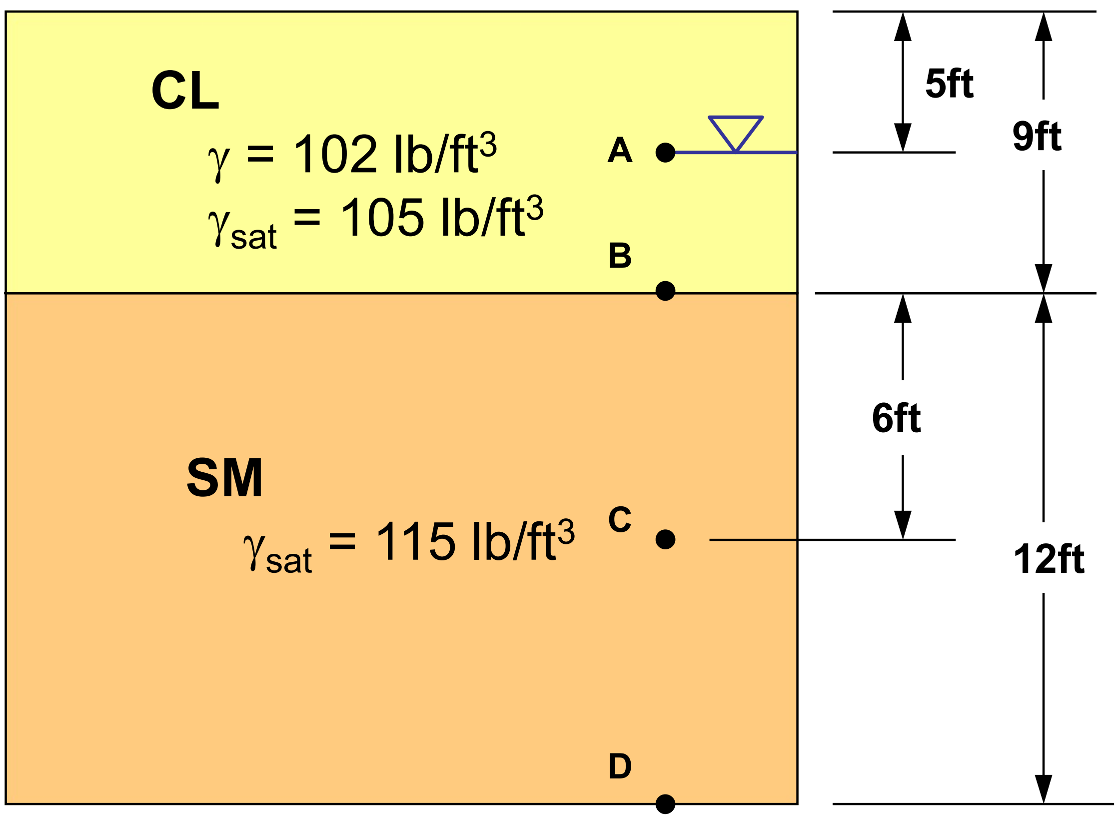
$$(\gamma_{sat} - \gamma_w) = \gamma'$$

γ' = Submerged unit weight of soil

EFFECTIVE STRESS CONCEPT

NO SEEPAGE: EXAMPLE PROBLEM

GIVEN SOIL PROFILE (NTS):



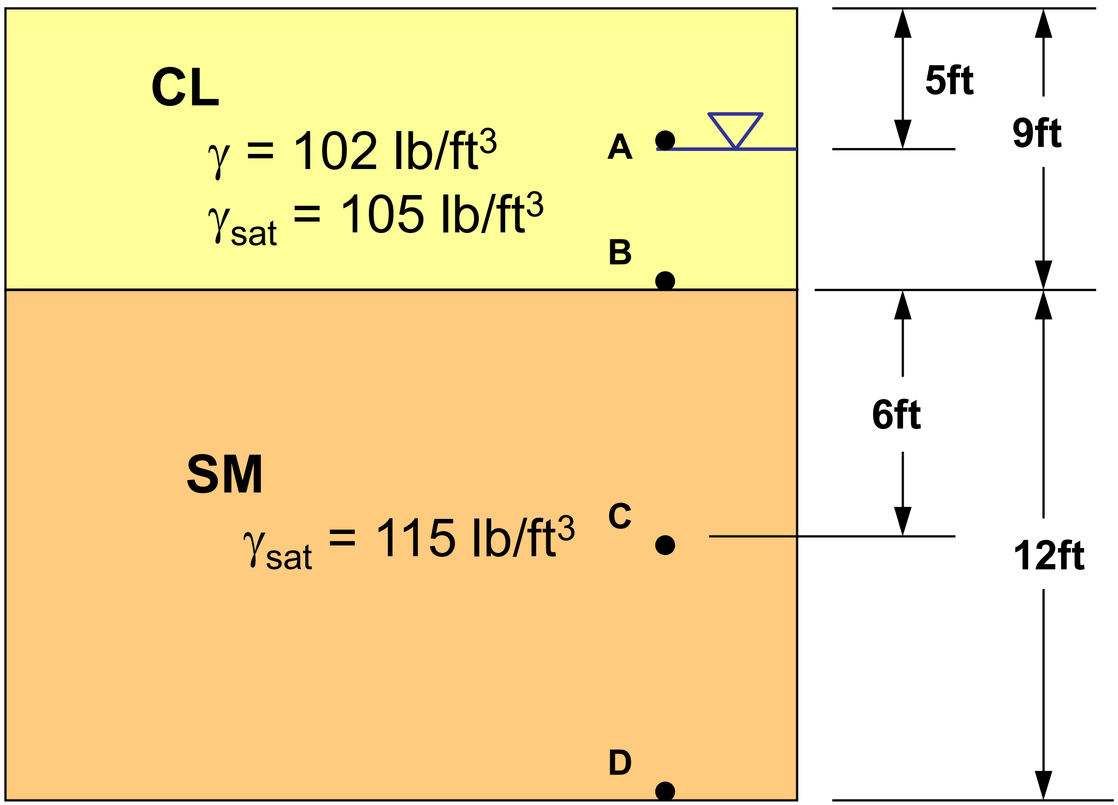
FIND:

Total and Effective Stresses at Pts. A, B, C, & D.

EFFECTIVE STRESS CONCEPT

NO SEEPAGE: EXAMPLE PROBLEM

GIVEN SOIL PROFILE (NTS):



FIND:

Total and Effective Stresses at Pts. A, B, C, & D.

@ Point A:

$$\sigma_A = \gamma_{CL} \times Z_A = 102\frac{lb}{ft^3}(5\,ft)$$

$$\sigma_A = 510\frac{lb}{ft^2}$$

$$\sigma'_A = \sigma_A - u_A$$

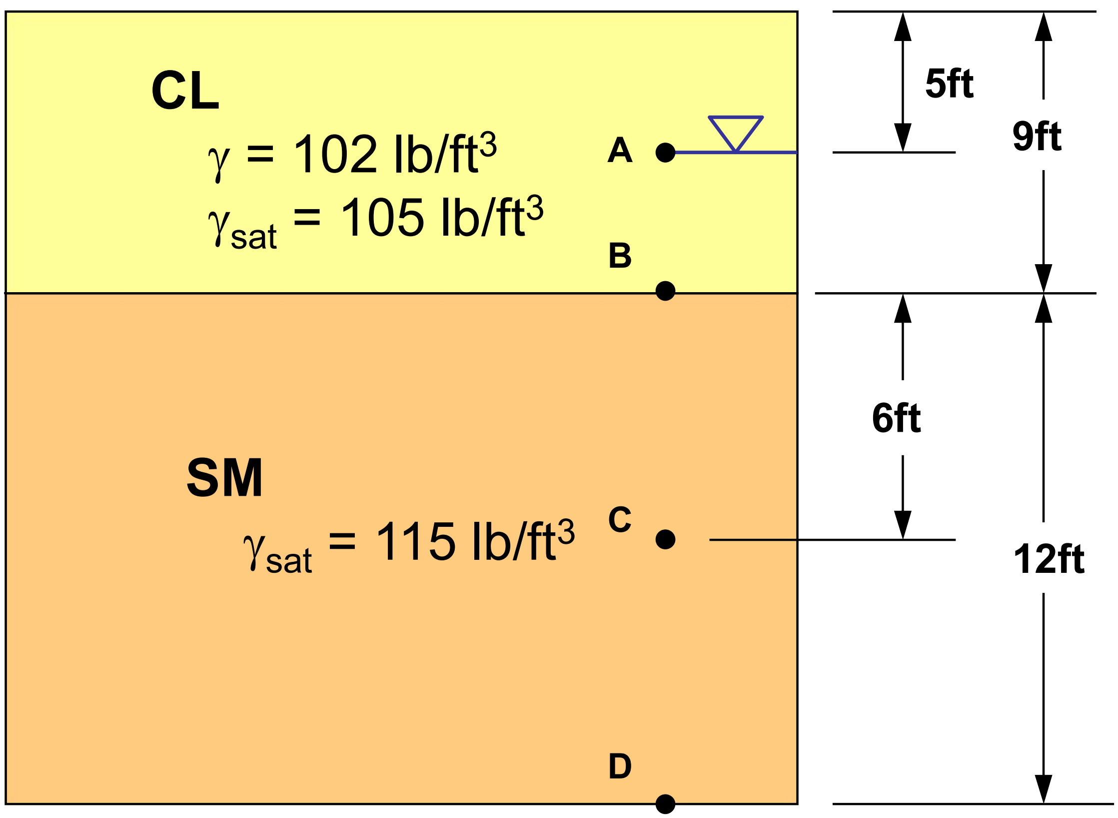
$$u_A = 0$$

$$\therefore \sigma'_A = \sigma_A = 510\frac{lb}{ft^2}$$

EFFECTIVE STRESS CONCEPT

NO SEEPAGE: EXAMPLE PROBLEM

GIVEN SOIL PROFILE (NTS):



@ Point B:

$$\sigma_B = \sigma_A + (\gamma_{sat,CL} X 4\,ft)$$

$$\sigma_B = 510\frac{lb}{ft^3} + 105\frac{lb}{ft^3}(4\,ft)$$

$$\sigma_B = 930\frac{lb}{ft^2}$$

$$\sigma'_B = \sigma_B - u_B$$

$$u_B = \gamma_w \times 4\,ft = 62.4\frac{lb}{ft^3} \times 4\,ft = 250\frac{lb}{ft^2}$$

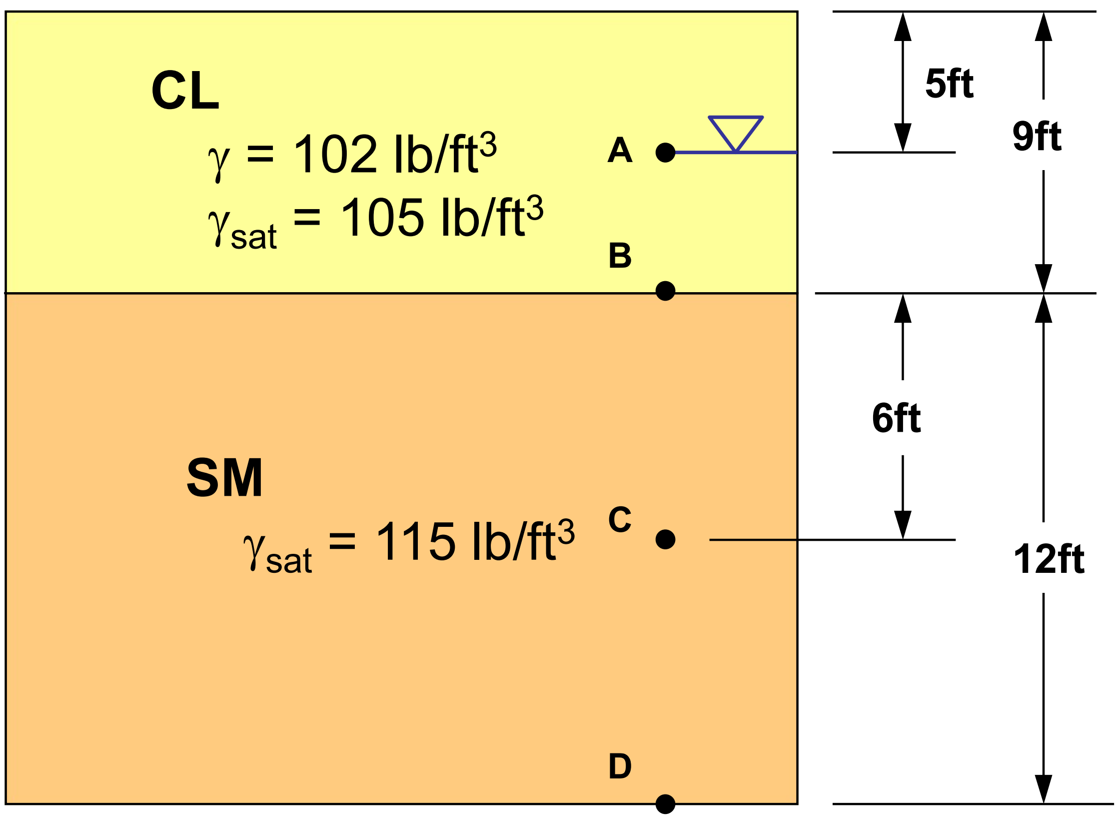
$$\sigma'_B = \sigma_B - u_B = 930\frac{lb}{ft^2} - 250\frac{lb}{ft^2}$$

$$\sigma'_B = 680\frac{lb}{ft^2}$$

EFFECTIVE STRESS CONCEPT

NO SEEPAGE: EXAMPLE PROBLEM

GIVEN SOIL PROFILE (NTS):



@ Point C:

$$\sigma_C = \sigma_B + (\gamma_{sat,SM} \times 6\,ft)$$

$$\sigma_C = 930\frac{lb}{ft^3} + 115\frac{lb}{ft^3}(6\,ft)$$

$$\sigma_C = 1620\frac{lb}{ft^2}$$

$$\sigma'_C = \sigma_C - u_C$$

$$u_C = \gamma_w \times 10\,ft = 62.4\frac{lb}{ft^3} \times 10\,ft = 624\frac{lb}{ft^2}$$

$$\sigma'_C = \sigma_C - u_C = 1620\frac{lb}{ft^2} - 624\frac{lb}{ft^2}$$

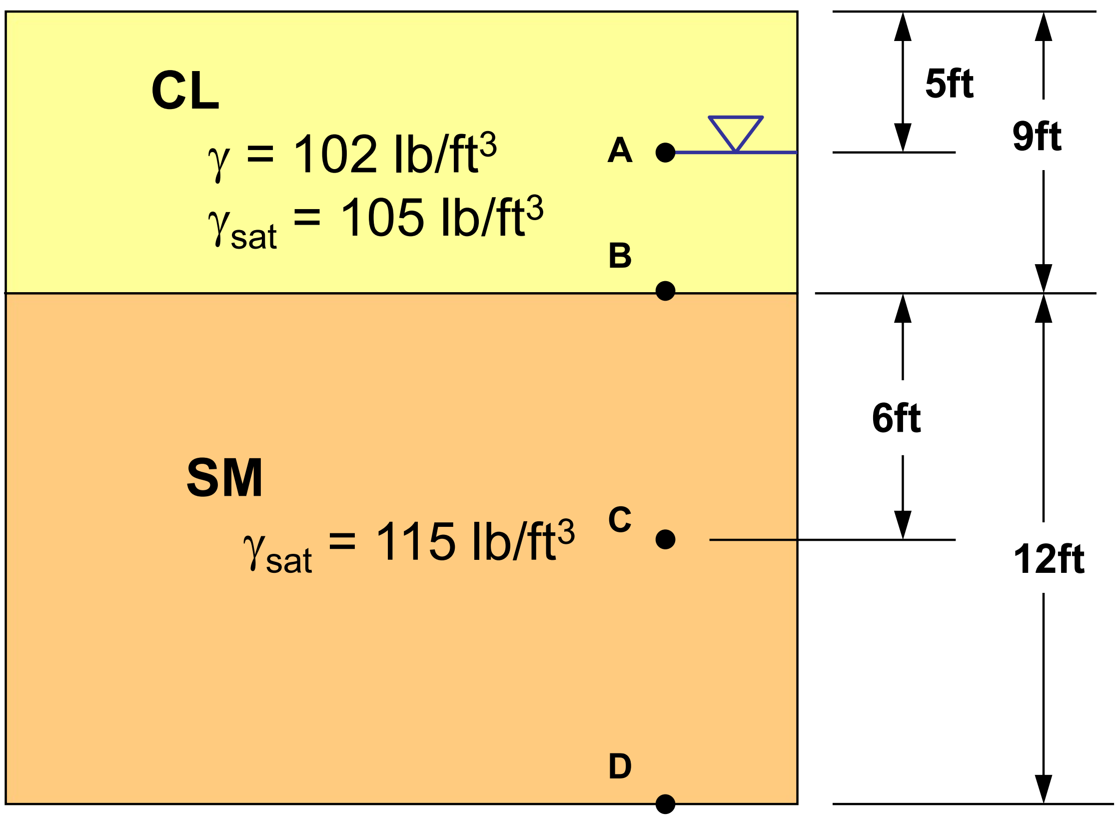
$$\sigma'_C = 996\frac{lb}{ft^2}$$

$$\sigma'_C = 1000\frac{lb}{ft^2} \quad \text{(round to nearest 5 psf)}$$

EFFECTIVE STRESS CONCEPT

NO SEEPAGE: EXAMPLE PROBLEM

GIVEN SOIL PROFILE (NTS):



@ Point D:

$$\sigma_D = \sigma_B + (\gamma_{sat,SM} \times 12\,ft)$$

$$\sigma_D = 930\frac{lb}{ft^3} + 115\frac{lb}{ft^3}(12\,ft)$$

$$\sigma_D = 2310\frac{lb}{ft^2}$$

$$\sigma'_D = \sigma_D - u_D$$

$$u_D = \gamma_w \times 16\,ft = 62.4\frac{lb}{ft^3} \times 16\,ft = 998\frac{lb}{ft^2}$$

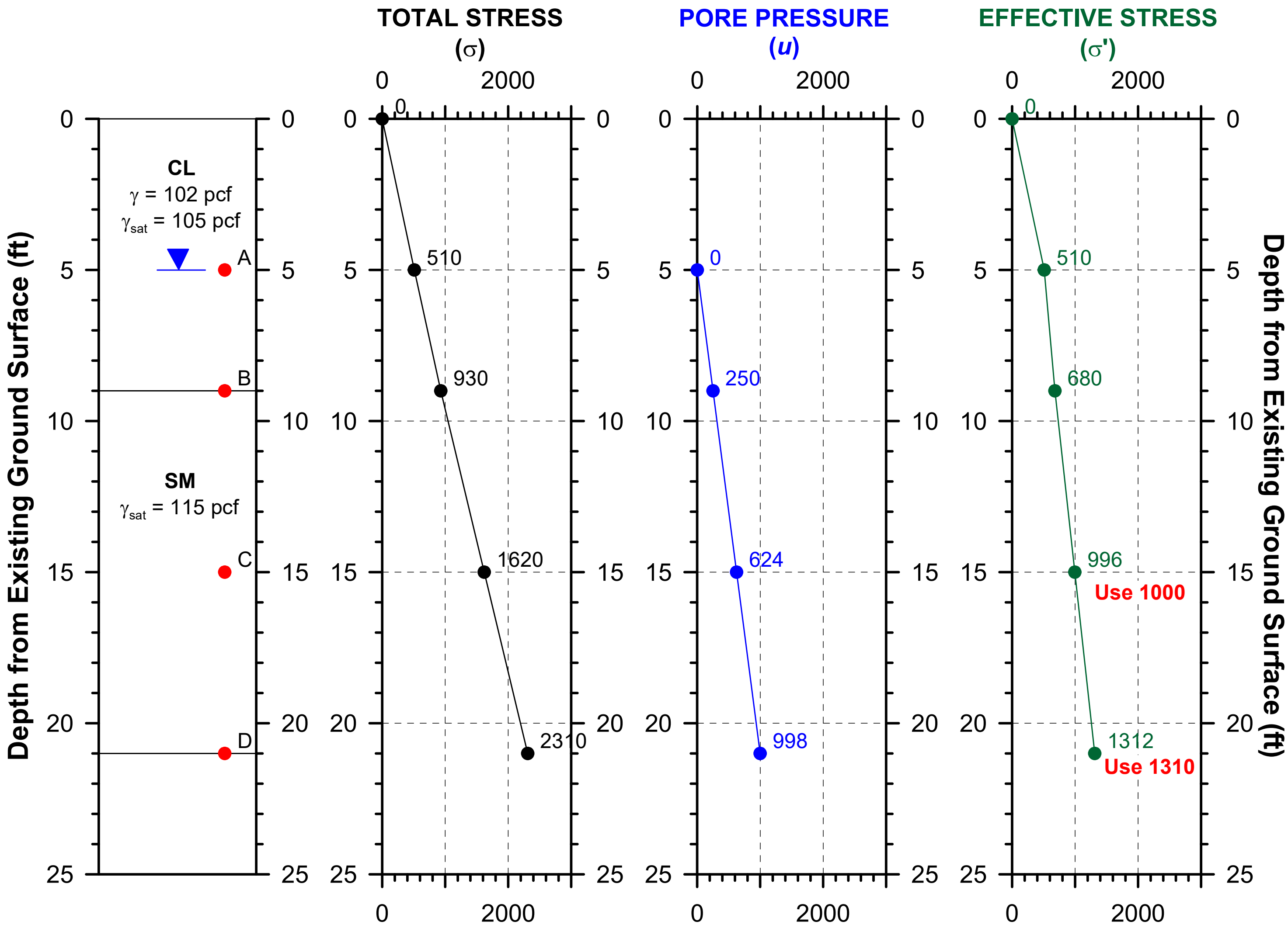
$$\sigma'_D = \sigma_D - u_D = 2310\frac{lb}{ft^2} - 998\frac{lb}{ft^2}$$

$$\sigma'_D = 1312\frac{lb}{ft^2}$$

$$\sigma'_D = 1310\frac{lb}{ft^2} \quad \text{(round to nearest 5 psf)}$$

Effective Stress Concept

No Seepage: Example Problem



EFFECTIVE STRESS CONCEPT

UPWARD SEEPAGE

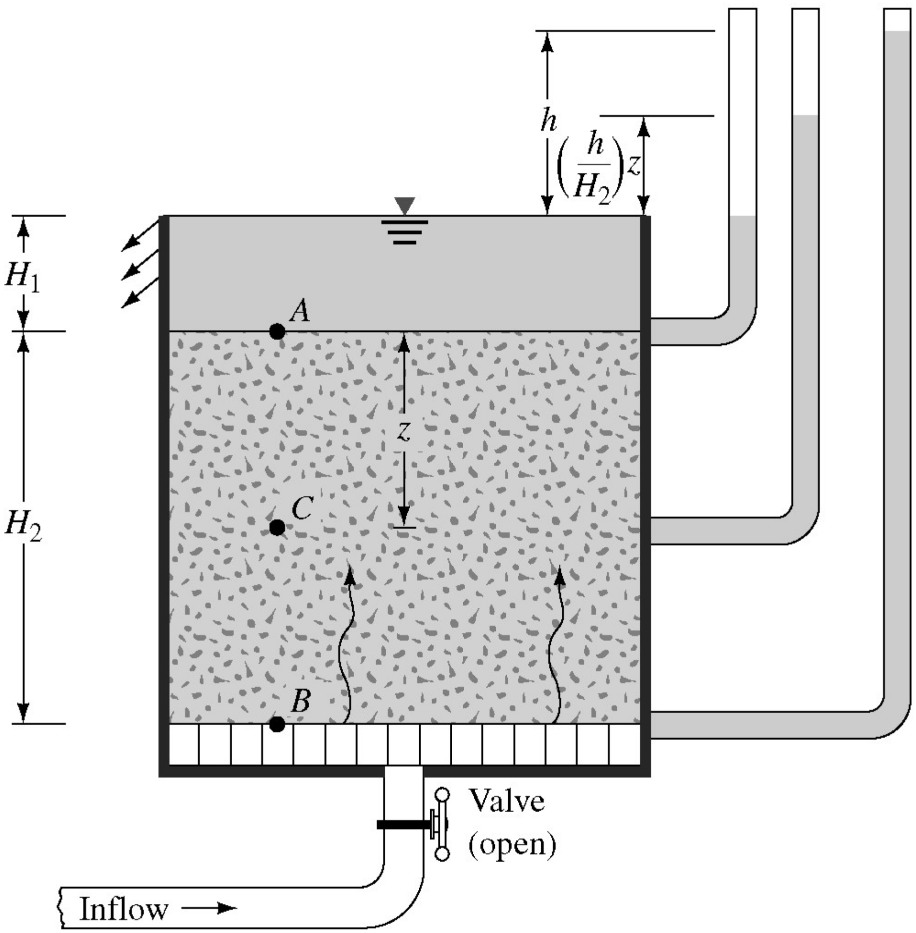


Figure 6.3a. Das FGE (2005).

Stresses @ Point A:

$$\sigma_A = H_1\gamma_w$$

$$u_A = H_1\gamma_w$$

$$\sigma'_A = \sigma_A - u_A = 0$$

Stresses @ Point B:

$$\sigma_B = H_1\gamma_w + H_2\gamma_{sat}$$

$$u_B = (H_1 + H_2 + h)\gamma_w$$

$$\sigma'_B = \sigma_B - u_B$$

$$\sigma'_B = (H_1\gamma_w + H_2\gamma_{sat}) - (H_1 + H_2 + h)\gamma_w$$

$$\sigma'_B = H_2(\gamma_{sat} - \gamma_w) - h\gamma_w$$

$$\sigma'_B = H_2\gamma' - h\gamma_w$$

EFFECTIVE STRESS CONCEPT

UPWARD SEEPAGE

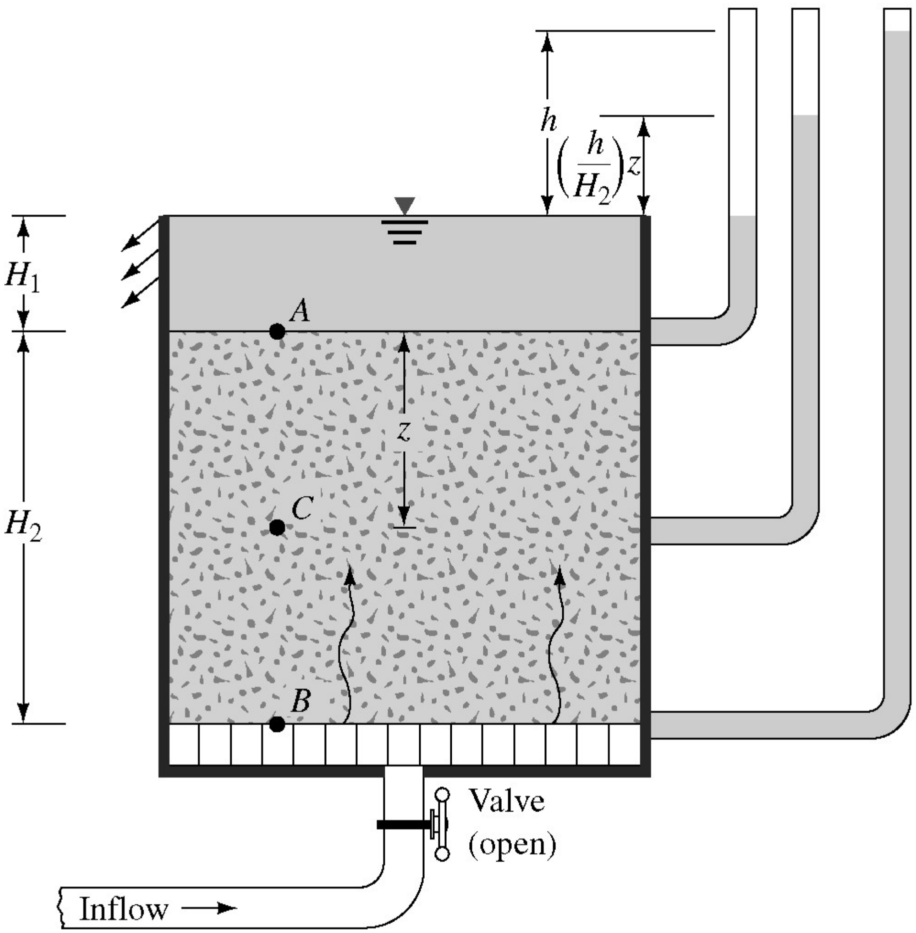


Figure 6.3a. Das FGE (2005).

Stresses @ Point C:

$$\sigma_C = H_1\gamma_w + z\gamma_{sat}$$

$$u_C = (H_1 + z + \frac{h}{H_2}z)\gamma_w$$

$$\sigma'_C = \sigma_C - u_C$$

$$\sigma'_C = z(\gamma_{sat} - \gamma_w) - \frac{h}{H_2}z\gamma_w$$

$$\sigma'_C = z\gamma' - \frac{h}{H_2}z\gamma_w$$

NOTE: $i = \dfrac{h}{H_2} = \dfrac{\text{Change in Head}}{\text{Length of Water Flow}}$

$$\therefore \sigma'_C = z\gamma' - iz\gamma_w$$

EFFECTIVE STRESS CONCEPT

UPWARD SEEPAGE

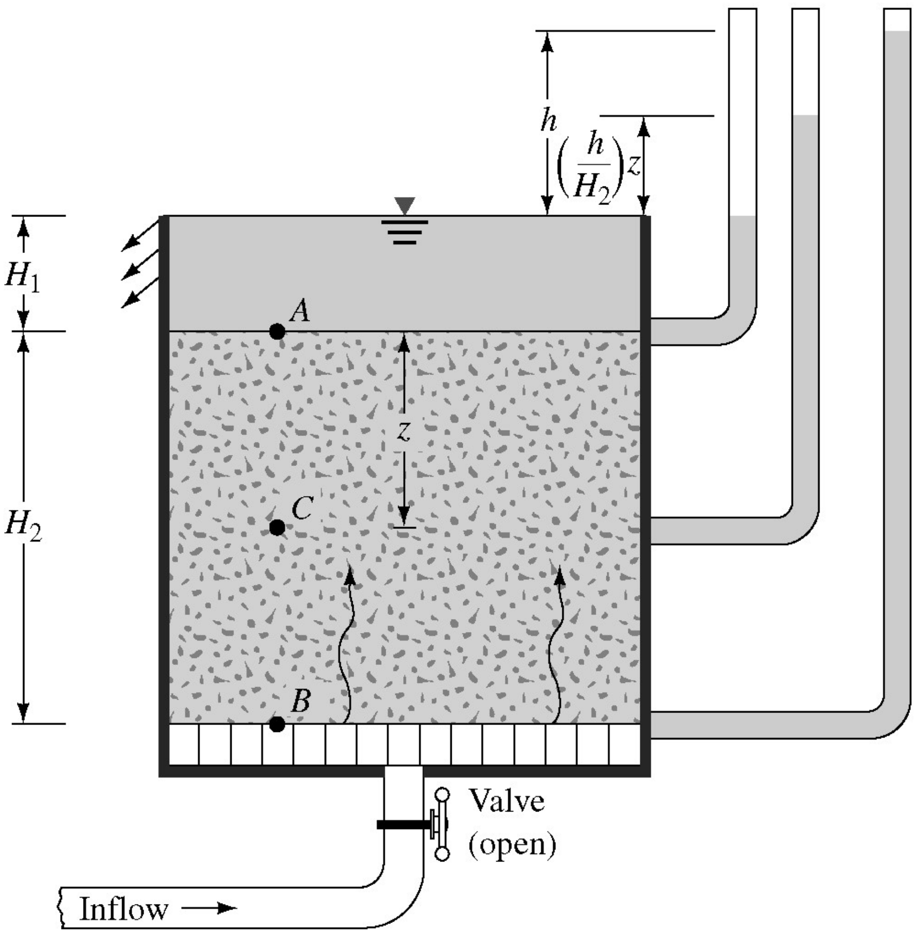


Figure 6.3a. Das FGE (2005).

CRITICAL HYDRAULIC GRADIENT
(i_{cr})

$$\sigma'_C = z\gamma' - i_{cr} z \gamma_w = 0$$

NO EFFECTIVE STRESS!

Known as Boiling or Quick Condition

$$i_{cr} = \frac{\gamma'}{\gamma_w}$$

For Most Soils:
i_{cr} ranges from 0.9 to 1.1, with an average of 1

EFFECTIVE STRESS CONCEPT

UPWARD SEEPAGE

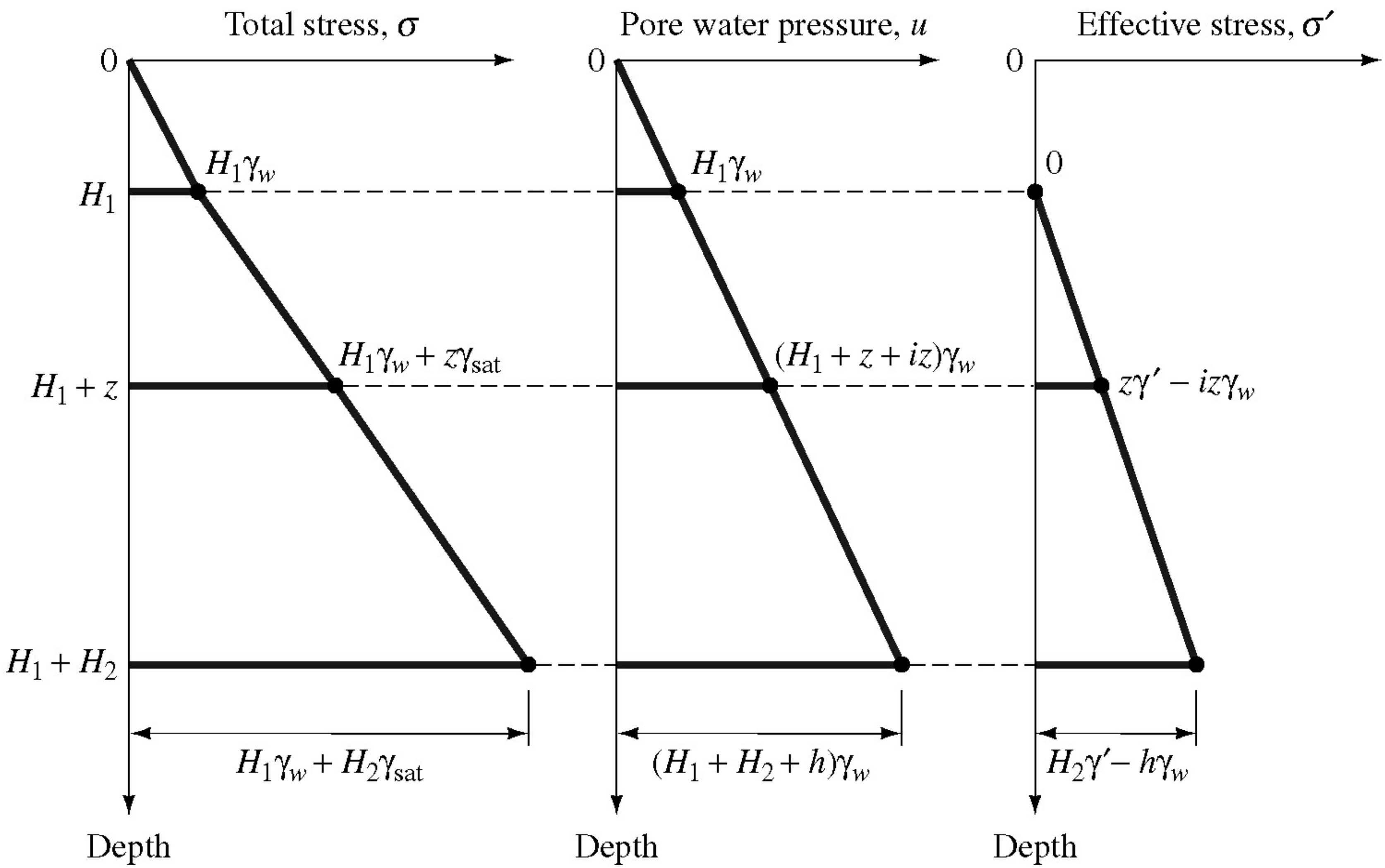


Figure 6.3b. Das FGE (2005).

EFFECTIVE STRESS CONCEPT

DOWNWARD SEEPAGE

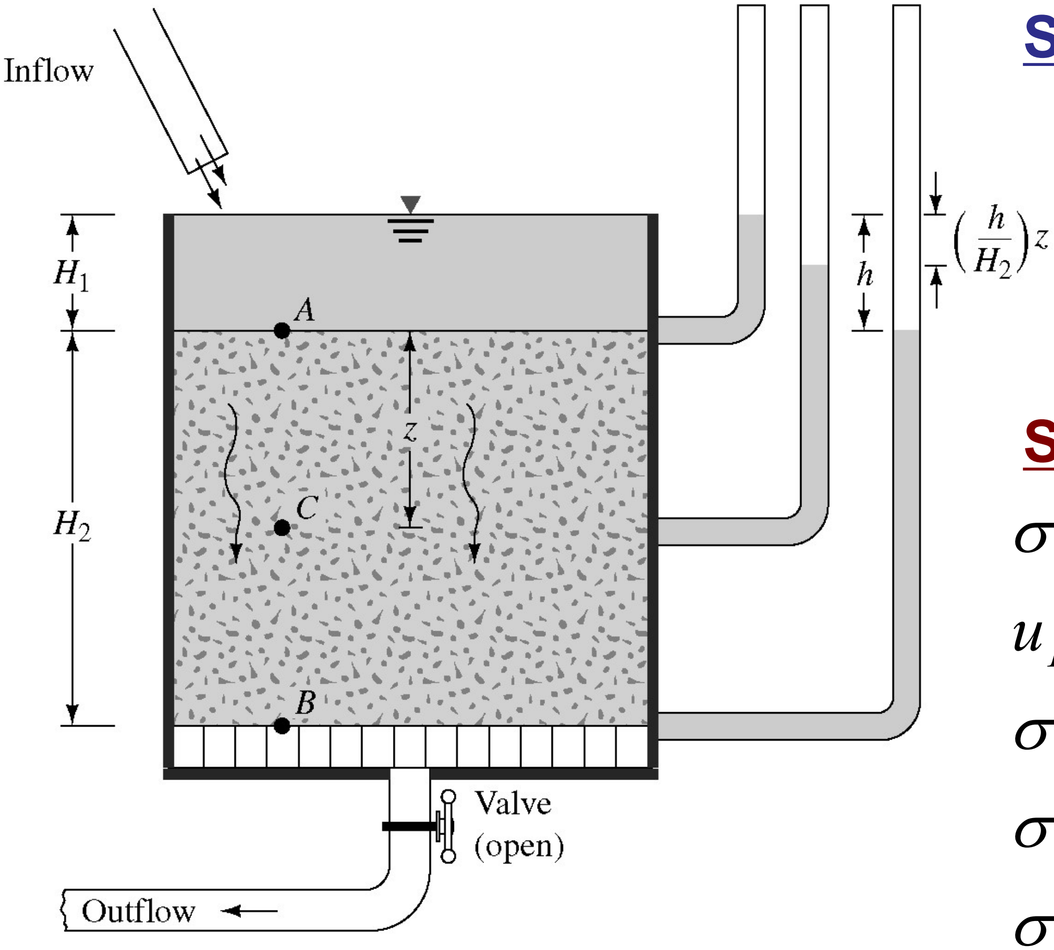


Figure 6.4a. Das FGE (2005).

Stresses @ Point A:

$$\sigma_A = H_1\gamma_w$$

$$u_A = H_1\gamma_w$$

$$\sigma'_A = \sigma_A - u_A = 0$$

Stresses @ Point B:

$$\sigma_B = H_1\gamma_w + H_2\gamma_{sat}$$

$$u_B = (H_1 + H_2 - h)\gamma_w$$

$$\sigma'_B = \sigma_B - u_B$$

$$\sigma'_B = (H_1\gamma_w + H_2\gamma_{sat}) - (H_1 + H_2 - h)\gamma_w$$

$$\sigma'_B = H_2(\gamma_{sat} - \gamma_w) + h\gamma_w$$

$$\sigma'_B = H_2\gamma' + h\gamma_w$$

EFFECTIVE STRESS CONCEPT

DOWNWARD SEEPAGE

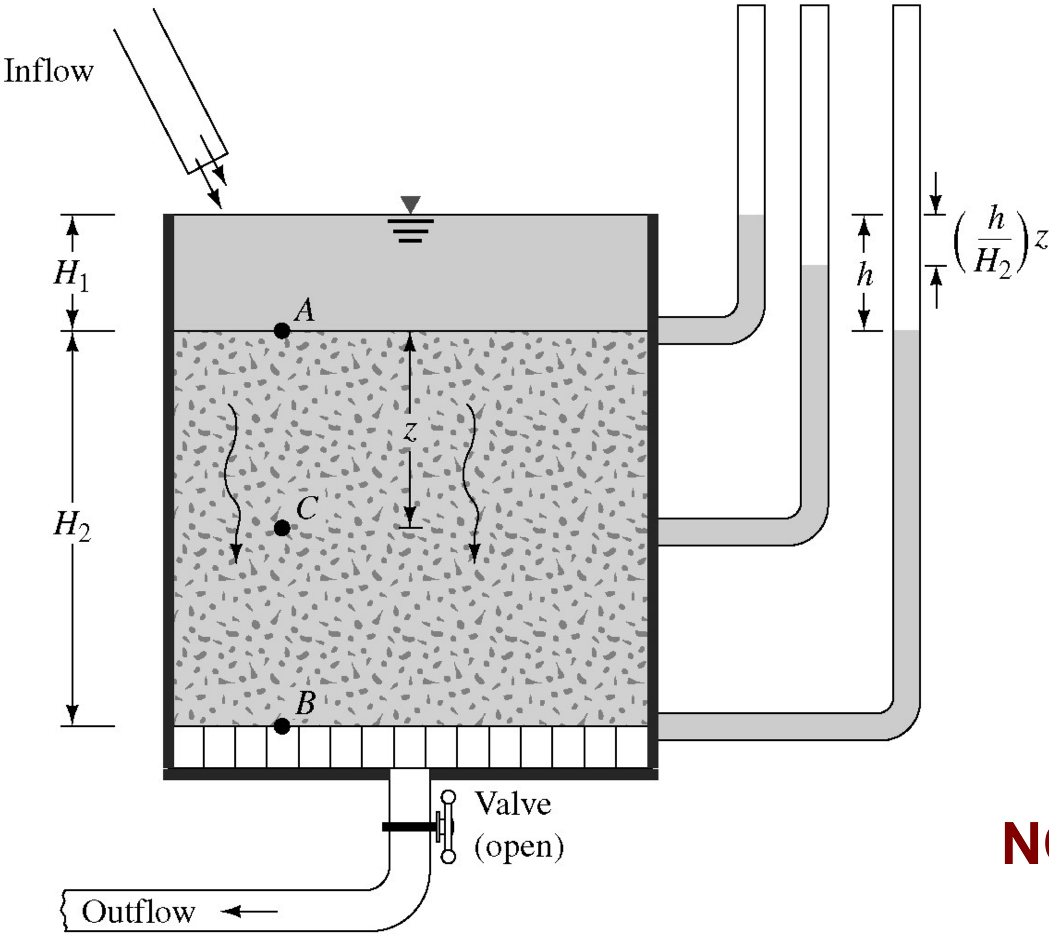


Figure 6.4a. Das FGE (2005).

Stresses @ Point C:

$$\sigma_C = H_1\gamma_w + z\gamma_{sat}$$

$$u_C = (H_1 + z - \frac{h}{H_2}z)\gamma_w$$

$$\sigma'_C = \sigma_C - u_C$$

$$\sigma'_C = H_1\gamma_w + z\gamma_{sat} - (H_1 + z - \frac{h}{H_2}z)\gamma_w$$

$$\sigma'_C = z\gamma' + \frac{h}{H_2}z\gamma_w$$

NOTE: $i = \frac{h}{H_2} = \frac{\text{Change in Head}}{\text{Length of Water Flow}}$

$$\therefore \sigma'_C = z\gamma' + iz\gamma_w$$

EFFECTIVE STRESS CONCEPT

DOWNWARD SEEPAGE

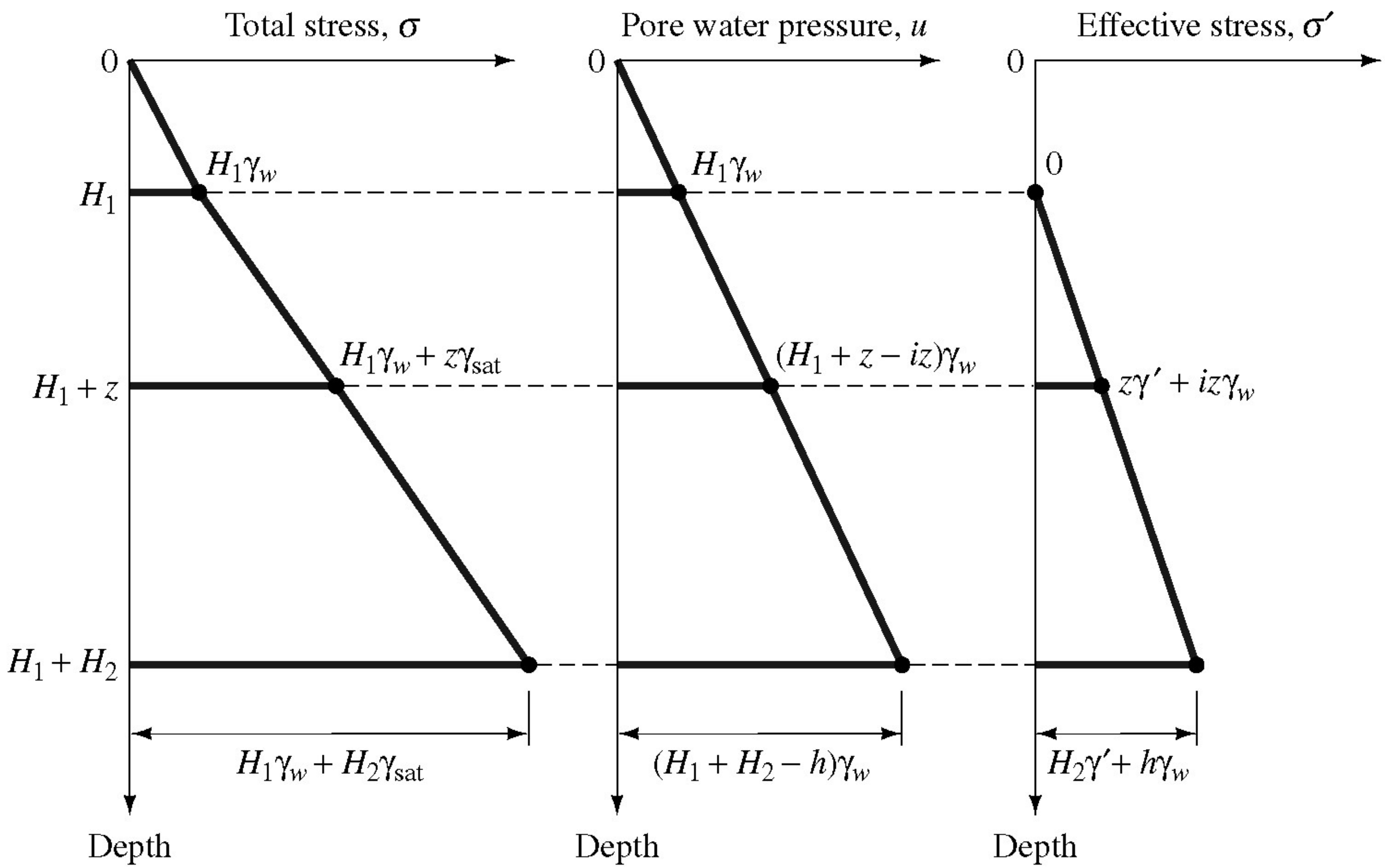


Figure 6.4b. Das FGE (2005).

EFFECTIVE STRESS CONCEPT

PARTIALLY SATURATED SOIL

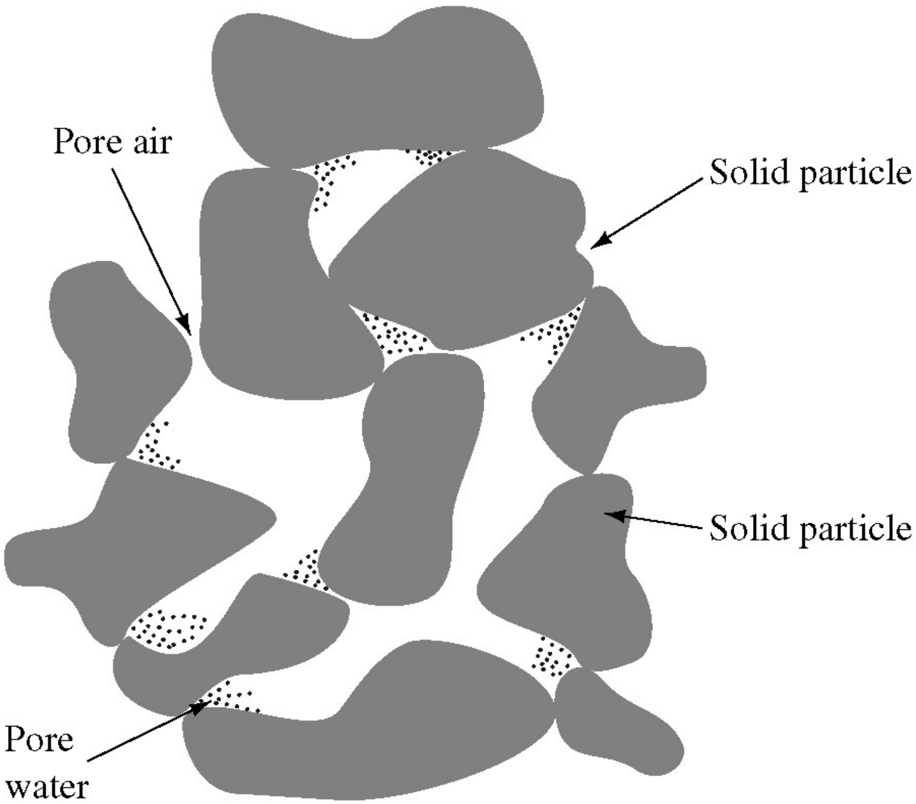


Figure 6.6. Das FGE (2005).

$$\sigma' = \sigma - u_a + \chi(u_a - u_w)$$

Where:

u_a = Pore Air Pressure
u_w = Pore Water Pressure
χ = Fraction of unit cross-sectional area of soil occupied by water.
χ = 0 for dry soil; 1 for saturated soil.

χ depends on degree of saturation (S). Also influenced by soil structure.

CAPILLARY RISE IN SOILS

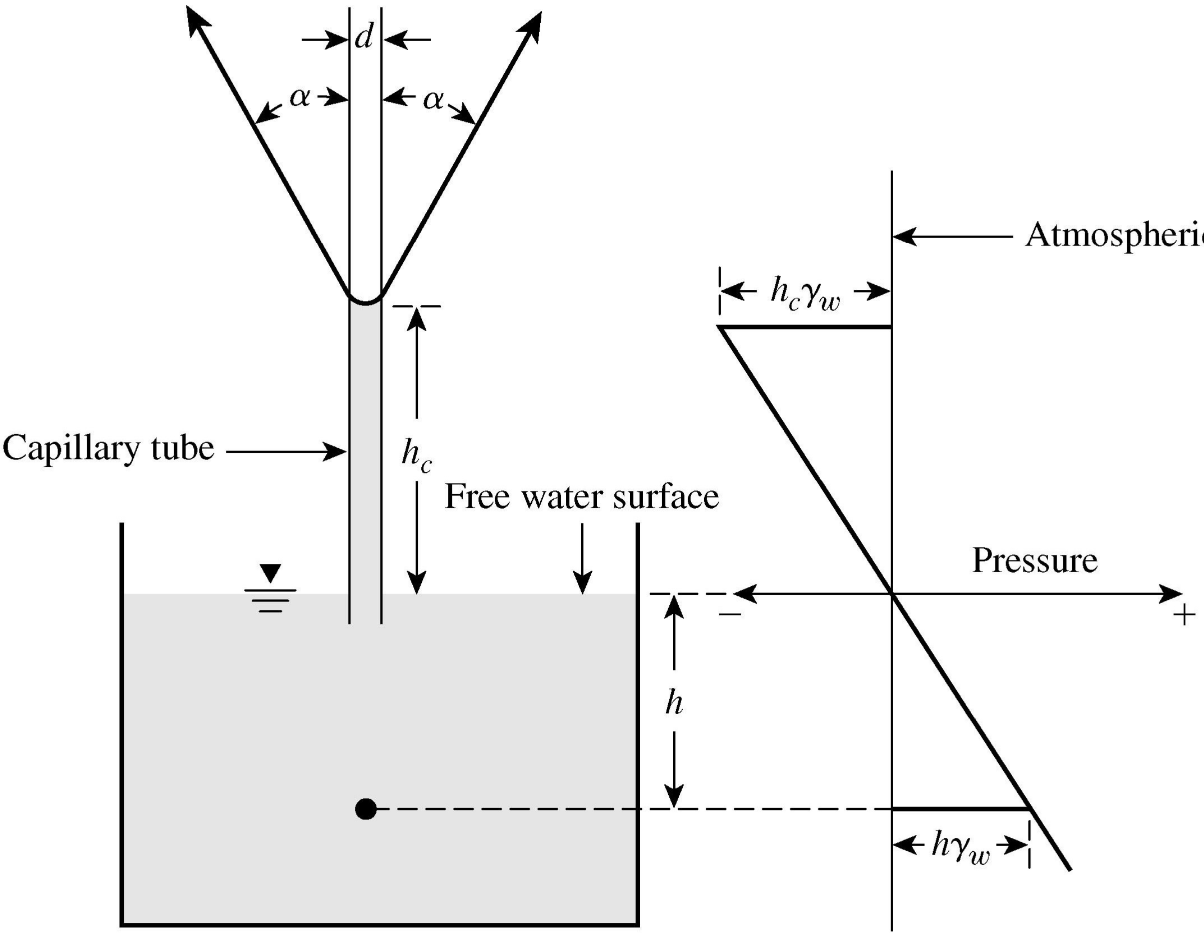


Figure 8.19. Principles of Geotechnical Engineering, Das (2006).

Summing Forces in Vertical Direction

$$\left(\frac{\pi}{4}d^2\right)h_c\gamma_w = \pi d T \cos\alpha$$

$$h_c = \frac{4T\cos\alpha}{d\gamma_w}$$

Where:

- T = Surface Tension
- α = Angle of Contact
- d = Capillary Tube Diameter

T, α, γ_w remain constant

$$h_c \propto \frac{1}{d}$$

CAPILLARY RISE IN SOILS

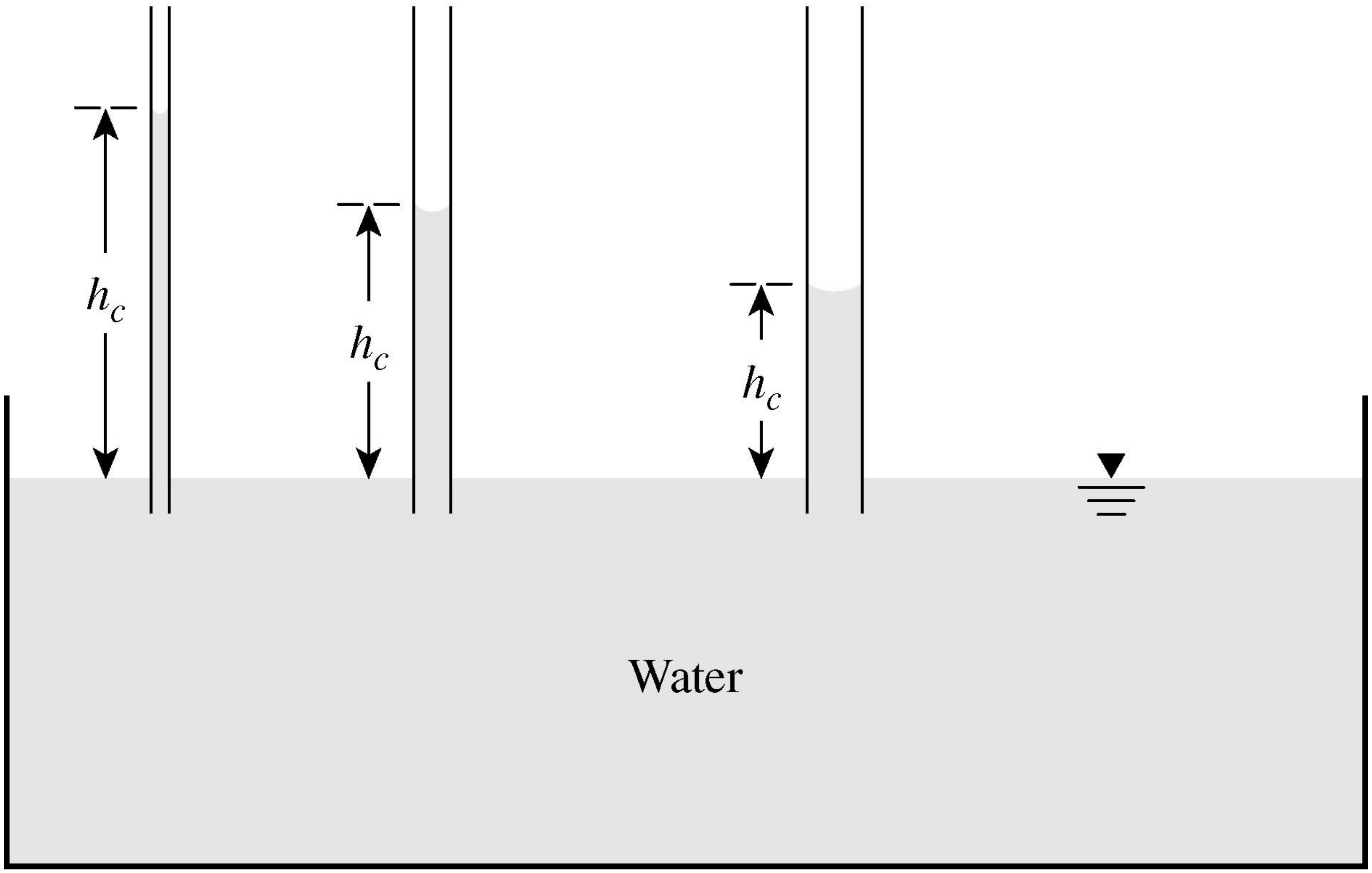
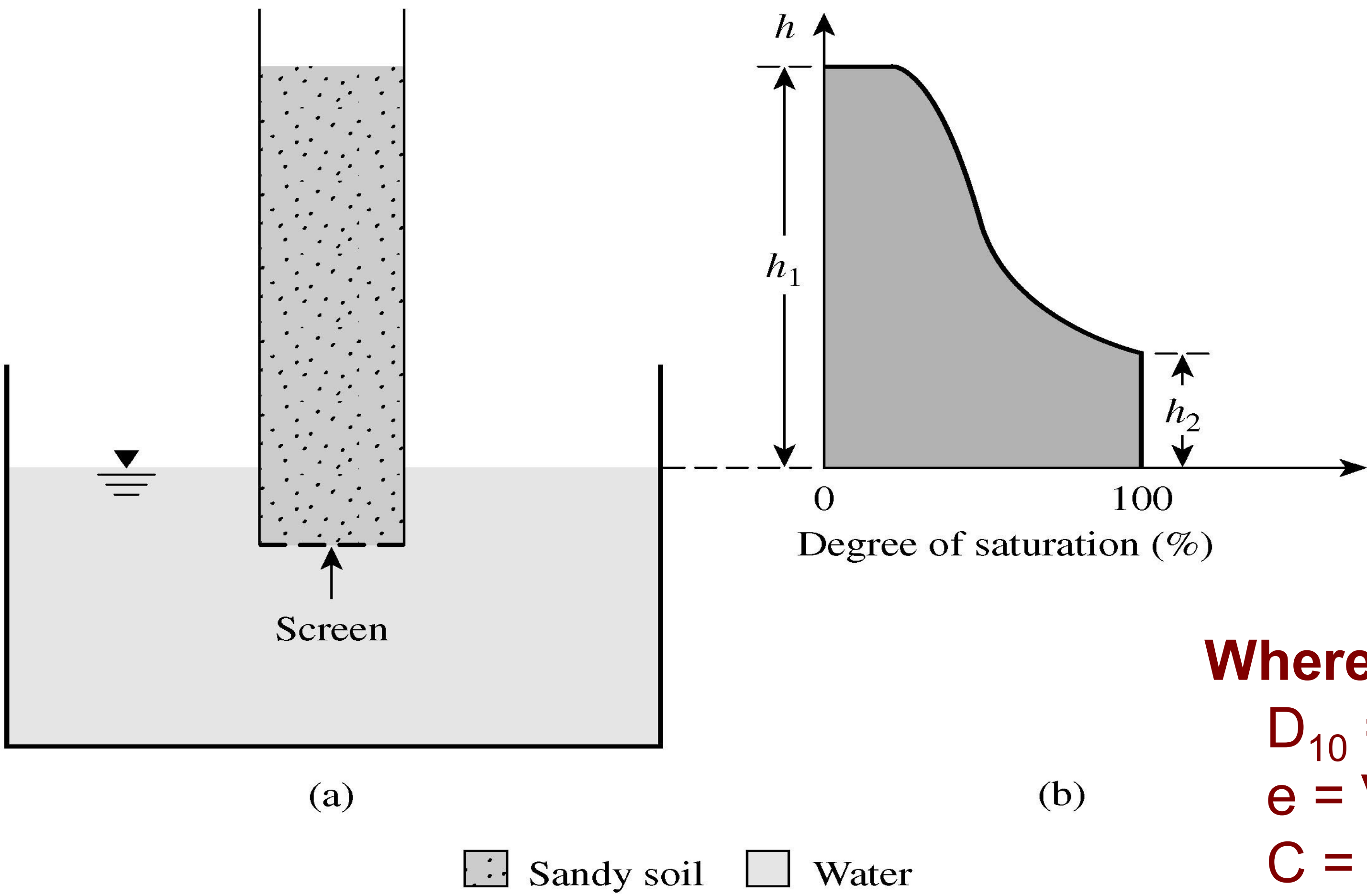


Figure 8.20. Principles of Geotechnical Engineering, Das (2006).

$$h_c \propto \frac{1}{d}$$

CAPILLARY RISE IN SOILS



Hazen (1930)

$$h_1 = \frac{C}{eD_{10}}$$

Where:

- D_{10} = Effective Size (mm)
- e = Void Ratio
- C = Constant (ranging from 10 mm^2 to 50 mm^2)

Capillary Rise in Soils

Table 8.2 (Das, PGE 2006). Approximate Range of Capillary Rise in Soils.

Soil Type	Range of Capillary Rise	
	m	ft
Coarse Sand	0.1 – 0.2	0.3 – 0.6
Fine Sand	0.3 – 1.2	1 – 4
Silt	0.75 – 7.5	2.5 – 25
Clay	7.5 - 23	25 - 75

Effective Stress in Capillary Zone

$$\sigma' = \sigma - u$$

Saturated: $u = -h\gamma_w$

Partially Saturated: $u = -h\left(\frac{S}{100}\right)\gamma_w$

SEEPAGE FORCE

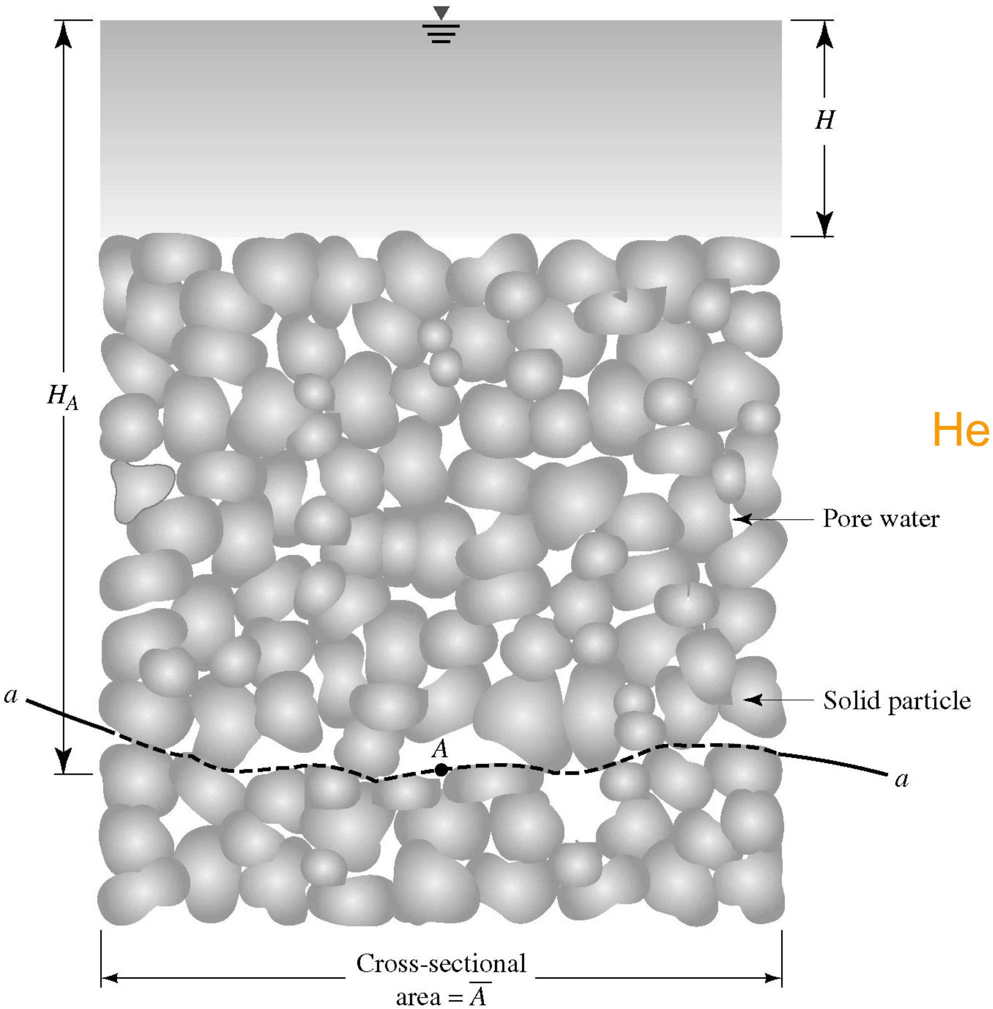


Figure 6.1. Das FGE (2005).

WITH NO SEEPAGE (i.e. STATIC CONDITIONS)

$$\sigma' = (H_A - H)(\gamma_{sat} - \gamma_w)$$

Height of soil column = *z*

$$(\gamma_{sat} - \gamma_w) = \gamma'$$

γ' = Submerged unit weight of soil

Effective Stress: $\sigma' = z\gamma'$

Effective Force: $P_1' = z\gamma' A$

Where:

A = Area

SEEPAGE FORCE: UPWARD SEEPAGE

	EFFECTIVE STRESS	EFFECTIVE FORCE
NO SEEPAGE:	$\sigma' = z\gamma'$	$P_1' = z\gamma' A$
W/ SEEPAGE:	$\sigma' = z\gamma' - iz\gamma_w$	$P'_2 = (z\gamma' - iz\gamma_w)A$

DECREASE OF TOTAL FORCE DUE TO SEEPAGE:

$$P_1' - P_2' = iz\gamma_w A$$

SEEPAGE FORCE PER UNIT VOLUME:

$$\frac{P_1' - P_2'}{\text{(Soil Volume)}} = \frac{iz\gamma_w A}{zA} = i\gamma_w$$

SEEPAGE FORCE SUMMARY

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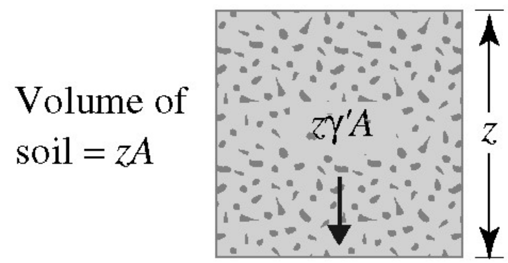
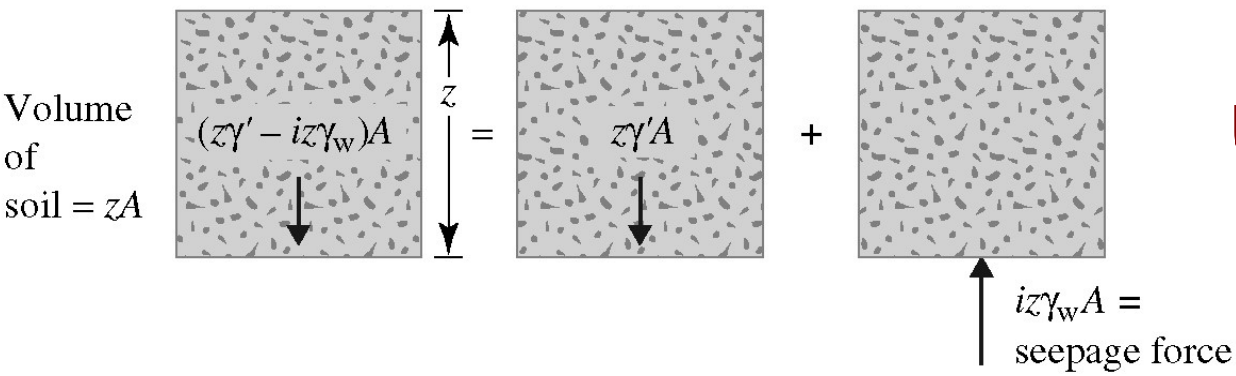


UPWARD SEEPAGE



DOWNWARD SEEPAGE