Abstracts

Thirty-Sixth Annual Workshop in Geometric Topology

University of Wisconsin–Milwaukee

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Principal Lectures

Triangulations, homology cobordisms, and Floer theories

Ciprian Manolescu (University of California, Los Angeles)

Titles and abstracts of the individual talks are as follows:

Talk 1: Triangulations of manifolds

Which manifolds are triangulable? Can any two triangulations of the same manifold be made equivalent after subdivision? I will survey the history of these questions, and explain the role of the three dimensional cobordism group in the study of triangulations.

Talk 2: Seiberg-Witten theory and homology cobordism

I will explain how invariants from gauge theory give information about homology cobordism. In particular, Pin(2)-equivariant Seiberg-Witten Floer homology can be used to prove that there are no elements of order 2 and Rokhlin invariant one in homology cobordism. This implies that non-triangulable manifolds exist in all dimensions at least five.

Talk 3: Involutive Heegaard Floer homology

I will describe an invariant of 3-manifolds called involutive Heegaard Floer homology, which gives further information about homology cobordism. The invariant was constructed in joint work with Kristen Hendricks, and has recently led to a proof (by Dai-Hom-Stoffregen-Truong) that the homology cobordism group has an infinite-rank summand.

Twenty Minute Talks

Spaces for which cohomological dimension equals dimension

Ric Ancel (UW-Milwaukee)

It is known that for compact ANRs, cohomological dimension (over $\mathbb{Z}$) equals dimension. We discuss the background and the confused provenance of this result. We then explain how to expand its scope to a class of locally compact spaces which is rife with non-ANRs – the locally compact approximate C-spaces.

Coarse Alexander Duality

Arka Banerjee (UW-Milwaukee)

Classical Alexander Duality says cohomology of complement of a subspace in Euclidean space is determined by the homology of the subspace. John Roe introduced the notion of coarse (co)homology of a metric space which encodes information about the large scale geometry of the space. This is joint work with Boris Okun.

From EDT0L languages to generating functions

Eric Freden (Southern Utah University)

Lindenmayer systems (L-systems) constitute grammars for generating formal languages. Unlike the grammars seen most often in computer science or combinatorics, L-systems are highly parallel and have been used to model biological organisms and chemical processes. Their use in combinatorics goes something like this: given a stratified combinatorial structure (e.g. the word problem in some group or a triangulation of an infinite geometric object), encode the structure as a formal language of words, find a grammar that generates that language, compute the number of number of words of length n for each $n > 0$.

This talk explores a new method to compute the number of words of length $n > 0$ for the generalized L-system family known as EDT0L. Several examples will be shown.

What is sheaf cohomology?

Greg Friedman (Texas Christian University)

This will be a very brief, but hopefully gentle, introduction to sheaf cohomology and some of the things it is good for. No prior experience with sheaves will be expected or permitted.

Discrete harmonic maps between hyperbolic surfaces

Jonah Gaster (McGill University and UW-Milwaukee)

Celebrated work of Eels-Sampson and Hartman asserts the existence of a harmonic diffeomomorphism in any homotopy class of maps between a pair of homeomorphic compact hyperbolic surfaces. The study of harmonic maps has since been vastly generalized in the work of Gromov-Schoen, Korevaar-Schoen, and Jost, with wide-ranging application. With the goal of producing the harmonic map in practice, we pursue a discrete version of the theory. Along the way towards showing convergence of the discrete harmonic maps to the smooth one, we estimate a positive modulus of convexity for the discrete energy functional (a property not known for the smooth energy functional). These ideas are implemented in a user-friendly computer program that I will present. This is joint work with Brice Loustau and Léonard Monsaingeon.

Boundaries of spaces

Pawel Grzegrzolka, (University of Tennessee)

To investigate large-scale properties of spaces (e.g., metric spaces, coarse spaces) topologists often assign a topological space (usually called "a boundary" or "a space at infinity") to the original space. That assigned topological space typically reflects many large-scale properties of the original space. In this talk, we will introduce a common language for studying the boundaries of many spaces, including proper metric spaces, locally compact Hausdorff spaces, and complete proper CAT(0) spaces. First, we will introduce coarse proximities, which capture the intuitive notion of two sets being "close at infinity." Then, we will construct boundaries of coarse proximity spaces. Finally, we will give 4 natural coarse proximity structures whose boundaries are the Higson, Freudenthal, Gromov, and Visual boundary.

Fibrations of $\mathbb{R}^3$ by lines

Michael Harrison (Lehigh University)

Is it possible to cover 3-dimensional space by a collection of lines, such that no two lines intersect and no two lines are parallel? More precisely, does there exist a fibration of $\mathbb{R}^3$ by pairwise skew lines? We discuss a topological classification of these skew fibrations, closely related to the classification of great circle fibrations/Hopf fibrations. We briefly summarize some recent results regarding contact structures on $\mathbb{R}^3$ which are naturally induced by skew fibrations and also briefly discuss the problem without the skew condition, and the problem in higher dimensional space.

Trisections of 4-manifolds

Adam Howard (Montana State University)

We'll discuss some diagrammatic techniques for studying 3-manifolds and then move on to see the analogous tools for studying 4-manifolds.

Dynamics and Topology of Conformally Anosov Contact 3-Manifolds

Surena Hozoori (Georgia Institute of Technology)

We will use Hofer's work in contact dynamics and certain Conley-Zehnder index computations to give the first contact topological results about conformally Anosov contact structures. Conformally Anosov flows, introduced by Thurston-Eliashberg and Mitsumatsu, seem to be more of topological interest than their more well studied special case, Anosov flows and it is natural to ask about the consequences of having such dynamical property on the Reeb vector field of a given contact 3-manifold. We will see such contact manifolds are universally tight, irreducible and do not admit exact cobordism to the tight sphere. Such study also serves motivations from Riemannian geometry of contact structures and in particular contributes to the Chern-Hamilton conjecture on the energy functional of Riemannian structures compatible with a contact 3-manifold.

Describing surfaces and isotopies in 4-manifolds via banded unlinks

Mark Hughes (Brigham Young University)

There are a number of well-established ways to represent knotted surfaces and isotopies between them in S^4, including motion pictures with movie moves, or broken surface diagrams with Roseman moves. In this talk I will discuss another method of representing surfaces in 4-space via banded unlink diagrams, which can also be used to describe surfaces in an arbitrary oriented 4-manifold X. I will present a set of moves which are sufficient to relate any two banded unlink presentations of isotopic surfaces in X, which generalizes a theorem in S^4 due to Swenton. As an application of this theorem we prove that bridge trisections of surfaces in 4-manifolds are unique up to perturbations. This is joint work with Seungwon Kim and Maggie Miller.

Countable approximation of topological G-manifolds in dimensions three and four

Qayum Khan (Indiana University)

Let G be a matrix group. Topological G-manifolds with Palais-proper action have the G-homotopy type of countable G-CW complexes. This generalizes E Elfving's dissertation theorem for locally linear G-manifolds (1996). Also we improve the Bredon–Floyd theorem (1960) from compact to all Lie groups G.

Braid index of knotted surfaces

Sudipta Kolay (Georgia Institute of Technology)

Braided surfaces play a similar role in understanding the knotted surfaces in $\mathbb{R}^4$, as classical braids for knots in $\mathbb{R}^4$. In this talk, I will introduce the notion of braided surfaces, and discuss properties of braid index of knotted surfaces and contrast it with that of classical knots.

Smooth and symplectic isotopy on rational 4-manifolds

Jun Li (University of Michigan)

We study rational 4-manifolds and its symplectomorphism group. Analogous to the 2-dimensional case, one can think about the symplectic Torelli group and symplectic mapping class group, where the former is the subgroup of the symplectomorphism group fixing homology and the latter is the symplectomorphism group mod symplectic isotopy. We'll show how the symplectic mapping class groups of rational 4-manifolds are related to braid groups. Also, McDuff-Salamon asked whether the symplectic Torelli group of any rational 4-manifold is smoothly isotopic to identity. And we answer it in the positive.

Cheeger-Gromov L^2 rho-invariants of 3-manifolds

Geunho Lim (Indiana University)

In this talk, we first recall the definition and properties of Cheeger-Gromov L^2 rho-invariants including the existence of universal bounds. Cha gave lower bounds for the complexities of 3-manifolds using universal bounds for Cheeger-Gromov L^2 rho-invariants. We discuss Cha's methods and how to improve Cha's lower bounds for elliptic 3-manifolds.

Approximate Inverse Limits and $(\mathbf{m}, \mathbf{n})$-dimensions

Matthew Lynam (East Central University)

In 2012, V. Fedorchuk, using m-coverings, introduced the notion of the *(m,n)-dimension* of a space. It generalizes covering dimension. Here we are going to look at this concept in the setting of approximate inverse systems of compact metric spaces. We give a characterization of (m, n)-$\dim X$, where X is the limit of an approximate inverse system, strictly in terms of the given system.

Coarse Extension Theory and some C*-algebras

Atish Mitra (Montana Technological University)

Coarse Extension Theory attempts to understand the coarse geometry of spaces through extensions of certain classes of functions between them. In this talk we give an overview of this and reformulate this problem in C^*-algebraic terms.

Genera of knots in $\mathbb{C}P^2$

Jacob Pichelmeyer (Kansas State University)

We define the $\mathbb{C}P^2$ genus of a knot K to be the least genus among all orientable surfaces smoothly and properly embedded in $\mathbb{C}P^2$ with boundary K. Through fusion, we are able to construct slice discs directly for several knots of low crossings. By combining genus bounds and degree obstructions of Ozvath and Szabo, Kronheimer and Mrowka, Gilmer and Viro, Rohlin, and Yasuhara, we are able to obstruct sliceness of a knot in $\mathbb{C}P^2$ when information is known about it's mirror. To clearly illustrate these methods, we will consider the particular knots 7_3, $m7_3$ and compute their $\mathbb{C}P^2$-genera. This material is the basis of the speaker's current dissertation work and is ongoing, with a promising improvement to obstruction being currently explored.

Salem numbers and arithmetic hyperbolic groups

John Ratcliffe (Vanderbilt University)

This talk is about discrete subgroups of the group of isometries of hyperbolic n-space. In this talk, I will discuss a direct relationship between certain algebraic integers, called Salem numbers, and translation lengths of hyperbolic elements of arithmetic hyperbolic groups of the simplest kind. All non-cocompact arithmetic hyperbolic groups are of the simplest kind, and in even dimensions, all arithmetic hyperbolic groups are of the simplest kind. As an application, we determine a sharp lower bound for the length of a closed geodesic in a non-compact arithmetic hyperbolic n-orbifold for each dimension n. Generalized Baumslag-Solitar groups that are fundamental groups of compact orientable 3-manifolds are classified. This work is joint with Vincent Emery and Steven Tschantz.

Generalized Suspension Theorem in Extension Theory

Leonard Rubin (University of Oklahoma)

In 1991, A. Dranishnikov proved that for each CW-complex K and metrizable compactum X with $X \tau K$, it is true that $X \times I \tau \Sigma K$. Here, ΣK means the suspension of K in the CW-category, and by $X \tau K$ we mean that K is an absolute extensor for

X. We have generalized this result so that X could be either a stratifiable space or a compact Hausdorff space. Since all metrizable spaces are stratifiable, then either way, our result generalizes Dranishnikov's.

Right-angled mock reflection surfaces

Timothy Schroeder (Murray State University)

A right-angled mock reflection group (RAMRG) is a generalization of a Coxeter group in the sense that it acts on a CAT(0) cube complex with $\mathbb{Z}_2$-edge stabilizers. We give a combinatorial description of finite index, torsion-free subgroups of RAMRGs; and, in dimension two, classify the resulting quotient surfaces. This is collaborative work with Rick Scott of Santa Clara University.

The Schur Multiplier of a Generalized Baumslag-Solitar Group

Mathew Timm (Bradley University)

The second homology group of a group is also called the Schur Multiplier. A result of D. Robinson gives that the Schur Multiplier for a generalized Baumslag-Solitar group is free abelian. Its rank can be expressed in terms of the graph used to define the group. We give topological proofs of Robinson's results.

An infinite rank summand of the homology cobordism group

Linh Truong (Columbia University)

We show that the homology cobordism group of integer homology three-spheres contains an infinite rank summand. The proof uses an algebraic modification of the involutive Heegaard Floer package of Hendricks-Manolescu and Hendricks-Manolescu-Zemke. This is inspired by Hom's techniques in the setting of knot concordance. (his is joint work with Dai, Hom and Stoffregen.

A-infinity minimal models of differential graded algebras

Jiawei Zhou (University of California, Irvine)

For a formal differential graded algebra, if extended by an odd degree element, we prove that the extended algebra has an A-infinity minimal model where only m2 and m3 are non-trivial operations. As an application, the A-infinity algebras constructed by Tsai, Tseng and Yau on formal symplectic manifolds satisfy this property. Also, we expand the result of Miller and Crowley-Nordstrom for k-connected manifold. If the dimension n is smaller that or equal to $(l+1)k+2$, then its de Rham complex has an A-infinity minimal model with operations $mp = 0$ for all $p \geq l$.