Instructions: No note or calculators are allowed. Answers must be supported by work on your exam sheets. Answers with little or no supporting work will receive little or no credit. Work must be neat, organized and easily interpreted.

A page of useful information has been included on the last page of the exam.

Please circle your final answers.

Notes:
0! = 1 and if \( n > 0 \) then \( n! = 1 \times 2 \times 3 \times \cdots \times n \)

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1. Evaluate the integrals:
   a. (6 pts) \( \int_0^\pi \frac{\sin x}{2 - \cos x} \, dx \)
   b. (6 pts) \( \int x^2 \sin x \, dx \)
c. (6 pts) \[
\int \frac{5x - 3}{x^2 - 2x - 3} \, dx
\]

d. (6 pts) \[
\int_0^\infty \frac{1}{1 + x^2} \, dx
\]
2. (10 pts) Use trigonometric substitution to evaluate

\[ \int \frac{x^3}{\sqrt{1-x^2}} \, dx. \]
3. (6 pts) Sketch and shade in the region bounded by the curves

\[ y = x, \quad y = \frac{1}{x^2} \quad \text{and} \quad x = 2. \]

Find the area of this region.

4. (6 pts) Find the area of the surface generated by revolving the curve below about the x-axis.

\[ y = \sqrt{2x - x^2}, \quad \frac{1}{2} \leq x \leq \frac{3}{2} \]
5. Let $R$ be the region in the first quadrant bounded above by $y = 2x$ and below by $y = x^2$.

   a. (6 pts) Find the volume of the solid generated when $R$ is revolved about the $x$-axis.

   b. (6 pts) Find the volume of the solid generated when $R$ is revolved about the $y$-axis.
6. Determine whether the following series **converge** or **diverge**. Justify your answer by referencing AND applying the appropriate convergence test. Show that the requirements of the test being used are satisfied.

a. (6 pts) \[ \sum_{k=1}^{\infty} \frac{k+1}{k^2 \sqrt{k}} \]

b. (6 pts) \[ \sum_{n=0}^{\infty} \frac{(-1)^n \cdot n^{23}}{n^{23}+1} \]
7. (10 pts) Find the radius and interval of convergence for the power series
\[ \sum_{k=0}^{\infty} \frac{(2016)^k x^k}{k!} . \]

a. Radius of convergence: ________________________________

b. Interval of convergence: ________________________________
8. (6 pts) Find the Taylor series generated by \( f(x) = x^4 \) centered at \( x = 2 \).

9. (6 pts) The Maclaurin series for \( \tan^{-1} x \) is

\[
\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots, \quad |x| \leq 1
\]

Find the Maclaurin series for \( g(x) = x \tan^{-1} x \). Give your answer in **summation form**.
10. (2 pts each) Find the Cartesian coordinates of the following points given in polar coordinates.
   a. \((r, \theta) = (-1, \pi)\)
   b. \((r, \theta) = \left(5, \frac{2\pi}{3}\right)\)

11. (2 pts each) Find the polar coordinates, \(0 \leq \theta < 2\pi\) and \(r \geq 0\), of the following points given in Cartesian coordinates.
   a. \((x, y) = (4\sqrt{3}, 4)\)
   b. \((x, y) = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)\)
Disk Method: \( V = \int_a^b \pi [R(x)]^2 \, dx \)

Washer Method: \( V = \int_a^b \pi ([R(x)]^2 - [r(x)]^2) \, dx \)

Shell Method: \( V = \int_a^b 2\pi rh(x) \, dx \)

Surface Area: \( S = \int_a^b 2\pi f(x) \sqrt{1 + (f'(x))^2} \, dx \)

Arc Length Formula: \( L = \int_a^b \sqrt{1 + (f'(x))^2} \, dx \)

Useful Trigonometric Identities: \( \cos^2 \theta = \frac{1 + \cos 2\theta}{2}; \sin^2 \theta = \frac{1 - \cos 2\theta}{2}; \sin 2\theta = 2 \sin \theta \cos \theta \)

The \( n \)-th Term Test for Divergence: \( \sum_{n=1}^{\infty} a_n \) diverges if \( \lim_{n \to \infty} a_n \) fails to exist or is different from zero.

The Limit Comparison Test: Let \( \sum a_n \) and \( \sum b_n \) be series with positive terms and suppose \( L = \lim_{n \to \infty} \frac{a_n}{b_n} \).

a) If \( L \) is finite and \( L > 0 \), then the series both converge or both diverge.

b) If \( L = 0 \) and \( \sum b_n \) converges, then \( \sum a_n \) converges.

c) If \( L = \infty \) and \( \sum b_n \) diverges, then \( \sum a_n \) diverges.

Ratio Test: Let \( \sum a_n \) be a series with nonzero terms and suppose \( \rho = \lim_{n \to \infty} \frac{|a_{n+1}|}{|a_n|} \).

a) If \( \rho < 1 \), the series converges absolutely.

b) If \( \rho > 1 \) or \( \rho = \infty \), the series diverges.

c) If \( \rho = 1 \), then the test is inconclusive, use a different test.

Alternating Series Test: An alternating series \( \sum_{n=1}^{\infty} (-1)^n u_n \) or \( \sum_{n=1}^{\infty} (-1)^{n+1} u_n \) converges if the following three conditions are satisfied:

1) \( u_n > 0 \) for all \( n \geq N \)  
2) \( \lim_{n \to \infty} u_n = 0 \)  
3) \( u_n \geq u_{n+1} \) for all \( n \geq N \) for some \( N \)

The \( n \)-th Taylor polynomial for \( f \) about \( x = a \) is \( p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!} (x - a)^k \)

The Taylor series for \( f \) about \( x = a \) is \( \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!} (x - a)^k \)

Symmetry Tests for Polar Graphs

1. Symmetry about the x-axis: If the point \((r, \theta)\) lies on the graph, then \((r, -\theta)\) or \((-r, \pi - \theta)\) also lies on the graph.

2. Symmetry about the y-axis: If the point \((r, \theta)\) lies on the graph, then \((r, \pi - \theta)\) or \((-r, -\theta)\) also lies on the graph.

3. Symmetry about the origin: If the point \((r, \theta)\) lies on the graph, then \((-r, \theta)\) or \((r, \theta + \pi)\) also lies on the graph.

Area Enclosed by a Polar Curve: \( A = \frac{1}{2} \int_{a}^{b} r^2 \, d\theta \)

Arc Length of a Polar Curve: \( L = \int_{a}^{b} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta \)