Instructions: No note or calculators are allowed. Answers must be supported by work on your exam sheets. Answers with little or no supporting work will receive little or no credit. Work must be neat, organized and easily interpreted.

A page of useful information has been included on the last page of the exam.

Please circle your final answers.

Notes:
0! = 1 and if \( n > 0 \) then \( n! = 1 \times 2 \times 3 \times \cdots \times n \)

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1. Evaluate the integrals:
   a. $(6 \text{ pts}) \int_0^1 x\sqrt{3x^2 + 1} \, dx$
   b. $(6 \text{ pts}) \int \cos^3(x) \, dx$
c. (6 pts) \( \int x \cos(7x) \, dx \)

d. (6 pts) \( \int \frac{30}{x^2 - 25x + 100} \, dx \)
2. (5 pts) Suppose that you use the substitution $x = 5 \tan \theta$ on an integral $I = \int f(x) \, dx$ and the integral in terms of $\theta$ is equal to

$$I = \sin \theta + \sec \theta + C$$

Find an expression for $I$ in terms of $x$ that does not include any inverse trigonometric functions.

3. (5 pts) Write the following improper integral in terms of the limit of a proper integral. Then determine whether the integral converges or diverges. If the integral converges, evaluate it.

$$\int_{0}^{\infty} e^{4x} \, dx$$
4. (6 pts) Sketch and shade in the region in the first quadrant bounded by the curves $y = \sqrt[3]{x}$ and $y = x^3$. Find the area of this region.

5. (6 pts) Sketch and shade in the region bounded by the x-axis and the curves $y = \sqrt{x}$, $x = 1$ and $x = 3$. Find the volume generated by revolving this region about the x-axis.
6. (6 pts) Let $R$ be the region in the first quadrant bounded above by $y = 2x$ and below by $y = x^2$. Set up, **but do not evaluate**, the sum of integrals that represent the perimeter of $R$.

![Graph of the region R]

7. (6 pts) The value of $\pi$ can be calculated as follows:

$$\pi = 4 \int_0^1 \frac{1}{1 + x^2} dx$$

Use Simpson’s Rule with $n = 4$ to estimate the value of $\pi$. Do **not** simplify your answer.
8. Determine whether the following series converge or diverge. Justify your answer by referencing AND applying the appropriate convergence test. Show that the requirements of the test being used are satisfied.

a. (6 pts) \( \sum_{k=0}^{\infty} \frac{5^k}{1+6^k} \)

b. (6 pts) \( \sum_{n=0}^{\infty} \frac{n^5}{n^5+2016} \)
9. (6 pts) Find the radius of convergence for the power series

\[ A = \sum_{k=1}^{\infty} \frac{(x - 2)^k}{k^3 3^k}. \]
10. (8 pts) The power series

\[ A = \sum_{n=0}^{\infty} \frac{(-1)^n(x - 1)^{2n+1}}{2n + 1} \]

has interval of convergence \(0 \leq x \leq 2\).

a. For which values of \(x\) does the power series \(A\) converge absolutely? Justify your answer by referencing AND applying the appropriate convergence test. Show that the requirements of the test being used are satisfied.

b. For which values of \(x\) does the power series \(A\) converge conditionally?
11. Maclaurin and Taylor series
   a. (6 pts) Find the Taylor series generated by $f(x) = x^4$ centered at $x = -1$.

   b. (5 pts) Using the fact that the Maclaurin series for
      \[ \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n + 1)!} x^{2n+1}, \]
      find the Maclaurin series for $2 \sin x \cos x$. Give your answer in **summation form**.
      (Hint: A trigonometric identity is very useful here.)
12. (5 pts) On the grid provided, plot the polar curve

\[ r = \sin 2\theta. \]

On the grid provided, mark and label the nine points on the curve where

\[ \theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi. \]

NOTE: \( \sqrt{2}/2 \approx 0.7 \quad \sqrt{3}/2 \approx 0.9 \)
Disk Method: \( V = \int_a^b \pi [R(x)]^2 \, dx \)  
Washer Method: \( V = \int_a^b \pi ([R(x)]^2 - [r(x)]^2) \, dx \)

Shell Method: \( V = \int_a^b 2\pi r(x) h(x) \, dx \)  
Surface Area: \( S = \int_a^b 2\pi f(x) \sqrt{1 + (f'(x))^2} \, dx \)

Arc Length Formula: \( L = \int_a^b \sqrt{1 + (f'(x))^2} \, dx \)

Useful Trigonometric Identities: \( \cos^2 \theta = \frac{1 + \cos 2\theta}{2} \); \( \sin^2 \theta = \frac{1 - \cos 2\theta}{2} \); \( \sin 2\theta = 2\sin \theta \cos \theta \)

The \( n \)-th Term Test for Divergence: \( \sum_{n=1}^\infty a_n \) diverges if \( \lim_{n \to \infty} a_n \) fails to exist or is different from zero.

The Limit Comparison Test: Let \( \sum a_n \) and \( \sum b_n \) be series with positive terms and suppose \( \rho = \lim_{n \to \infty} \frac{a_n}{b_n} \).

a) If \( \rho \) is finite and \( \rho > 0 \), then the series both converge or both diverge.

b) If \( \rho = 0 \) and \( \sum b_n \) converges, then \( \sum a_n \) converges.

c) If \( \rho = \infty \) and \( \sum b_n \) diverges, then \( \sum a_n \) diverges.

Ratio Test: Let \( \sum u_n \) be a series with nonzero terms and suppose \( \rho = \lim_{n \to \infty} \frac{|u_{n+1}|}{|u_n|} \).

a) If \( \rho < 1 \), the series converges absolutely.

b) If \( \rho > 1 \) or \( \rho = \infty \), the series diverges.

c) If \( \rho = 1 \), the series may converge or diverge.

Alternating Series Test: An alternating series \( \sum_{n=1}^\infty (-1)^n a_n \) or \( \sum_{n=1}^\infty (-1)^{n+1} a_n \) converges if the following three conditions are satisfied:

1) \( a_n > 0 \) for all \( n \)  
2) \( \lim_{n \to \infty} a_n = 0 \)  
3) \( a_n \geq a_{n+1} \) for all \( n \geq N \) for some \( N \)

The \( n \)-th Taylor polynomial for \( f \) about \( x = a \) is \( p_n(x) = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!}(x-a)^k \)

The Taylor series for \( f \) about \( x = a \) is \( \sum_{k=0}^\infty \frac{f^{(k)}(a)}{k!}(x-a)^k \)

Symmetry Tests for Polar Graphs

1. Symmetry about the x-axis: If the point \((r, \theta)\) lies on the graph, then \((r, -\theta)\) or \((-r, \pi - \theta)\) also lies on the graph.

2. Symmetry about the y-axis: If the point \((r, \theta)\) lies on the graph, then \((r, \pi - \theta)\) or \((-r, -\theta)\) also lies on the graph.

3. Symmetry about the origin: If the point \((r, \theta)\) lies on the graph, then \((-r, \theta)\) or \((r, \theta + \pi)\) also lies on the graph.

Area Enclosed by a Polar Curve: \( A = \frac{1}{2} \int_\alpha^\beta r^2 \, d\theta \)

Arc Length of a Polar Curve: \( L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta \)