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# Theory of Banded Waveguides

This article describes *banded waveguides*, a way of synthesizing sounds made by solid objects and an alternative method for treating two- and three-dimensional objects. It belongs to the synthesis algorithms known as physical models, and in particular, it is a departure from waveguide synthesis.

Physical modeling of musical instruments is a synthesis technique that is well established in computer music. Physical models are historically related to computationally expensive algorithms (Ruiz 1969) but have become more efficient with faster methods such as waveguide synthesis (Smith 2003). Digital waveguide models provide discrete-time models of distributed media such as vibrating strings, bores, horns, and plates.

We begin by outlining related synthesis methods with emphasis on traditional waveguide synthesis, which motivated the creation of this new structure. To simulate sustained and transient excitations such as striking, bowing, and rubbing, different excitation models are also proposed in this article. Instruments that have been modeled using banded waveguides are discussed in a companion article (Essl et al. 2003).

Computer Music Journal, 28:1, pp. 37–50, Spring 2004


## Digital Waveguides

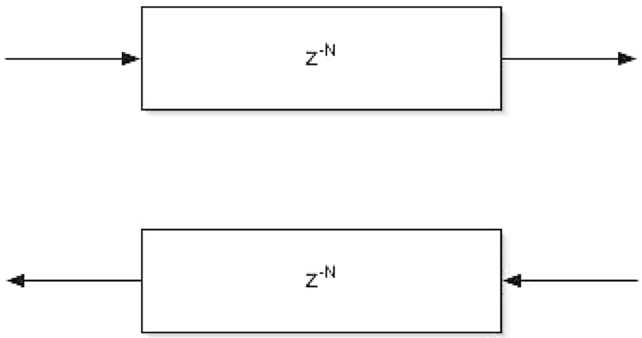
Figure 1 shows a one-dimensional digital waveguide. A lossless digital waveguide is a bidirectional delay line at some wave impedance, and each delay line element contains a sampled traveling-wave component (Smith 2003).

Efficient physical models of vibrating strings, wind instruments, and other quasi-harmonic systems have been implemented using digital waveguides. For a review of physical models using digital waveguides, see Smith (2003) and the references therein.

### Digital Waveguide Strings

Using digital waveguides, it is easy to create a physical model of a vibrating string. The structure of this model is shown in Figure 2. In this case, the delay line represents the sampled string in which traveling waves propagate. The low-pass filter is used to model losses along the string and at the ex-

Figure 1. A digital waveguide formed by two delay lines of N samples each with opposite wave propagation directions.




tremities. For simplification, we assume that the string is excited at one extremity. This corresponds to the original Karplus–Strong algorithm (Karplus and Strong 1983).

**Advantages and Disadvantages of One-Dimensional Waveguides**

Digital waveguides provide an efficient synthesis tool for quasi-harmonic resonators, which include vibrating strings when there is negligible or weak dispersion. In situations where stiffness is noticeable but not high, such as in piano strings, all-pass filters have been used to model the inharmonicity of overtones (Rocchesso and Scalcon 1996). The role of all-pass filters is to create a frequency-dependent propagation velocity, resulting in partials that are stretched in frequency.

For very stiff systems such as rigid bars, however, a combination of waveguides and all-pass filters provides a less efficient structure for sound synthesis. Some inharmonic structures such as bells (Karjalainen, Välimäki, and Esquef 2002) and higher-dimensional structures (Rocchesso and Dutilleux 2001) have recently been modeled using all-pass filters. In these cases, other synthesis techniques, such as spectral modeling synthesis (Serra 1986) or modal synthesis (Adrien 1991), have been used. Another approach to modeling complex resonators in higher dimensions is to use the waveguide mesh (van Duyne and Smith 1993), a generalization of the digital waveguide described in the following section.

## The Digital Waveguide Mesh

Figure 3 shows a two-dimensional digital waveguide mesh. It is a regular array of digital one-dimensional waveguides arranged along each perpendicular dimension, interconnected at their crossings by scattering junctions *J*. In the figure, each waveguide is one sample long. In addition to

Figure 2. A simplified waveguide model of a vibrating string. The delay line of N samples represents the string, and "LP" represents the low-pass filter that accounts for losses.

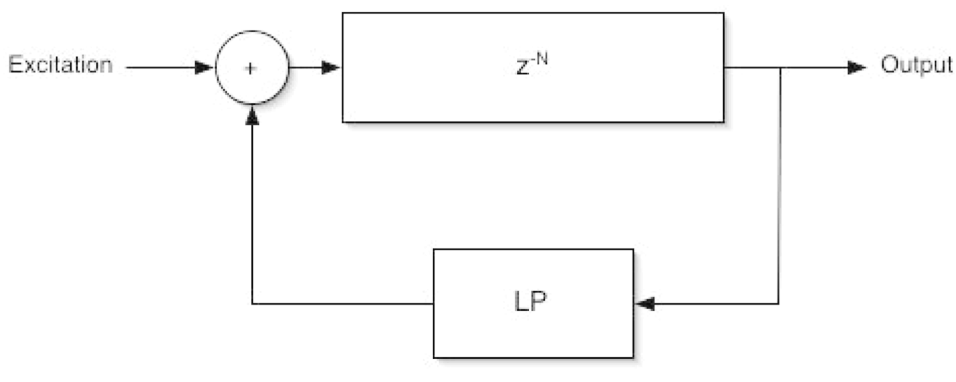
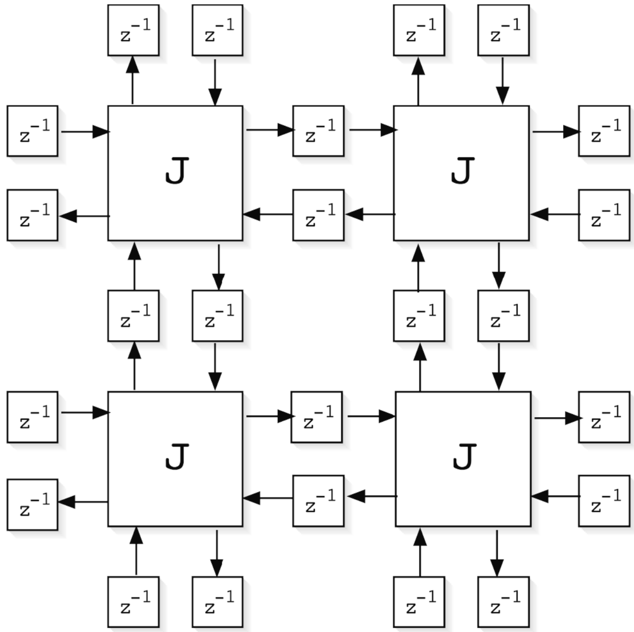


Figure 3. *A digital waveguide mesh.* J *represents scattering junctions connected by single-sample delays.*




the rectilinear mesh shown in Figure 3, other mesh geometries have been explored, such as triangular and tetrahedral.

## Advantages and Disadvantages of Waveguide Meshes

Digital waveguide meshes are a synthesis technique usually adopted to generate large numbers of modes. They are especially useful for modeling the high frequency modes of complex resonators. The computational cost of the waveguide mesh, however, may not allow the mesh to be used in real-time applications, especially in the case of large meshes, which are necessary to model resonators like reverberant rooms.

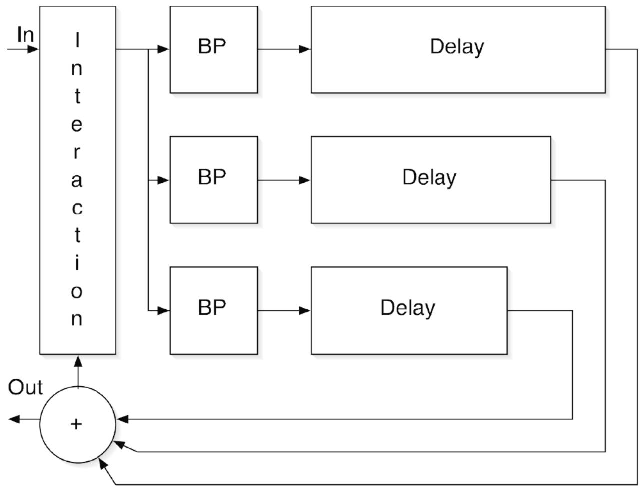
## Other Related Methods

A large class of real-world objects of interest for musical and non-musical sound simulation belongs to the category of highly inharmonic objects, not necessarily with a high number of modes. Examples include bars and metal objects. In these cases, a synthesis technique called *modal synthesis* is used, in which the sound simulation is based on the modal frequencies of objects. These modal frequencies are typically derived from measurements (van den Doel, Kry, and Pai 2001) or from the general equations used for finite element methods. A technique similar to modal synthesis is the *spectral modeling approach*, in which the spectral evolution of the partials of the object of interest is taken into account.

These methods are efficient and work well for all types of linear interactions, i.e., interactions that can be well described by impulsively adding energy to the system. For many complex interactions, however, a different description is desirable. A typical example of this is a sustained interaction such as bowing. In these situations, the interaction depends on the physical state (e.g., force, velocity, and displacement) of the object, like multiple bounces of interacting objects or strongly non-linear interactions like stick-slip friction. Moreover, synthesis techniques such as spectral modeling and modal synthesis do not directly simulate the way waves propagate in the vibrating object.

Another computational physical modeling technique called the *finite difference method* consists of ''discretizing'' differential equations that describe the object in space and time. The resulting system of equations then is solved. Depending on the type of ''discretization,'' this method is also known as the *finite* or *boundary-element method* (Chaigne and Doutaut 1997; Doutaut, Matignon, and Chaigne 1998; O'Brien, Cook, and Essl 2001). At present, these methods are computationally expensive. In addition, these methods are prone to instability owing to numeric imprecision, as disturbances are propagated by algebraic operations and truncation errors of multiplications accumulate over the number of multiplications. Waveguide synthesis methods propagate disturbances using lossless delay lines that are implemented by moving pointers, hence are not affected by accumulated truncation errors. Numerical errors resulting from algebraic operations are confined to digital filters's simulating losses owing to propagation, scattering, or boundary reflections. Because numerically ro-

Figure 4. A banded waveguide structure as proposed in Essl and Cook (1999).




bust digital filter structures are known, waveguide synthesis is well-suited for simulating highly resonant oscillatory phenomena.

In the following section, we propose banded waveguides as a synthesis technique offering unique advantages for certain kinds of modeling problems.

## Theory of Banded Digital Waveguides

As the name suggests, in banded waveguide synthesis, the spectrum of a passive or impulsively excited vibrating system is divided into frequency bands, each band containing primarily one resonant mode. For each band, digital waveguides are used to model the dynamics of the traveling wave and the resonant frequency of the mode. By retaining the wave dynamics, the synthesis algorithm can be used with nonlinear excitation models like violin bow friction models or reed models. Furthermore, by separating the spectrum, the method allows modeling of idiophones like marimba bars and bells.

An elementary banded waveguide structure is depicted in Figure 4. Compared to the waveguide structure of Figure 2, a single delay line is replaced by a number of delay lines (in Figure 4, three are depicted, but this number is not essential to the model), an interaction model has been added, and the low-pass filter has been replaced by a band-limited operation as discussed next.

The diagram shows four separate components that must be defined. It shows band-limiting (band-pass) operations (''BP''), delay operations (''delay'') and an interaction operation (''interaction''). In addition, the number of band-pass/delay pairs must be defined.

The most simple of these operations is the delay. Its function is simply to delay digital signals by a certain amount of time. The corresponding linear digital filter transfer function is

$$H_d(z) = z^{-d} \tag{1}$$

where $d$ is the delay-time. If the delay $d$ is an integer, this structure can be implemented efficiently using circular buffers (Dodge and Jerse 1985). Any non-integer fraction can be implemented using fractional delay techniques (Laakso, Välimäki, Karjalainen, and Laine 1996).

The band-limiting operation is achieved by using band-pass filters, among which we choose second-order resonant filters, because they are the simplest and most efficient (Steiglitz 1996). As will be discussed later, the band limiting is not a critical operation, and hence a computationally inexpensive second-order filter is sufficient. In essence, the important parameters are the center frequency of the pass band and the gain at that frequency, while the bandwidth is typically less critical. Among the second-order options offered in Steiglitz (1996) , we take for illustrative purposes the two-pole version. Its transfer function is given by:

$$H_2(z) = \frac{1 - z^{-2}}{1 - (2R\cos\theta)z^{-1} + R^2 z^{-2}} \tag{2}$$

where $R$ and $\theta$ are free parameters of the poles that relate to bandwidth $B$, center frequency $\psi$, and gain $A_0$ in the following way (Steiglitz 1996):

$$R \approx 1 - B/2 \tag{3}$$

$$\cos\theta = \frac{2R}{1 + R^2}\cos\psi \tag{4}$$

$$A_0 = (1 - R^2)\sin\theta \tag{5}$$

Hence the free parameters are length of the delay line $d$, resonance frequency $\psi$, gain of the band-pass $A_0$, and the bandwidth $B$.

The gain $A_0$ is a damping factor that relates to the multiplicative loss per period of oscillation. The length of the delay line is usually tuned to the frequency $f_m$ of a mode to be modeled. The relationship between $f_m$, sampling frequency $f_s$ and delay line length $d$ is:

$$d = f_s / f_m \tag{6}$$

To calculate the band-pass parameter $R$, the same modal frequency $f_m$ is used and converted into radians:

$$\psi = 2\pi f_m / f_s \tag{7}$$

Among these parameters, only the bandwidth $B$ does not have a strict physical interpretation. It is usually chosen to sufficiently reject other modes of the comb response of a feedback delay-line filter.

The idea behind banded waveguides is to create a hybrid of modal and waveguide synthesis in which each waveguide models the propagation of waves around a particular spatial mode of the system. For this reason, as shown in Essl et al. (2003), banded waveguides are useful for modeling instruments such as struck bars in which many modes are excited initially, but only a few strongly inharmonic modes are dominant after the attack. The original application of banded waveguides was the separation of a normal waveguide structure into a superposition of band-limited waveguide structures that accommodate different wave-propagation speeds at different frequencies in resonating medium such as a bar.

Although this step is the core of the generalization, in typical applications of banded waveguides, a second step is taken. The frequency bands are centered around the dominant modal frequencies of the instrument to be modeled. This step is motivated by the principle of ''closed wavetrains.'' This principle states that a mode occurs when a traveling wave closes onto itself and the frequencies of the modes can be derived by finding the waves that close onto themselves with the same phase. This principle is well known (see Cremer, Heckl, and Ungar 1988) and is important in chaos theory and modern dynamical theory (Essl 2002). More details on the historical development of these ideas can be found in Essl and Cook (2003).

Using this principle, the design of banded waveguides becomes straightforward. The modes of a particular instrument are analyzed using spectrum analysis or modal analysis methods, and the parameters of each mode are used to configure a banded waveguide structure to create the synthesis method. If a waveguide creates a mode, any multiple of the length of that waveguide will also create that same mode.

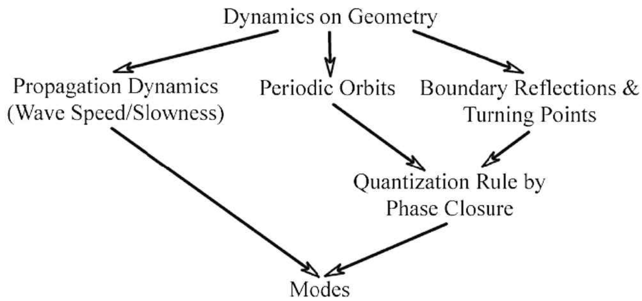
There is some ambiguity in the relationship of waveguide length to the created mode. The choice of waveguide length can be expected to influence transients, while the late impulse response remains the same (a single mode). Unless precise attack reconstruction is desired, this ambiguity can be ignored. This ambiguity can often be resolved if knowledge of the mode shapes or the wave-propagation speed is available.

The band-pass filter eliminates neighboring peaks in the comb-filter response created by the closed delay-line loop. We have found that a simple second-order resonant filter works well in many situations. If the wave-propagation speed is constant for all frequencies, then these band-pass filters are not required. In this case, banded waveguides are equivalent to conventional waveguide synthesis using one feedback delay line.

The banded waveguide model has a number of desirable properties. First, if memory use is not restricted, it has approximately the same computational complexity as modal synthesis, because the delay lines only add a small constant number of operations per time step independent of their length (Dodge and Jerse 1985).

Moreover, the model is ''bowable'' in the same way that waveguides are ''bowable,'' because the ''pulse timing,'' which synchronizes the bowed string stick-slip process, is preserved to some extent, unlike with modal synthesis (Avanzini, Serafin, and Rocchesso 2002; Essl 2002). Finally, it inherits the numerically desirable properties of waveguide synthesis. An interpretation of the banded waveguide principle and its relationship to

Figure 5. Relationship of geometry and dynamics to modes.




the literature of dynamical systems can be found in Essl (2002).

**Spatial Information in Banded Waveguides**

The relationship between mode frequency and speed of the traveling wave responsible for that mode is not unique. It depends both on the distance the wave must travel and the time it takes to travel that distance. The additional information to remove this ambiguity can either be measured or derived from constituent equations. When the traveling speed $c$ and the traveling length $x$ are known, the traveling time $d$ can uniquely be calculated as $d = x/c$. If this information is not known, we can take a good guess and use that guess in the simulation. The system's spectrum will be modeled correctly, but the response time may differ. This affects only the transient response of the system, as the modes "come in" or "speak" either too fast or too slowly, but once the mode is established, the mode remains unaffected. For nonlinear interactions such as bowing, this means that an object may lock to a mode more quickly or slowly than expected, but it will not affect the fact that it locks to the mode. Another way to understand the same point is by considering the effect of the ambiguity on the time-domain, steady-state response. The ambiguity only affects how many full periods of modes are stored in a delay line; hence, at the bowing point, the time-domain function looks the same, as they are self-similar over any multiple of full period oscillations. Therefore, the result sounds and behaves qualitatively correct after some onset latency.

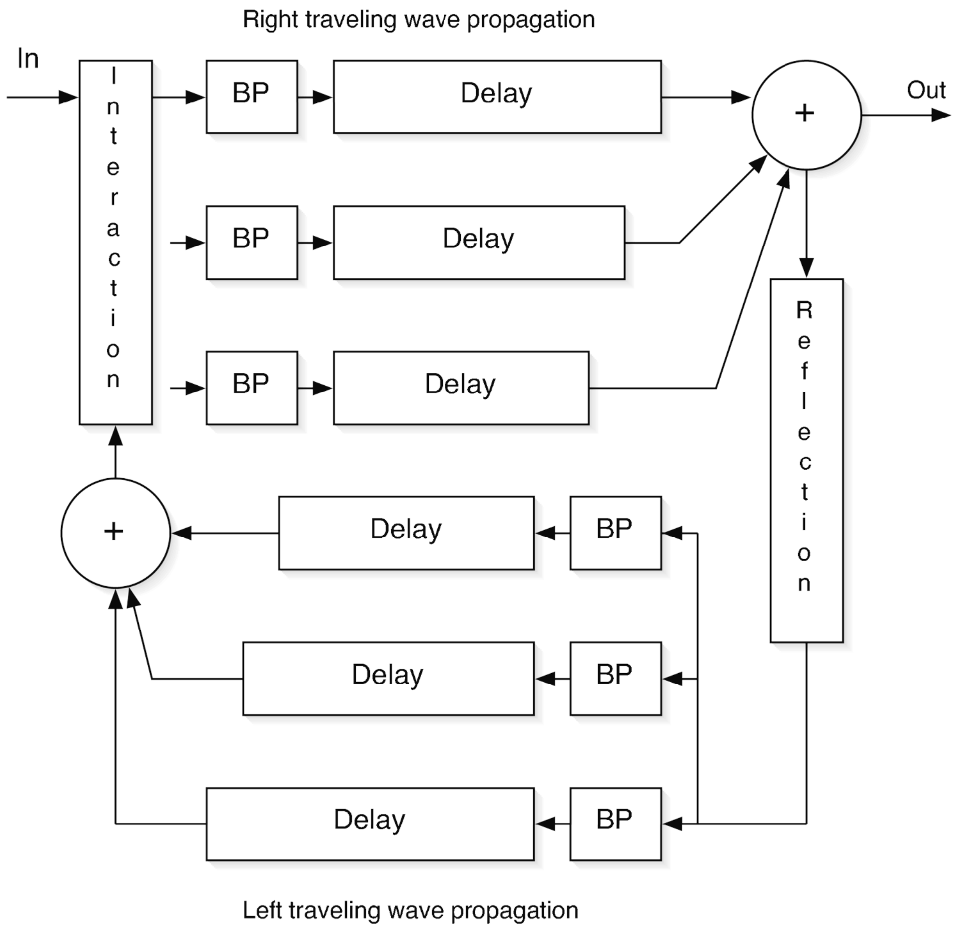
If the goal is to simulate existing musical instruments, then one issue is how to optimize the integer multiple delay-lengths to best match the physics of the instrument. The connection between geometry, dynamics, and modes as primary components of sound production is summarized in Figure 5. The dynamics of a given geometry can be decomposed into three aspects: the speed of propagation of traveling waves, the length and topology of closed paths (here called *periodic orbits*), and the additional phase changes that occur while a wave traverses the path owing to boundaries and other effects. The combination of the last two aspects defines the effective path length, which in turn defines what frequencies of traveling waves will form standing waves and hence "quantize" the path. (These are called quantization rules in physics; see Brack and Bhaduri [1997].) Together with the speed, the actual frequency can then be calculated.

The connection between components in Figure 5 can follow two directions. First, following the direction of the arrows corresponds to a construction of a simulation from complete dynamical and geometrical description. In this case, the tasks are (1) to find a closed-loop path; (2) to identify boundary conditions and turning points and their phase contributions to path length; and (3) to find wave propagation speed characteristics.

However, this is not necessarily the most practical approach. Finding closed-loop paths can be difficult, and the theoretical dynamics of a complex instrument may not be known. Hence, it may be difficult to find the wave propagation speed characteristics. Finally, this construction is not precise; it is only approximate. Thus we propose that modal measurements are always used to allow precise tuning, and the path-length construction is used to disambiguate the delay lines's lengths.

As banded waveguides are a spectrally decomposed version of digital waveguide synthesis, the same constructions for digital waveguides still hold. For example, Figure 6 shows two concatenated, banded waveguide structures that illustrate the propagation of waves to a reflection point. The reflection interaction is explicitly modeled, as is the propagation back to the starting point. Note how by splitting traveling paths, interaction and observation

*Figure 6. A banded waveguide system including explicit modeling of the reflection.*




points can be made different, as can also be seen in the location of the output in Figure 6.

## Banded Digital Waveguides in Higher Dimensions

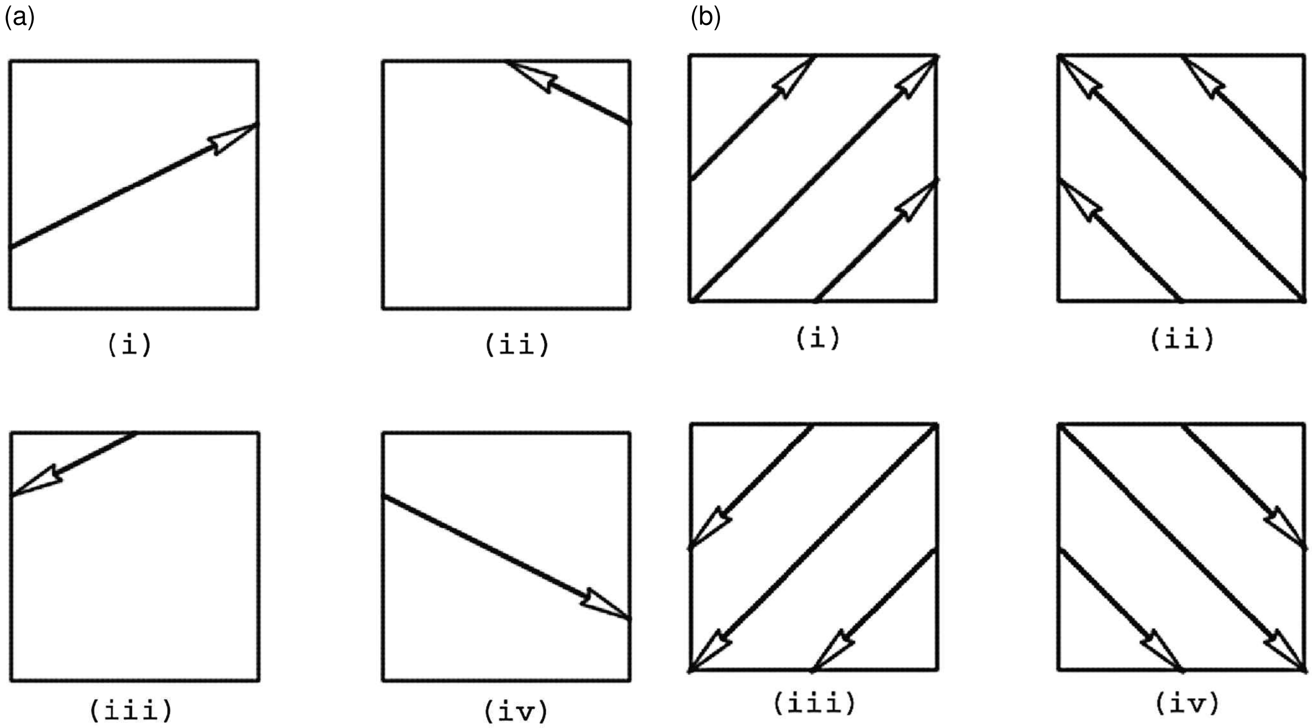
The principle of closed wavetrains states that modes and closed paths correspond. This is also true in higher dimensions, in which case spatial information of a banded waveguide model of an instrument whose resonator is two- or three-dimensional requires finding which geometric paths correspond to which mode.

We start with the notion of ray paths in higher dimensions as opposed to wave-front propagation for a number of reasons. Waves traveling on rays can be ''discretized'' using waveguide structures, which are computationally very efficient and well understood. Also, a principle directly linking ray paths and resonance—the principle of closed wavetrains—is known and easily applicable. Finally, a body of literature exists following this approach. For a detailed discussion of the historic development of this literature, see Essl (2002) and Essl and Cook (2003) and the references therein.

Solutions for square, circular, and elliptical membranes are well known. Musical instruments are usually highly symmetric, so one can hope that a precise or at least an approximate construction can often be found.

It should be noted that in general, this is however a difficult problem and relates to Kac's famous

Figure 7. (a) Reflections of a single ray; (b) types of reflections for ray families.




question "Can you hear the shape of a drum?" (Kac 1966). It has since been found that for certain asymmetric, non-smooth boundaries that a spectrum does not uniquely match one geometric shape (Driscoll 1997).

Finding these paths has been studied in various contexts; specifically, it was proposed by Rocchesso as a synthesis paradigm (1995). The constructions for the square and circular membrane were presented by Keller and Rubinow (1960).

**Rectangular Membranes**

As an example, let us examine a rectangular domain. Starting with a family of parallel rays "shooting off" at a certain angle, geometric reflection just means an inversion of the travel direction normal to the boundary. For our present discussion, the intuitive notion of a ray as a line trajectory in space suffices. A rigorous treatment can be found in Keller and Rubinow (1960) and Chapman, Lawry, Ockendon, and Tew (1999).

This behavior is depicted in Figure 7. Figure 7a shows the first four reflections of a ray (i–iv). Figure 7b shows the same general behavior for a whole family of rays mirroring the same directional states given in *i* to *iv*. This picture should be interpreted as states rather than as actual ray paths, meaning there are four possible propagation configurations that change depending on reflections at the boundary. Any family of rays with an angle different than 90° will eventually reflect and change directions. As a ray propagates through the domain, it can reflect many times until it closes onto itself, but it still can only be in one of these configurations. This repeating configuration is usually interpreted topologically and then used to study resonant behavior of closed paths. This constructions will not be presented here but can be found in Essl (2002).

Obviously, these ray-families share the same closed path-length and integer ratios of bounces in the up–down to left–right dimension, yielding closed paths of finite length. (Irrational ratios yield infinitely long closed paths.) By combining the path length of the closed loop with the phase change

*Figure 8. Topological transformation of two independent geometric variables as coordinates of a rectangle (i), to bent planes (ii, iii), and to angle and radius of a circle (iv).*

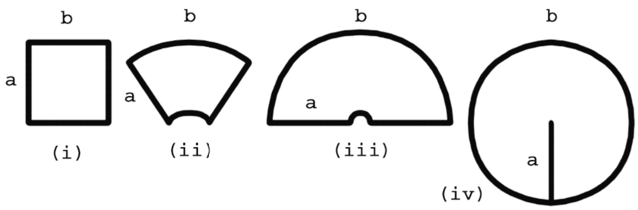
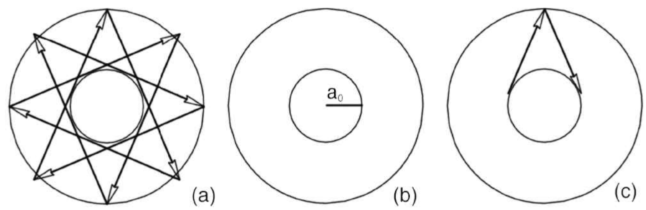



*Figure 9. Path construction on the circular domain (cf. Keller and Rubinow 1960, Figures 3 and 5). (a) Closed path touching the interior circular region; (b) a purely circular path; (c) path containing rays traveling from interior circular region to boundary and back.*




owing to wave reflection conditions at the boundary, the frequency of the mode can be calculated.

For each closed traversal in one dimension, the phase changes twice. If $\theta$ is the angle of the ray family from its horizontal dimension, $a$ is the length of the rectangular region in that dimension, and $k$ is the sought wave number, then the total reversed phase between two reflections will be $2ka \cos\theta$. The closure condition demands that the traversal phase is a multiple of $2\pi$. The same also holds for the vertical direction of length $b$. Hence we get

$$2ka \cos \theta = 2\pi n_1 \tag{8}$$

$$2kb \sin \theta = 2\pi n_2 \tag{9}$$

Eliminating the angle and calculating the frequency $f$ from the wave number with the speed of sound $c$ yields

$$f = \frac{c}{2\pi}\ k = \frac{c}{2}\sqrt{\frac{n_1^2}{a^2} + \frac{n_2^2}{b^2}} \tag{10}$$

where $n_1$ and $n_2$ are integers. This result corresponds to the well-known solution for modal frequencies of the rectangular membrane. See also Zhou and Chen (1991) for a similar calculation.

### Circular Membrane

Paths of rays on a circular membrane can be constructed in a related fashion. Both examples have two independent spatial dimensions. In fact, a topological mapping of a rectangle to a disk can be achieved by shrinking one edge to point size while forming a closed circle with the opposite edge and "gluing" the remaining edges together, as depicted in Figure 8. Such a topological mapping is called *homeomorphism* and is not limited to these particular shapes.

Again, all rays with the same reflection angle at the boundary belong to a family of rays with the same resonant properties. Any ray with a non-zero radial propagation component (see Figure 9a) thus alternates between reflecting at the boundary and an enclosed circular region. An integer ratio between radius and angular component yields finite length paths.

## Application to Non-Physical Entities

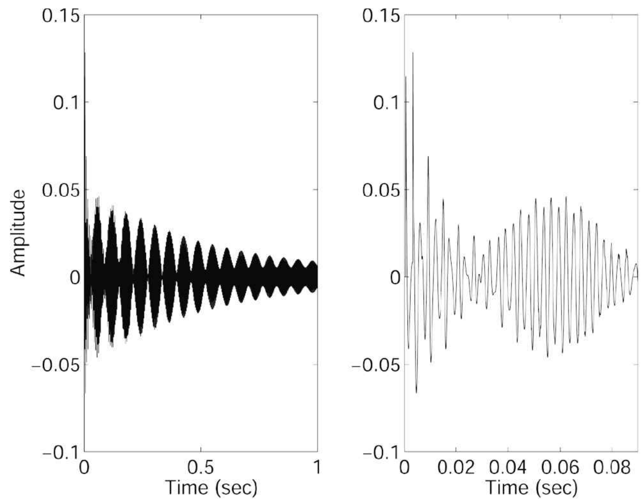
The abstract structure, and even the filter interpretation thereof, does not necessarily need a physical interpretation. In this case, banded waveguides can be seen as a purely abstract synthesis method with certain properties. The choice of parameters then becomes aesthetically motivated rather than physically motivated.

An example of non-physical application of this structure, discovered by accident, is the following: if the sum of the band-limited delay lines is numerically integrated (accumulated sum), and the appropriate "physical" interpretation of the result of reconstruction becomes displacement, we obtain a quantity that is not properly physically informed. If this integrated displacement is then fed into a standard bowing interaction model, rich, chaotic sounds can be produced in a stable manner. Non-physical applications of banded waveguides are discussed in more detail in Essl (2003).

### Beating Banded Waveguides

Beating occurs in objects, both physical and non-physical, for various reasons. The modes of the ob-

Figure 10. (Left) Evolution of an isolated simulation of a beating mode pair; (right) initial transient and the first beating period.




ject itself can be so close that they form a beating pair. Without this small detuning between modes, a phenomenon called *degeneracy* appears in which two different modes have almost the same frequency. The interaction may also be low in frequency and hence have a beating envelope with respect to the sound produced by the instrument. Beating has been modeled in the context of coupled piano strings (Bank 2000) and plucked strings with sympathetic coupling (Karjalainen, Välimäki, and Tolonen 1998). In this article, we describe another possibility of implementing beating for banded waveguides.

The beating modes combined with weak damping pose a challenge. For two neighboring banded wave paths whose center frequencies converge, the respective frequency bands start to overlap strongly. This means that energy will contribute to traveling waves in both bands simultaneously. Beating can be implemented as two banded waveguides slightly detuned in frequency.

To guarantee stability within the frequency region, the sum gain of both waveguides cannot exceed unity, as both are added together for interaction or feedback. More specifically, the gain of the respective banded wave paths can be calculated from the maximum of the overlapping band-pass filter amplitude characteristics. This maximum must be tuned to the desired gain, and the respective gains of the band-pass filters are adjusted by the weight of the overlap.

The simulation of an isolated beating mode pair can be seen in Figure 10. The relative ratio between the modes is 1:1.05.

Essl et al. (2003) presents an application of beating banded waveguides to the Tibetan bowl.

## Banded Waveguide Mesh

As described in the previous section, banded waveguides are an efficient synthesis technique for inharmonic structures where the number of modes is relatively low. In situations in which complex resonators with many modes are considered, an extension of banded waveguides called the *banded waveguide mesh* has been proposed (Serafin, Huang, and Smith 2001).

Figure 11 shows the structure of a banded waveguide mesh. The banded waveguide mesh is a generalization to multiple dimensions of the banded waveguide to efficiently implement a complex resonator without using too many banded waveguides.

As in the case of banded waveguides, we divide the spectrum of a vibrating system into frequency bands. For frequency bands where a single resonance is present, a one-dimensional digital waveguide is used. For bands where resonances are more complex, we use either a two-dimensional or three-dimensional digital waveguide mesh whose dimensions are chosen to match statistically and psychoacoustically the resonances of the modeled object. This creates a connection of banded waveguides and waveguide meshes. As in the one-dimensional case, reflection filters are included in the structure to achieve desired decay characteristics.

Let $f_c$ be the cutoff frequency above which an adequately high concentration of modes appears in the spectrum, and let $f_{0m}$ be the fundamental frequency of the mesh, i.e., the lowest mode generated by the mesh. In all applications, $f_c \geq f_{0m}$, so the role of the waveguide mesh is to model resonances above $f_c$.

Note that the higher the value of $f_{0m}$, the smaller the dimensions of the waveguide mesh and the more efficient the implementation. The choice of

Figure 11. A banded waveguide mesh with two one-dimensional digital waveguides and a digital waveguide mesh.

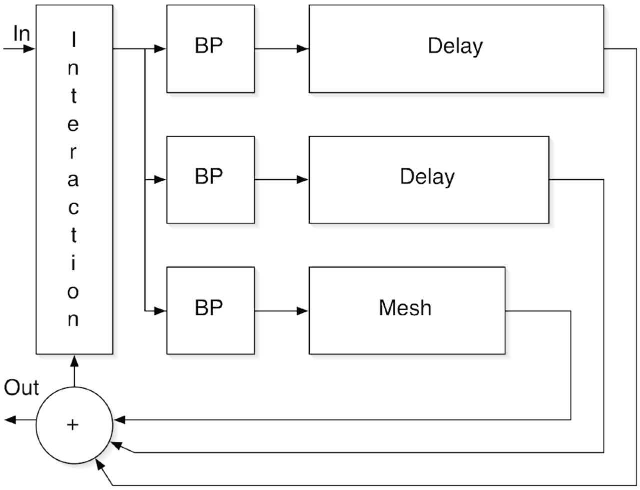



Figure 12. Exciter and resonator connected in a feed-forward loop.


$f_c$, therefore, is an important decision that affects the resulting computational cost.

The banded waveguide mesh has been applied to the modeling of bowed plates and bowed cymbals, as described in Essl et al. (2003). A structure similar to the banded waveguide mesh has also been used to model the body of complex resonators such as the violin (Huang, Serafin, and Smith 2000).

So far, we have described only models for the resonating part of musical instruments. In the following section we describe how the excitations are modeled and coupled to the banded waveguides.

## Modeling the Excitation

The banded waveguides described in the previous section can be excited either by a sustained or a transient mechanism. In this section, we examine how to model transient inputs such as hitting the resonators and sustained inputs such as frictional interactions between dry surfaces.

### Modeling a Transient Excitation

While attempting to simulate complex transient excitations, we first experimented with physical models proposed in the literature (Marhefka and Orin 1999; Avanzini and Rocchesso 2001; van den Doel, Kry, and Pai 2001), but we noticed they were not able to reproduce faithfully the strength of the impact between hard surfaces such as a metal and a hard mallet, as in the case of the instruments proposed in Essl et al. (2003). We therefore decided to use a spectral approach. Using a force hammer, we recorded the impulse response of the instruments while struck at different positions and with different excitation forces. We then analyzed the frequency response to detect the main resonance frequencies. By using inverse filtering through second-order notch filters, we removed such frequencies from the impulse response, which gave us samples of different excitations at different positions (i.e., the residual). The residual obtained was modified through a filtering procedure in the synthesis step according to the input parameters, i.e., the excitation force and position.

Such transient excitation was fed into the resonator in a feed-forward loop, as shown in Figure 12.

A similar approach is also proposed in Cook (2002). Here, the excitation duration is modeled with a filter excited by noise.

### A Physical Model of the Sustained Excitation

Banded waveguides can be excited through a ''stick-slip'' process that is similar to the one of a violin bow exciting a string. The frictional interaction between dry surfaces is a phenomenon of interest to various fields of engineering in which friction is considered an unpleasant source of noise and instabilities that must be reduced.

In the literature, velocity-dependent friction models have been used for their simplicity yet ability to reproduce most of the phenomena that derive from friction. Recently, dynamic friction models have been proposed (Hayward and Armstrong 2000) and applied to the sonic simulation of rigid bodies in frictional contact (Avanzini, Serafin, and Rocchesso 2002).

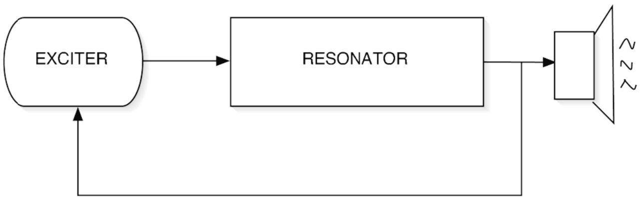



*Figure 13. Exciter and resonator connected in a feedback loop.*

In this article, we use the velocity-dependent friction curve:

$$\mu = \mu_d + \frac{(\mu_s - \mu_d)v_0}{v_0 + v_{rel}} \tag{11}$$

where $\mu_d$ and $\mu_s$ are the static and dynamic friction coefficients, respectively, $v_0$ is the initial velocity of the excitation, and $v_{rel}$ represents the relative velocity between the exciter and the resonator.

The values of the friction coefficients that depend on the characteristics of the materials in contact are taken from Rabinowicz (1995). In this friction model, the unknown variable is the velocity of the waves propagating in the resonator. By using the same time-domain simulation first proposed in McIntyre, Schumacher, and Woodhouse (1983), it is possible to obtain the new values of the dynamic friction coefficient and the velocity waves by solving a system of two equations. The first is a linear equation that represents the velocity waves propagating along the resonant structure, and the second is a nonlinear equation that is the friction function of Equation 11.

Despite its simplicity, the friction model of Equation 11 gives satisfactory results from a perceptual point of view. Moreover, this model allows us to analytically solve the coupling between the friction curve and the waves propagating along the waveguides. This is an important advantage for efficiency in real-time implementation and for avoiding numerical errors. The waveguide resonator is coupled to the friction excitation in a feedback loop as shown in Figure 13.

## Conclusions

In this article, we proposed banded waveguides as an efficient technique to model complex resonators with few modes. The theory of banded waveguides is by no means a closed chapter. Many open questions regarding the scope and applicability of this technique remain. For instance, what kind of geometries can be modeled usefully using banded waveguides? Although the difference between banded waveguides and traditional waveguides is clear, some questions may still be raised on the advantages of banded waveguides versus modal synthesis. First of all, in banded waveguides, the excitation's position does not need to be imposed on the modes of the structure by varying their relative amplitudes, but rather it derives naturally from the structure of the model itself. Moreover, in banded waveguides, the transient attack is enriched by the fact that, in each waveguide, the fundamental frequency of each mode and its harmonics appear for few msec before the band-pass filtering takes effect. In this way, we achieved interestingly rich interactions between rubbed surfaces without using a complex friction model. A companion article Essl et al. (2003) presents a number of successful applications developed by the authors so far. We hope that with future explorations of theory and applications, we will learn more about this and see the range of applications, advantages, and limitations be clearly and fully explored.

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