Technology Diffusion, Product Differentiation and Environmental Subsidies

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Abstract

This paper explores the relationship between environmental subsidies, the diffusion of a clean technology, and the degree of product differentiation in an imperfectly competitive market. Like others, we show that the subsidy succeeds in reducing environmental damage only when the substitution effect (the reduction in pollution associated with the clean technology) exceeds the output effect (the extent that the subsidy increases output). Here, we add product differentiation and diffusion dynamics. When the substitution effect dominates, environmental damage decreases monotonically during the diffusion process. The extent of technology diffusion (the degree to which clean technology replaces dirty) is decreasing in the degree of product differentiation. Further, as products become closer substitutes, it is more likely that the subsidy will reduce environmental damage. Finally, the subsidy for clean technology will spill over to the remaining dirty producers, increasing their profit as well. In a free-entry equilibrium, the subsidy decreases pollution when product differentiation is low compared to the relative pollution intensity of the clean technology.

KEYWORDS: environmental policy, technology diffusion, product differentiation, subsidies, evolutionary game theory

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1 Introduction

Technological change is often seen as the driving force in mitigating or solving environmental problems. Policy intervention that spurs the development, adoption and diffusion of new, environmentally benign technologies therefore holds great appeal for environmental authorities. Since firms’ response to policy is not instantaneous, an understanding of both the short run and long run impacts of environmental policy on the adoption and subsequent diffusion of clean technology is extremely important (Jaffe et al., 2002). In this respect, policymakers have various instruments at their disposal, ranging from direct regulation (command-and-control strategies) to market-based instruments, such as taxes, subsidies and tradable pollution permits. However, the ranking of these different policy instruments appears to be ambiguous when markets are imperfectly competitive (e.g., Requate, 2005). Therefore, we do not aim to establish a policy instrument ranking, but rather closely examine the role of subsidies as a technology diffusion policy in the presence of imperfect competition.

For several reasons, we examine the linkage between subsidies and diffusion. First, actual policy most often takes the form of subsidies (OECD, 2006). At the same time, there is far less literature on subsides than there is on pollution taxes, which tend to be the instrument of choice for economic theorists. Second, because technology diffusion currently ranks high on the policymakers’ agenda, a better understanding of the relationship between policy and diffusion is required (Stoneman, 2002). Third, subsidies can be particularly beneficial in markets characterized by imperfect competition since they reduce the output distortion. Fourth, technology diffusion is a largely neglected area in the environmental economics literature, despite the fact that it is ‘[…] generally realised that it is the process of diffusion, or the use of technology that creates productive potential and competitiveness […]’ (Stoneman and Dieder, 1994, pg. 918).

This paper explores the relationship between subsidies, the diffusion of a clean technology, and the degree of product differentiation in an imperfectly competitive market. Firms base their technology adoption decision on the difference in profit between the two technological modes, where the technology adoption decision and the subsequent technology diffusion process is affected by the subsidy. One of our main findings is that a subsidy stimulates the diffusion of clean technology in the long run, the subsidy also spills over to firms that do not adopt the clean technology (remaining dirty producers), hence increasing their long run profits as well. Our analysis supports earlier results (e.g., Baumol and Oates, 1988; Fullerton and Mohr, 2003) that show the subsidy succeeds in reducing environmental damage only when the substitution effect (the reduction in pollution per unit, associated with the use of the clean technology) exceeds the output effect (the extent that the
subsidy increases output). However, we also show that the extent of diffusion, and
the likelihood that the substitution effect will dominate, decreases with the degree
of product differentiation. In a free-entry equilibrium, we find that the subsidy
decreases pollution only if the degree of product differentiation is sufficiently low
compared to the relative pollution intensity of the clean technology.

In addition to analyzing the negative externalities and other welfare compo-
nents in response to environmental policies in a differentiated market setting (e.g.,
Moraga-González and Padrón-Fumero, 2002; Bansal and Gangopadhyay, 2003;
Verhoef and Nijkamp, 2003; Kuhn, 2005; Conrad, 2005; Rodríguez-Ibeas, 2006),
we stress the significance of the reciprocal influence of technology diffusion on the
optimal subsidy scheme and the subsequent output decisions. Analyzing the ef-
effect of subsidies in such a strategic environment is particularly interesting because
in addition to addressing the output distortion, subsidies may also create greater
incentives for early technology adoption, hence facilitating technology diffusion.

The endogenous dynamics intrinsically embedded in technology diffusion (the
change in dirty and clean technology adopters over time) implies that firms switch
technologies to increase profits. However, at the same time, the relative profitabil-
ity of employing the different technological modes changes as diffusion gradually
advances (e.g., Reinganum, 1981). That is, the relative profitability of the different
technologies is contingent on the current distribution of employed technologies as
well as on the external market conditions, such as demand, the degree of product
differentiation, costs, and policy (environmental subsidies).

In a policy framework, Samaniego (2006) has shown that this dynamic feature
is very important, since the (aggregate) effect of subsidies depends greatly upon
the pattern of technology adoption (i.e., the diffusion dynamics), and the endoge-
nous response of firms is an important channel for the final effects. Our model also
captures these endogenous responses during the dynamic process of technology
diffusion. However, in contrast to Samaniego (2006), who models more general in-
dustrial subsidies to failing plants in a dynamic general equilibrium framework, we
explore the influence of these reciprocal conditions given that consumers are aware
of the firm’s environmental orientation through the level of product differentiation.

Evolutionary game theory provides a general framework to analyze the full im-
pact of environmental policy. The short run impact of subsides is illustrated by the
Cournot-Nash equilibrium for a given level of diffusion. In the short run the subsidy
creates a profit difference across technologies. The profit difference is eroded along
the adjustment path as firms adopt that technology which has a profit advantage. At
an evolutionary equilibrium a subsidy for the clean technology also impacts dirty
firms. An evolutionary game framework simultaneously allows for short run, long
run and adjustment dynamics as well as the partial and general equilibrium effects
of environmental policy in an imperfectly competitive framework with product differentation.

Evolutionary games analyze strategic interaction over time incorporating the elements monotonicity, Game Against Nature (GAN) and inertia (Friedman, 1991, 1998). In our technology adoption/diffusion game, monotonicity implies that the lower payoff technology will become less prevalent in the long run, or may even be eliminated. In this respect, the dynamics refers to firms’ technology adoption decision over time, as well as the strategic interaction between these firms in the output market. As such, our evolutionary game model provides insight into the adjustment process towards the industry’s diffusion equilibrium under different subsidy rules and market conditions.\(^1\) The adjustment dynamic is attached to the GAN condition, which means that firms do not systematically attempt to influence other firm’s future behavior. Finally, since the technology adoption across the population of firms is not instantaneous, the diffusion of technology is a gradual process in our model. This gradual technological advancement represents the inertia element.\(^2\)

By incorporating inertia into our model we rule out radical technological regime shifts. This fits with the energy-efficiency paradox, which contends that the diffusion of cost-effective energy-saving technologies (or environmentally benign technologies more broadly) follows a gradual rather than a radical pattern (e.g., Jaffe and Stavins, 1994). In the light of environmental subsidies and diffusion, it is essentially these types of technologies that we have in mind. Jaffe and Stavins (1994) show that firms may choose to postpone the adoption decision if subsidies (or tax credits) increase rapidly over time, because firms then receive a higher subsidy at a later moment. On the other hand, empirical evidence reveals that subsidies do stimulate technology diffusion. For instance, Jaffe and Stavins (1995) and Kemp (1997) show that subsidies enhance the diffusion of thermal home insulation.

Given the reciprocal nature of technology diffusion, we show that subsidies have a positive effect on the incentive to adopt the clean technology in the short run, implying that optimal output and profits of clean firms are positively affected by the subsidy, hence reducing the relative profitability of dirty firms. However, in the long run, when technology diffusion has reached its stable equilibrium (saturation level), subsidizing clean output indirectly generates a positive spillover to the dirty

\(^1\)Following Requate and Unold (2003), our model has the endogenous feature that in the long run the technology does not necessarily diffuse fully across the industry, as is commonly assumed in some literature on instrument ranking and adoption incentives (see references cited therein).

\(^2\)Evolutionary game theory can also be used as a selection device to eliminate those (Nash) equilibria that are not robust given some stability criterion. For the purpose of our paper we take a broader view and wish to model diffusion dynamics in a market environment where firms do not systematically try to attempt each other’s payoff in a repeated game setting (see Friedman, 1991, 1998).
firms. In this diffusion equilibrium profits of clean and dirty firms are equal, and no firm has an incentive to change technology. In the long run the relative prevalence of dirty firms decreases, implying less intense competition at the diffusion equilibrium than during the process of technology diffusion. Due to this reduced pressure in competing for the dirty good, the subsidy positively affects the profits of dirty firms in the long run. Furthermore, we provide explicit conditions for the interrelationship between product differentiation, environmental subsidies and technology diffusion.

The rest of the paper is structured as follows. Section 2 presents the model and the short run equilibrium. In section 3, the dynamics of technology diffusion and the stability properties of the diffusion process are determined given the subsidy and degree of product differentiation. Section 4 extends the model by analyzing the relationship between subsidies and diffusion with free entry, and considers an alternative lump-sum subsidy on adopting the clean technology. The optimal subsidy for linear damage and the welfare implications of the subsidy for damage in general form are examined in section 5. Section 6 concludes, discussing policy implications and future research.

2 The model

Consider a polluting industry with a fixed number of firms, \( n \). Each firm \( i = 1, 2, \ldots, n \) can produce their goods by using a single linear technology, denoted by \( k \): ‘dirty’ \( (k = d) \) or ‘clean’ \( (k = c) \). Then, given technology choice \( k = d, c \), a firm produces output \( q_k \) accordingly. To ensure that technology diffusion evolves gradually, it is assumed that once a firm has adopted a specific type of technology it continues using this technology in the short run, i.e. a firm cannot ‘jump’ between technologies in the short run. Given the fixed number of firms \( n \), define \( s \) as the fraction of firms using the clean technology and \( 1 - s \) as the fraction of firms using the dirty technology. Both technologies emit a hazardous pollutant \( e_k \), assumed to be proportional to output:

\[
e_k = \epsilon_k q_k, \quad k = d, c \text{ and } \epsilon_c < \epsilon_d.
\]

(1)

To investigate both the short run and long run dynamic impact of subsidies on technology diffusion, we assume that consumers are able to differentiate goods by firms’ choice of either a ‘clean’ or ‘dirty’ technology.\(^3\) Product differentiation may take different forms in this framework. The traditional form of differentiation is

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\(^3\)There is literature that shows that consumers differentiate goods with respect to the associated impact on the environment, and as such are willing to pay a price premium for goods that are produced in an environmentally friendly fashion (e.g., Levin, 1990; Cairncross, 1992).
where goods are imperfect substitutes in consumption. An example is organic and traditional agriculture; another is hybrid vehicles compared to internal combustion. Additionally, products may be differentiated by consumers’ perception of the production process, even when the good is homogenous. One example of homogenous output product differentiation is ‘green’ electricity versus traditional power generation.4 Our analysis allows for both types of differentiation.

Given consumers perceive the two goods differently, based on their effect on environmental quality, the inverse demand functions for the two types of goods can be written as follows (Dixit 1979):

\[ P_k(Q_k, Q_{-k}) = \alpha_k + \beta Q_k - \delta Q_{-k}, \]

where \( P_k \) denotes the price of the good produced with technology \( k = d, c \) and \( Q_k = \sum_{i \in k} q_i \) is the level of aggregate output supplied in submarket \( k \). Parameters \( \beta > 0 \) and \( \delta > 0 \) measure the direct and cross-price effects respectively. Without loss of generality, we normalize \( \beta \) to one, so that the cross effect \( \delta \in [0, 1] \). For \( \delta = 0 \) the goods are unrelated in the eyes of consumers and for \( \delta = 1 \) the goods are perfect substitutes. Values for \( \delta \) on the interval \( 0 < \delta < 1 \) measure the degree of substitutability between the dirty and the clean good, or, alternatively, the degree of product differentiation. Product differentiation increases as \( \delta \) approaches zero. The intercept term \( \alpha_k \) determines which good enjoys a price premium at equal output of each type. Since we will concentrate on the impact of the subsidy on the supply side of the market, our model is one of horizontal product differentiation, i.e. there is no heterogeneity across consumers that allows for an (objective) quality ranking of the two goods.5

Production costs of a firm that produces output \( q_k \) through employing technology \( k \) are assumed to be proportional in output:

\[ C_k(q_k) = \begin{cases} 
  c_d q_d & \text{if } k = d \\
  (c_c - \phi) q_c & \text{if } k = c 
\end{cases} \]

The dirty firm faces constant marginal costs \( c_d > 0 \). However, the governmental agency may provide a per-unit subsidy \( \phi \) on the clean good. Hence, the effective marginal cost of a unit of clean output equals \( c_c - \phi \).

4Some utility companies in the US are selling electricity as differentiated by production method. For instance, Central Vermont Public Service is a utility that provides electricity to 158,000 customers in Vermont. About 4,000 customers pay a premium of 4 cents per kilowatt hour for electricity produced from a renewable source (cow manure). The other 154,000 customers pay about 12.5 cents for electricity produced from conventional coal-fired power plants. Similarly, Alliant Energy charges a premium of 2 cents per kilowatt hour for renewable energy to customers in Wisconsin, Iowa and Minnesota (New York Times, 2008).

5See e.g. Giannakas and Yiannaka (2008) for a model that includes both horizontal and vertical differentiation.
Define the parameter \( \theta_k \equiv \alpha_k - c_k > 0 \) \( (k = d, c) \) as the choke price and cost margin (e.g., Dixit, 1979). We assume fixed costs to be zero; however, non-zero fixed costs will not affect the short run Cournot game with a fixed number of firms. In this respect, one could assume the existence of sufficient barriers to entry due to high fixed costs. In section 4 we relax these assumptions and allow for non-zero fixed costs that differ by technology, and we allow for free-entry.

A firm makes two decisions. The long run decision is the choice of technology, examined in the next section. In the short run, a firm with technology \( k \) chooses an output level \( q_k \geq 0 \) to maximize profit \( \pi_k \), taking the output decisions of other firms as given:

\[
\max_{q_k} \pi_k = P_k q_k - C_k.
\]  

(4)

Substituting (2) and (3) into (4) and simultaneously solving for \( q_d \) and \( q_c \) yields the following firm-level Nash equilibrium quantities:

\[
q_d(s) = \frac{\theta_d(1 + sn) - \delta(\theta_c + \phi)sn}{\Delta},
\]

\[
q_c(s) = \frac{(\theta_c + \phi) ((1 - s)n + 1) - \delta\theta_d(1 - s)n}{\Delta},
\]

(5)

where \( \Delta = s(1 - s)n^2(1 - s^2) + n + 1 > 0 \) for \( s, \delta \in [0, 1] \).

In section 3 we examine the parameter restriction in terms of \( \delta \), which determines the existence of an interior equilibrium \( \delta = f(\theta_c, \theta_d, \phi, n) \).

Profit for a clean firm in the short run, taking \( s \) as given, is simply \( \pi_c = (P_c - c_c + \phi)q_c \), or \( \pi_c = (q_c)^2 \). Similarly, profit for a dirty firm is \( \pi_d = (q_d)^2 \). The corresponding market-level outputs at the Nash equilibrium are \( Q_d = (1 - s)nq_d \) and \( Q_c = snq_c \) for the dirty and clean submarket respectively.

Since output \( q_k \) is explicitly contingent on diffusion \( s \), it is straightforward to see that the relative profitability of the two technologies depends on the state variable \( s \) as well as demand and cost conditions. Outputs, profits, pollution and welfare depend on the state of clean technology diffusion, \( s \). Given this endogeneity it is particularly interesting to explore the diffusion dynamics and the associated equilibrium.

3 Diffusion dynamics and equilibrium properties

Recall that \( s \) reflects the extent by which the clean technology has diffused in the industry. We are particularly interested in how welfare, specified in section 5, changes.

\(^6\Delta = n + 1 \) for \( s = 0, s = 1 \) or \( \delta = 1 \). Otherwise, \( \Delta > n + 1 \).
as a function of this diffusion process, which in turn depends on the market characteristics and the subsidy scheme.

The evolutionary process that drives the change in the diffusion of clean technology $s$ is expressed by the relative profitability of the two alternative technologies. Define the profit differential as:

$$
\Pi_D(s) \equiv \pi_c(s) - \pi_d(s).
$$

From (6) it directly follows that when $\Pi_D(s) > ( < ) 0$, the clean technology yields a higher (lower) profit than the dirty technology. Following a continuous time deterministic framework, for any initial state $s(0) = s_0$ the diffusion dynamics can be expressed as $\dot{s} \equiv ds/dt = H(s, z)$, where $H$ is a dynamic that is written as a function of the state of diffusion $s$ and a vector $z$ that summarizes the external market conditions (demand and cost structure). Friedman (1991) introduces the term compatibility to express the dynamic $H$ in terms of the payoffs. In our framework, compatibility implies that the clean (dirty) technology displaces the dirty (clean) technology if using the clean (dirty) technology yields higher profit than using the dirty (clean) technology, i.e. $\dot{s} \leq 0 \iff \Pi_D(s) \leq 0$. In other words, if the profit from adopting the clean technology exceeds the profit of firms applying the dirty technology, the fraction of clean firms will increase, and vice versa.

The long run evolutionary equilibrium of the diffusion process is where $\Pi_D(s) = 0$. The relevant diffusion equilibrium is:

$$
s^* = \frac{(\theta_c + \phi)(1 + n) - \theta_d(1 + \delta n)}{n(1 - \delta)(\theta_c + \theta_d + \phi)} \in [0, 1].
$$

The denominator of (7) is strictly positive, other than the case of perfect substitutability between the clean and dirty good ($\delta = 1$). The parameters determine if there is a Hawk-Dove game (interior diffusion equilibrium) or a Prisoner’s Dilemma, implying that either adoption of the dirty technology ($s^* = 0$) or adoption of the clean technology ($s^* = 1$) is the dominant strategy (e.g., Weibull, 1995). An interior equilibrium is obtained when substitutability is low, compared to the $\theta$-advantage. Hence, we have:

**Proposition 1** For $\delta \in \left[ \frac{\theta_c + \phi(1+n) - \theta_d}{n\theta_d}, \frac{\theta_d(1+n) - (\theta_c + \phi)}{n(\theta_c + \phi)} \right]$, there is a unique interior diffusion equilibrium $s^* \in (0, 1)$. If $\theta_c + \phi < \theta_d$ and $\delta \geq \frac{\theta_c + \phi(1+n) - \theta_d}{n\theta_d}$, then the long run diffusion equilibrium is $s^* = 0$ (all firms employ the dirty technology). If $\theta_c + \phi > \theta_d$ and $\delta \geq \frac{\theta_d(1+n) - (\theta_c + \phi)}{n(\theta_c + \phi)}$, then the long run diffusion equilibrium is $s^* = 1$ (all firms employ the clean technology).

The other root is: $s^* = \frac{(\theta_c + \phi)(1+n) - \theta_d(1 + \delta n)}{n(\theta_c - \theta_d + \phi)(1 + \delta)} \notin [0, 1]$. 

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7The other root is: $s^* = \frac{(\theta_c + \phi)(1+n) - \theta_d(1 + \delta n)}{n(\theta_c - \theta_d + \phi)(1 + \delta)} \notin [0, 1]$. 

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From (7) it is also straightforward to derive the following corollary:

**Corollary 2** If $\theta_c + \phi = \theta_d$ then $s^* = 0.5 \forall \delta \in [0, 1]$ (the industry is equally mixed with 50 percent clean firms and 50 percent dirty firms). If $\delta = 1$ then $s^*$ in (7) is undefined and if $\delta = 0$ then $s^* = \frac{\theta_c(1+n) - \theta_d}{n(\theta_c + \theta_d)}$.

The subsidy changes the relative value of the margins. This solution has a unique interior diffusion equilibrium, implying that more substitutable products mean smaller differences in the $\theta$’s. Alternatively, larger differences in the $\theta$’s can be supported as an interior equilibrium for greater product differentiation. As expected, a marginal increase of the subsidy stimulates the adoption of the clean technology, hence yielding a higher degree of diffusion in the long run (7) accordingly:

$$\frac{ds^*}{d\phi} = \frac{[2 + n(1 + \delta)]\theta_d}{n(1 - \delta)(\theta_c + \theta_d + \phi)^2} > 0.$$

In what follows, we restrict attention to the case of a unique interior diffusion equilibrium (Proposition 1), implying that not all firms necessarily adopt the clean technology in the long run.

Taking a closer look at the diffusion equilibrium as shown in (7), we can derive the following:

**Proposition 3** An increase of the subsidy $\phi$ enhances diffusion for all degrees of product differentiation. The effect is greater for a higher degree of substitutability (high $\delta$) between the goods.

**Proof.** See appendix. ■

The result is driven by relative competitive pressure in the clean and dirty submarkets. The higher the degree of product substitutability (thus lower degree of product differentiation), the more competition there will be in the market for the clean good. The subsidy increases the relative profitability of clean, inducing some dirty firms to switch technology, hence increasing competition in the clean sector and reducing competition in the dirty sector. More product differentiation implies greater competition across submarkets, which mitigates the effective reduction in competition in the dirty sector. A subsidy on adopting the clean technology has, therefore, a greater short run impact on the relative profitability of clean given a high degree of competition, which induces a higher degree of clean technology diffusion in the long run.
Furthermore, evaluating output and profits at the diffusion equilibrium value of \( s \) we find:

\[
q^*_c = q^*_d = \frac{\theta_c + \theta_d + \phi}{n(1+\delta) + 2},
\]

(9a)

\[
\pi^*_c = \pi^*_d = \left( \frac{\theta_c + \theta_d + \phi}{n(1+\delta) + 2} \right)^2,
\]

(9b)

which leads us to contend:

**Proposition 4** An output subsidy on producing the clean good generates a positive profit spillover to the dirty firms in the long run diffusion equilibrium.

As we see from (9a) and (9b), the diffusion equilibrium results in equal output levels and, therefore, equal profit for both the dirty and clean firm. Any subsidy on adopting the clean technology has an equal impact on the profit of the dirty firm at the diffusion equilibrium, even though there is a negative impact on dirty profit at the Nash equilibrium for a given state of diffusion \( s \) (see equation (5) where the Nash equilibrium output of the dirty firm is decreasing in the subsidy \( \phi \), and equations (9a) and (9b) where output and profits of both the dirty and clean firm are increasing in \( \phi \), respectively). Thus, increasing the subsidy reduces the relative Nash equilibrium profitability of the dirty firm, hence their prevalence decreases in the diffusion equilibrium. The remaining dirty firms face an effective reduction in competition (\( \delta < 1 \)), which has a positive effect on their profitability. In essence, it is the fact that the equilibrium value of diffusion, \( s^* \), is increasing in \( \phi \), which generates the profit spillover to the dirty firms.

An interesting implication of this result is that a dirty firm might actively lobby for a subsidy for clean output. This would put the dirty firm at a competitive disadvantage in the short run, but in the long run the firm would either remain dirty and see profit increase, or would adopt the clean technology and also see its profits increase. From a political economy perspective, this means that dirty firms will have no objection to subsidizing their clean rivals. All firms would be in favor of the subsidy, even though it directly only benefits one particular type.

From (10b) it is also clear that firm profits are increasing in product differentiation (lower \( \delta \)). However, the relationship between diffusion and product differentiation depends on the relative \( \theta \)'s. More specifically:

\[
\frac{\partial s^*}{\partial \delta} = \frac{(\theta_c + \phi - \theta_d)(n + 1)}{n(1-\delta)^2(\theta_c + \phi + \theta_d)} \geq 0 \iff \theta_c + \phi \geq \theta_d.
\]

(10)

Based on this we can contend the following:

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\(^8\)This result depends on the assumption that the number of firms is fixed. In Section 4 we relax this assumption.
Proposition 5. The equilibrium degree of diffusion, \( s^* \), is increasing in \( \delta \) if the clean firm has a margin advantage and is decreasing in \( \delta \) if the dirty firm has a margin advantage.

Proposition 5 states that for a sufficiently small subsidy that does not change the margin \( \theta_c + \phi \geq \theta_d \) inequality, an increase in the degree of product differentiation (reducing \( \delta \)) reduces the equilibrium degree of clean technology diffusion \( s^* \), even though the prior would be that diffusion is monotonic in \( \delta \); as we can see in (8).

The derivative (10) equals zero if \( \theta_c + \phi = \theta_d \), since \( s^* = 0 \) for all \( \delta \in [0, 1] \).

Thus far we have analyzed Propositions 3 and 5 separately. Next, we consider the linkage between these propositions by examining how the equilibrium value of diffusion relates to a change in both the degree of product differentiation and an increase of the subsidy level. Taking the cross-partial of (10) with respect to a change in the subsidy gives:

\[
\frac{\partial^2 s^*}{\partial \delta \partial \phi} = \frac{2\theta_d(n + 1)}{n(\delta - 1)^2 (\theta_c + \theta_d + \phi)^2} > 0, \tag{11}
\]

implying that the subsidy’s impact on the diffusion equilibrium is decreasing in the degree of product differentiation. An examination of equation (5) shows that the relative output of a clean firm, and thus relative profit, is increasing in the cross-partial.\(^9\) Thus, the subsidy has a greater impact on the relative profitability of clean when the goods are closer substitutes.

This is confirmed when taking a closer look at the marginal effects of the subsidy given the degree of product differentiation on aggregate clean and aggregate dirty output in equilibrium. The market output of the two types are:

\[
Q_c^* = \frac{(\theta_c + \phi)(n + 1) - \theta_d(1 + \delta n)}{(1 - \delta)(n(1 + \delta) + 2)}, \tag{12a}
\]
\[
Q_d^* = \frac{\theta_d(n + 1) - (\theta_c + \phi)(1 + \delta n)}{(1 - \delta)(n(1 + \delta) + 2)}. \tag{12b}
\]

Notice that \( Q_d^* \) is declining in \( \phi \) even though \( q_d^* \) is increasing in \( \phi \), which is due to the fact that the diffusion of clean technology is increasing in \( \phi \) (see (9a)). Now, differentiating (12a) and (13b) with respect to both \( \delta \) and \( \phi \), we obtain:

\[
\frac{\partial^2 Q_c^*}{\partial \delta \partial \phi} = \frac{2[n((1 + n)\delta + 1) + 1]}{(n(1 + \delta) + 2)^2 (1 - \delta)^2} > 0, \tag{13a}
\]
\[
\frac{\partial^2 Q_d^*}{\partial \delta \partial \phi} = \frac{-n^2(1 + \delta^2) - 2(n(1 + \delta) + 1)}{(n(1 + \delta) + 2)^2 (1 - \delta)^2} < 0. \tag{13b}
\]

\(^9\) \( \frac{\partial (q_c - q_d)}{\partial \delta \phi} = \frac{sn}{\Delta} + \frac{2s\phi(1 + n^2)\delta (1 - n + 1)}{\Delta^2} > 0. \)
which shows that the marginal effect of the subsidy on clean output at the diffusion equilibrium increases as products become closer substitutes. The converse holds for aggregate dirty output.

4 Two extensions

In this section we extend the model in two directions. First, we relax the assumptions of zero fixed costs and endogenize the number of firms. We assess the impact of product differentiation, the subsidy, and fixed costs on the free-entry number of firms at the diffusion equilibrium. We show that equal fixed costs drive the diffusion equilibrium towards the type with a margin advantage. Since the subsidy increases the number of firms at the diffusion equilibrium, we show that the subsidy decreases equilibrium pollution when product differentiation is low compared to the pollution intensity ratio (section 4.1). Second, we compare our previous results with a (lump-sum) subsidy for adopting the clean technology (section 4.2). A technology subsidy is equivalent to a fixed cost difference across types. We obtain the condition under which the technology subsidy results in an interior diffusion equilibrium, and we show that the two subsidy schemes are not equivalent.

4.1 Free-entry with identical fixed costs

Suppose fixed costs are non-zero and the number of firms is endogenous. Denote free-entry equilibrium results with a tilde (\(\tilde{\cdot}\)). Initially, we assume clean and dirty firms have the same fixed costs: \(F_c = F_d = F\). As usual, in a long run free-entry equilibrium, profits for both clean and dirty firms are zero. At a free-entry equilibrium, profits are:

\[
\pi^*_c = \pi^*_d = \left(\frac{\theta_c + \theta_d + \phi}{n(1 + \delta) + 2}\right)^2 - F = 0.
\]  

(14)

Solving (14) yields the free-entry equilibrium number of firms:

\[
\tilde{n}^* = \frac{\theta_c + \theta_d + \phi - 2\sqrt{F}}{1 + \delta}\sqrt{F}.
\]  

(15)

As expected, the subsidy increases the number of firms at the free-entry equilibrium, i.e. \(\frac{\partial \tilde{n}^*}{\partial \phi} = \frac{1}{(1 + \delta)\sqrt{F}} > 0\). Also, higher fixed costs reduces the number of firms at the free-entry equilibrium, i.e. \(\frac{\partial \tilde{n}^*}{\partial F} = -\frac{(\theta_c + \theta_d)}{(1 + \delta)^2\sqrt{F}} < 0\). With respect to the degree of product differentiation we find that \(\frac{\partial \tilde{n}^*}{\partial \delta} = \frac{2\sqrt{F} - (\theta_c + \theta_d + \phi)}{(1 + \delta)^3\sqrt{F}} < 0\) for \(\tilde{n}^* > 0\), since...
both require $F < \left(\frac{\theta_c + \theta_d + \phi}{2}\right)^2$. Hence, we can conclude that the free-entry number of firms is strictly increasing in the degree of product differentiation. The reason is that greater product differentiation increases profit, which in turn attracts more firms.

Evaluating (7) for the free-entry equilibrium number of firms (15) results in the following long run diffusion state:

$$e_s^* = \frac{\theta_c + \phi - \delta \theta_d - (1 - \delta)\sqrt{F}}{(1 - \delta)\theta_c + \theta_d + \phi - 2\sqrt{F}}. \quad (16)$$

The marginal effect of fixed costs on the free-entry diffusion equilibrium is:

$$\frac{\partial e_s^*}{\partial F} = \frac{(1 + \delta)(\theta_c + \phi - \theta_d)}{2(1 - \delta)\sqrt{F}\left(\theta_c + \theta_d + \phi - 2\sqrt{F}\right)^2} \geq 0 \iff \theta_c + \phi \geq \theta_d. \quad (17)$$

Given (17) we can now state:

**Proposition 6** The free-entry equilibrium degree of diffusion, $e_s^*$, is increasing (decreasing) in fixed costs, $F$, if clean (dirty) has a margin advantage.

This result depends on the same condition as Proposition 5. Increasing fixed costs results in fewer firms, which has a positive effect on the relative profitability of firms that have a margin advantage. More interestingly, compared to the diffusion equilibrium under a fixed number of firms (7), the introduction of identical fixed costs shifts the diffusion equilibrium under the free-entry case. With free-entry, identical fixed costs are not neutral, since an increase in fixed costs drives the free-entry equilibrium towards the type with a margin advantage. By contrast, identical fixed costs would not affect the diffusion equilibrium when the number of firms is held fixed.

Next, we examine the level of pollution at the free-entry equilibrium. With free entry the subsidy attracts more firms, which may lead to an increase in pollution via the output effect. Substituting (15) into (12a) and (13b) the aggregate output levels at the free-entry equilibrium are:

$$\tilde{Q}_c^* = \frac{\theta_c - \delta \theta_d + \phi - (1 - \delta)\sqrt{F}}{1 - \delta^2}, \quad (18a)$$

$$\tilde{Q}_d^* = \frac{\theta_d - \delta(\theta_c + \phi) - (1 - \delta)\sqrt{F}}{1 - \delta^2}. \quad (18b)$$

Pollution at the free-entry equilibrium is simply $\epsilon_c \tilde{Q}_c^* + \epsilon_d \tilde{Q}_d^*$, and the marginal effect of the subsidy on the free-entry level of pollution is then:

$$\frac{\partial(\epsilon_c \tilde{Q}_c^* + \epsilon_d \tilde{Q}_d^*)}{\partial \phi} = \frac{\epsilon_c - \delta \epsilon_d}{1 - \delta^2}, \quad (19)$$
from which we can state:

**Proposition 7** Pollution at the free-entry equilibrium is decreasing (increasing) in the subsidy if and only if \( \delta > (\lt) \frac{\epsilon_c}{\epsilon_d} \).

Thus, the subsidy reduces the free-entry equilibrium pollution level as long as the degree of product differentiation is sufficiently low (large \( \delta \)) relative to the pollution intensity ratio \( \frac{\epsilon_c}{\epsilon_d} < 1 \). If the goods are highly differentiated (small \( \delta \)) the subsidy will increase pollution at a free-entry diffusion equilibrium. There is little competition across types with highly differentiated goods, implying higher output and more pollution (consistent with Proposition 3). Note that, by definition, the pollution intensity ratio \( \frac{\epsilon_c}{\epsilon_d} < 1 \). As \( \frac{\epsilon_c}{\epsilon_d} \) approaches one, the clean technology is not so “clean” relative to dirty and the subsidy increases clean output and hence pollution.

### 4.2 Technology subsidy with asymmetric fixed costs

Thus far we have considered a subsidy per-unit of clean output. However, policymakers often directly subsidize the clean technology itself in order to stimulate adoption. Both types of subsidies encourage clean output (see, e.g. Fullerton and Mohr, 2003). As we will show, any difference in fixed costs with a fixed number of firms shifts the diffusion equilibrium towards the cheaper technology, but has no impact on the short-run equilibrium. We can therefore interpret a technology subsidy as equivalent to a difference in fixed costs. In order to obtain a given level of technology diffusion in the long run we can determine the fixed cost difference—that is, technology subsidy—needed to obtain that state.

Suppose fixed costs are asymmetric \( (F_c \neq F_d) \) either due to the technology subsidy or because of underlying cost differences. Following the profit differential in (6), define the fixed cost difference as \( F_D = F_c - F_d \). Allowing for unequal fixed costs, the profit differential (6) becomes \( \Pi_D(s) = \pi_c(s) - \pi_d(s) - F_D \). Then the fixed cost difference (or technology subsidy) \( F_D \) that leads to specific diffusion outcomes can be obtained. For full diffusion of the clean technology \( s = 1 \), the required technology subsidy is:

\[
F_{D,s=1} = \frac{\theta_c^2 - \theta_d^2 - 2n\theta_d(\theta_d - \delta\theta_c) - n^2(\theta_d - \delta\theta_c)}{(n + 1)^2},
\]

(20)

whereas to obtain the no diffusion state \( s = 0 \), the required technology subsidy is:

\[
F_{D,s=0} = \frac{\theta_c^2 - \theta_d^2 + 2n\theta_c(\theta_c - \delta\theta_d) + n^2(\theta_c - \delta\theta_d)}{(n + 1)^2}.
\]

(21)

From (20) and (21) we can deduce that:
Proposition 8 A technology subsidy $\omega = F_D \in (F_{D,s=1}, F_{D,s=0})$ will result in an interior diffusion equilibrium $s^* \in (0, 1)$.

As expected, a lower degree of product differentiation means that a smaller technology subsidy is needed to induce an interior diffusion equilibrium.

Compared to the output subsidy $\phi$, a technology subsidy $\omega$ is a relatively inefficient way to stimulate diffusion. A very large technology subsidy (compared to the profit level) is required to shift the equilibrium level of diffusion. Either type of subsidy reduces the number of dirty firms. Less competition in the dirty market improves the relative profitability of remaining dirty. Consequently, the technology subsidy $\omega$ would need to be very large in order to have the last dirty firm adopt the clean technology. Furthermore, profit is quadratic in the output subsidy, while the technology subsidy is linear.

Finally, in general, the output subsidy is not equivalent to a technology subsidy. For example, to obtain full diffusion ($s = 1$), the required technology subsidy is equation (20). The output subsidy required to obtain full diffusion, from equation (7), is:

$$\phi_{s=1} = \frac{\theta_d(n + 1) - \theta_c(1 + \delta n)}{1 + \delta n}.$$  \hspace{1cm} (22)

Since all firms are clean, aggregate output for $\phi_{s=1}$ is:

$$Q_c^* = \frac{n \theta_d}{1 + \delta n},$$  \hspace{1cm} (23)

resulting in the subsidy cost:

$$\phi_{s=1} Q_c^* = \frac{n \theta_d [\theta_d(n + 1) - \theta_c(1 + \delta n)]}{(1 + \delta n)^2},$$  \hspace{1cm} (24)

which is clearly different than (20).

5 Welfare and subsidies under different externality specifications

The policymaker chooses the subsidy $\phi$ to maximize social welfare. This value can be compared to the Pigouvian level. As usual, we define welfare ($W$) as the sum of industry profit ($\Pi_k$) (producer surplus) and consumer surplus ($CS_k$), and adjust this for the negative impact from the externality due to environmental damage ($D$) and the amount the government spends on subsidies ($V$):\textsuperscript{10}

$$W = \Pi_d + \Pi_c + CS_d + CS_c - D - V.$$  \hspace{1cm} (25)

\textsuperscript{10}From now on we refer to $V$ as the government’s ‘subsidy costs’.
This welfare function is rather intractable due to its nonlinear dependence on \( s \). We therefore examine social welfare and its elements at the equilibrium value of diffusion for different specifications of the negative externality, \( D \). To foster both the analytical tractability and transparency of the results, we will start by an examination of welfare in the absence of any externality (subsection 5.1) followed by a welfare analysis under externalities in general form (subsection 5.2), and finally linear externalities (subsection 5.3).

5.1 No externality

With no externalities, environmental damage is \( D = 0 \). At the diffusion equilibrium aggregate clean and dirty profit read \( \Pi_c^* = s^*n\pi_c^* \) and \( \Pi_d^* = (1 - s^*)n\pi_d^* \) respectively. Producer surplus at the equilibrium state of diffusion then equals:

\[
\Pi_c^* + \Pi_d^* = \frac{n(\theta_c + \theta_d + \phi)^2}{(n(1 + \delta) + 2)^2}.
\] (26)

It is straightforward to see that producer surplus is greater with higher product differentiation (lower \( \delta \)). The reason is that prices are increasing in product differentiation. Furthermore, as expected, producer surplus is increasing in the subsidy.

Collecting consumer surplus by market is rather difficult. However, we can combine the sum of consumer surplus \( (CS_d + CS_c) \) and the government’s subsidy costs, \( V \). At the diffusion equilibrium, where the latter is simply the subsidy per unit of clean output multiplied with the aggregate clean industry output (12a), i.e. \( V = \phi Q_c^* \). The marginal effect of the subsidy on the sum of consumer surplus minus the subsidy expenditures is negative\(^{11}\), i.e. the amount of money by the government spend on subsidizing clean output is greater than the benefit from the additional output to consumers. By relating this to the fact that producer surplus increases with the subsidy (see above), the overall effect of the subsidy on welfare, in the absence of externalities, is:\(^{12}\)

\[
\frac{\partial W^*}{\partial \phi} = \frac{n(1 - \delta)(\theta_c + \theta_d) - \phi((1 - \delta)n(n + 2) + 2)}{(1 - \delta)(n(1 + \delta) + 2)^2}.
\] (27)

The subsidy that maximizes welfare at the diffusion equilibrium with \( D = 0 \) is:

\[
\phi^*_{D=0} = \frac{n^2\theta_d \delta^2 + \delta [(\theta_d - \theta_c)n^2 + 4n\theta_d] + 2\theta_d - \theta_c(n + 2)^2}{n^2(1 + \delta) + 4n + 2},
\] (28)

\(^{11}\) \[ \frac{\partial[CS_c^* + CS_d^* - \phi Q_c^*]}{\partial \phi} = -\frac{[(1 - \delta)n(\theta_c + \theta_d) + \phi(n^2(1 + \delta) + 4n + 2)]}{(1 - \delta)n(1 + \delta) + 2} < 0. \]

\(^{12}\) With a non-zero externality, the optimal subsidy is the one that solves \( \frac{\partial[CS_c^* + CS_d^* - \phi Q_c^*]}{\partial \phi} + \frac{\partial[\Pi_c^* + \Pi_d^*]}{\partial \phi} = \frac{\partial D}{\partial \phi} \) for \( \phi \).
with $\frac{\partial \phi^*_D}{\partial \delta} > 0$ (see Appendix for proof). Without environmental damage from pollution, the optimal subsidy (28) just corrects the output distortion as a consequence of the imperfectly competitive market structure. As expected, the optimal subsidy in this case is unambiguously decreasing in the degree of product differentiation, i.e. the subsidy should increase when goods become closer substitutes. This is logical, because if goods become closer substitutes (less product differentiation), the corresponding short term Cournot-Nash quantities decrease. This requires a higher subsidy level in order to offset the reinforced distortion on output.

5.2 The subsidy’s impact on environmental damage

Consider environmental damage to be a strictly increasing function of aggregate emissions: $D(E)$, where $\frac{\partial D}{\partial E} \equiv D'(E) > 0$. Furthermore, assume that emissions are additively separable in dirty and clean output. From equation (1) aggregate emissions can be written as: $E = \epsilon_d(1 - s)nq_d + \epsilon_c snq_c$. The impact of the subsidy on environmental damage is then:

$$\frac{\partial D(E)}{\partial \phi} = \frac{\partial D}{\partial E} \frac{\partial E}{\partial \phi} = D'(E) \left[ \epsilon_d(1 - s)n \frac{\partial q_d}{\partial \phi} + \epsilon_c sn \frac{\partial q_c}{\partial \phi} \right].$$

(29)

Using (5) to evaluate the partials we have:

$$\frac{\partial D(E)}{\partial \phi} = \frac{D'(E)}{\Delta} \left[ -\epsilon_d(1 - s)n (\delta sn) + \epsilon_c sn ((1 - s)n + 1) \right],$$

(30)

where $\Delta > 0$ is defined after equation (5). Thus, environmental damage is increasing in the subsidy if:

$$\frac{\epsilon_d}{\epsilon_c} < \frac{(1 - s)n + 1}{\delta(1 - s)n}.$$  

(31)

From this we can derive the following proposition:

**Proposition 9** Environmental damage is decreasing (increasing) in the subsidy if the substitution effect dominates (is dominated by) the output effect.

**Proof.** See appendix. □

Proposition 9 isolates the interplay between the output (or scale) effect and the substitution effect as a consequence of technology diffusion. The substitution effect from switching between the two technological modes (here from dirty to clean) is just the ratio of the pollution intensiveness of the two technologies, $\epsilon_d/\epsilon_c > 1$. The output effect is just how much the level of output changes in the process of clean
technology diffusion as induced by the subsidy, viz. \(-\frac{\partial Q_d}{\partial \phi}\), which is exactly reflected by the right-hand-side of (31).\(^\text{(13)}\) Moreover, based on Proposition 9 one directly obtains:

**Corollary 10** The greater the degree of product differentiation (lower \(\delta\)), the greater the substitution effect needs to be for pollution to decrease in the subsidy.

The intuition is that if the degree of product differentiation in the market is relatively high and the clean technology does not differ much from the conventional dirty technology in terms of emission intensity, then emissions are likely to increase because the subsidy leads to higher output. Consequently, pollution will be relatively high, since emissions per unit of output are not very different across type. Conversely, as products become closer substitutes, the higher is the likelihood that the subsidy reduces environmental damage for a given emissions intensity ratio (substitution effect).

At the diffusion equilibrium we find a similar outcome. The subsidy effect on environmental damage is then simply (proof in the Appendix):

\[
\frac{\partial D(Q_d^*, Q_c^*)}{\partial \phi} = \epsilon_d \frac{\partial Q_d^*}{\partial \phi} + \epsilon_c \frac{\partial Q_c^*}{\partial \phi} \leq 0 \iff \frac{\epsilon_d}{\epsilon_c} \geq \frac{n + 1}{\delta(n + 1)}. \tag{32}
\]

Also from (32) we see that the subsidy has an adverse effect on environmental damage if and only if the substitution effect is dominated by the output effect. Finally, a direct comparison of (31) and (32) reveals:

**Proposition 11** If the substitution effect dominates the output effect at the diffusion equilibrium, then pollution will decrease monotonically as diffusion increases along the interval \(s \in [0, s^*]\).

### 5.3 The optimal subsidy under linear externalities

Finally, suppose that the damage from pollution is proportional to aggregate emissions (output):

\[
D = xE.
\]

The welfare maximizing subsidy \((\phi^*)\) at the diffusion equilibrium is:

\[
\phi^* = \frac{xf + n[(\theta_c + \theta_d)(1 - \delta) + x(\epsilon_d(3\delta + 1) - \epsilon_c(3 + \delta))]}{(1 + \delta)n(n + 2) + 2}, \tag{33}
\]

\(^\text{(13)}\)The output effect at the diffusion equilibrium is simply \(-\frac{\partial Q_d}{\partial \phi}\) = \(\frac{n+1}{\delta n+1} > 1\).
where \( f = n^2(1 + \delta) (\delta \epsilon_d - \epsilon_c) + 2 (\epsilon_d - \epsilon_c) \). Comparative statics reveals the following features of the optimal subsidy scheme (33). First, a higher degree of product differentiation has an ambiguous effect on the optimal subsidy and basically depends on the relative size of marginal damage \( x \):\(^{14}\)

\[
\frac{\partial \phi^*}{\partial \delta} = \frac{n [x g - 2(1 + n)^2(\theta_c + \theta_d)]}{2 + n(2 + n)(1 + \delta)} \leq 0 \iff \frac{x}{2(1 + n)^2(\theta_c + \theta_d)} \leq \frac{2 + n}{2(1 + n)^2 \epsilon_c + (2 + n ((2 + n)^2 + \delta(4 + n(2 + n(2 + \delta)))) \epsilon_d},
\]

where \( g = 2(1 + n)^2 \epsilon_c + (2 + n ((2 + n)^2 + \delta(4 + n(2 + n(2 + \delta)))) \epsilon_d \). Second, the impact of higher marginal damage on the optimal subsidy is also ambiguous and is essentially determined by the relative emission intensities of the two technologies:

\[
\frac{\partial \phi^*}{\partial x} = \frac{[2 + n(1 + \delta)] [(1 + \delta n) \epsilon_d - (1 + n) \epsilon_c]}{2 + n(n + 2)(1 + \delta)} \leq 0 \iff \frac{\epsilon_d}{\epsilon_c} \geq \frac{n + 1}{\delta n + 1}. \quad (35)
\]

This confirms the earlier result of the normalized externality case. By definition, the ratio \( \epsilon_d/\epsilon_c > 1 \) (the substitution effect). Since \( \delta \in [0, 1] \), the ratio \( \frac{n + 1}{\delta n + 1} > 1 \) (the output effect). Thus, if products are perfect substitutes (\( \delta = 1 \)) the latter term is equal to one and an increase of marginal damage has a positive effect on the optimal subsidy. If the goods produced by the two different technologies can be treated as unrelated (\( \delta = 0 \)), then the marginal effect on the optimal subsidy of an increase in marginal damage depends on relative difference between the emission intensities of the two technologies and the size of the industry. For instance, if the gap between the emission coefficients of the clean and dirty technology is relatively small and the size of the industry is rather big, then the optimal subsidy typically falls as the marginal damage from pollution increases. On the other hand, in case of a high degree of product differentiation (\( \delta \) close to zero) the optimal subsidy typically increases as a result of higher marginal damage if the clean technology is extremely environmentally friendly compared to the dirty technology (relatively large difference between \( \epsilon_d \) and \( \epsilon_c \) and the industry is less competitive.

In this respect, the effect of an increase in the output’s emission intensity is:

\[
\frac{\partial \phi^*}{\partial \epsilon_d} = \frac{x (1 + n \delta) (2 + n (1 + \delta))}{2 + n (n + 2)(1 + \delta)} > 0, \quad \frac{\partial \phi^*}{\partial \epsilon_c} = \frac{x (1 + n) (2 + n (1 + \delta))}{2 + n (n + 2)(1 + \delta)} < 0.
\]

\(^{14}\)All proofs are in the appendix.
From these expressions we can derive that for goods being perfect substitutes ($\delta = 1$), the optimal subsidy should exactly increase (decrease) with the marginal damage parameter $x$ for each marginal increase of $\epsilon_d (\epsilon_c)$. This is a Pigouvian result.\textsuperscript{15} So if environmental damage is linear, then we have a simple Pigouvian subsidy equal to marginal benefit rule, but the Pigouvian subsidy would be below the welfare maximizing subsidy (33) due to the existing imperfect market distortion.\textsuperscript{16} The subsidy partially corrects the output distortion. The output distortion is smallest when i) the industry is purely symmetric in terms of diffusion ($s = 0.5$), ii) the goods are close substitutes ($\delta$ close to $1$), and iii) neither good faces a $\theta$-advantage ($\theta_d = \theta_c$).

Furthermore,

$$\frac{\partial \phi^*}{\partial \theta_c} = \frac{\partial \phi^*}{\partial \theta_d} = \frac{n(1 - \delta)}{2 + n(2 + n)(1 + \delta)} > 0.$$  \hspace{1cm} (36)

The intuition here is that when either marginal costs $c_k$ ($k = d, c$) go down or demand increases due to an increase of the price premium $\alpha_k$ ($k = d, c$), output will expand\textsuperscript{17} and pollution will rise. As a result, the optimal subsidy should increase to stimulate clean technology diffusion in order to outweigh the negative impact from pollution. That is, the substitution effect should outweigh the output effect. This is interesting because the optimal subsidy is leading to lower output — thus not correcting the output distortion — and higher welfare by encouraging clean technology diffusion that reduces pollution by both the output and substitution effect.

6 Concluding remarks

This paper applies evolutionary game theory to explore the relationship between environmental subsidies, the diffusion of an environmentally benign technology and the degree of product differentiation in an imperfectly competitive market. Firms in this market can choose between two production technologies: dirty or clean, where the clean technology generates lower emissions per unit of output than the dirty technology. Firms base their technology adoption decision on the difference in profit that can be achieved under the two technological modes, where the technology adoption decision and the subsequent technology diffusion process is affected by the subsidy. Reciprocally, the diffusion dynamics affect the relative profitability of both the dirty and clean firms. These reciprocal conditions are investigated assuming that consumers recognize and value the firm’s environmental technology.

\textsuperscript{15}Note that this is only a Pigouvian subsidy at the diffusion equilibrium. Away from the diffusion equilibrium it will not be Pigouvian.

\textsuperscript{16}A proof of this is available from the authors by request.

\textsuperscript{17}This also comes out when differentiating output with respect to $\theta_k$:

$$\frac{\partial q_k}{\partial \theta_k} = \frac{\partial q_k}{\partial \theta_{-k}} = \frac{1}{2 + (1 + \theta)\eta} > 0.$$
One of the main results of our study is that subsidizing the adoption of clean technology in the short run generates a positive profit spillover to the firms that do not adopt the clean technology in the long run where the technological diffusion process is in equilibrium. The reason is that the subsidy affects the short run Cournot-Nash quantities of the clean firm positively and the dirty firm negatively. However, as diffusion of the clean technology gradually advances and ultimately reaches the long run equilibrium state, the Cournot-Nash quantities (and profits) of the dirty and clean firm coincide and are increasing in the subsidy. The effect comes down to the fact that the subsidy reduces the relative profitability of the dirty firm in the short run, implying that the prevalence of dirty firms also decreases in the long run. The remaining dirty firms subsequently face an effective reduction in competition in equilibrium, which has a positive effect on their profitability.

Furthermore, we find that the subsidy’s impact on the long run equilibrium number of clean firms is decreasing in the degree of product differentiation. The subsidy has a greater impact on the relative profitability of producing the environmentally friendly good as the goods become closer substitutes. This result is generated by the fact that the effective level of competition in the dirty output market is higher when the degree of product differentiation decreases.

With non-zero fixed costs, the free-entry equilibrium state of clean technology diffusion is increasing (decreasing) in the level of fixed costs and if the clean (dirty) firm has a margin advantage over the dirty (clean) firm. Furthermore, when fixed costs under both technologies are equal, the free-entry condition induces a shift in the equilibrium degree diffusion, which does not occur when the market size is fixed. Thus, equal fixed costs do not have a neutral impact on diffusion in the case of free-entry.

We also derive conditions that determine when subsidies lead to either an increase or decrease of environmental damage. We show that the ultimate outcome is driven by the interplay between a substitution effect and an output effect of technology diffusion. While the substitution effect reflects the relative pollution intensiveness of the technologies, the output effect shows how the subsidy changes output during the process of clean technology diffusion. A subsidy on clean output decreases (increases) environmental damage if the substitution effect dominates (is dominated by) the output effect. However, we show that the likelihood that the subsidy reduces environmental damage, for a given substitution effect, increases as products become closer substitutes. At the free-entry equilibrium, the subsidy increases pollution when goods are highly differentiated. This is caused by higher output levels due to less competition across product types. Our final result compares the output subsidy with a technology subsidy and it is shown that the latter subsidy type is a relatively inefficient way to stimulate the diffusion of clean technology under free entry.
The different demonstrations of the subsidy properties in relation to the degree of product differentiation has two main policy implications. First, given any consumer with a preference for dirty goods, complete diffusion of environmentally benign technologies within the industry need not be obtained in the long run, even though environmental subsidies stimulate the adoption of the clean technology. In this respect, our model identifies the role of differences in (net) absolute advantages between the clean and dirty good (i.e. the heterogeneity in the population of potential technology adopters) which explains to some extent the existence of the energy-efficiency paradox. In addition, the gradual adjustment of environmental technology diffusion is driven by the degree to which information about the different technology’s profitability spreads across the industry, and hence the firm’s behavior regarding the technology adoption decision.

Second, if the substitution effect dominates the output effect at the diffusion equilibrium, then environmental damage decreases monotonically during the process of technology diffusion. This rules out the possibility that a subsidy will increase emissions along the path to a long run equilibrium with lower emissions. Policymakers can simply compare the emission intensities to determine the substitution effect, with the number of firms and the degree of product differentiation which together determine the output effect. The policymakers’ information requirements are easily observable.

Appendix

Proof of proposition 3
It is sufficient to evaluate the diffusion equilibrium with respect to the subsidy as the degree of product substitutability $\delta$ approaches zero and 0.99 (since the diffusion equilibrium is undefined at $\delta = 1$). Evaluating (8), we find: $\lim_{\delta \to 0} \frac{d\sigma^*}{d\phi} = \frac{(2+n)\theta_d}{n(\theta_d+\theta_d+\phi)^2} < \lim_{\delta \to 0.99} \frac{d\sigma^*}{d\phi} = \frac{100(2+1.99n)\theta_d}{n(\theta_d+\theta_d+\phi)^2}$, since given equal denominators the numerator of the first limit ($\delta \to 0$) is smaller than the numerator of the second limit ($\delta \to 0.99$).

Proof of proposition 9
Starting point is equation (29):

$$\frac{\partial D(Q_d, Q_c)}{\partial \phi} = \epsilon_d \frac{\partial Q_d}{\partial \phi} + \epsilon_c \frac{\partial Q_c}{\partial \phi} = \frac{sn((1-s)n(\epsilon_c - \delta \epsilon_d) + \epsilon_c)}{sn(1-s)n(1-\delta^2) + n + 1}.$$ 

The denominator is positive, hence from the numerator it is easy to derive that environmental damage is decreasing (increasing) in the subsidy if the term $1 -$
s) n(\epsilon_c - \delta \epsilon_d) + \epsilon_c < (>)0. From this it is straightforward to derive that \frac{\partial D}{\partial \delta} \leq 0 \iff \frac{\epsilon_d}{\epsilon_c} \geq \frac{(1-s)n+1}{\delta(1-s)n}. The substitution effect, \frac{\epsilon_d}{\epsilon_c} > 1, and the output effect is (1-s)n+1. Thus, if the substitution effect exceeds the output effect, an increase of the environmental subsidy leads to a decrease in environmental damage, whereas environmental damage is increasing in the subsidy if the output effect dominates the substitution effect. □

**Proof sign equation (32)**
We need to determine when \frac{\partial D(Q^* d, Q^* c)}{\partial \delta} = \epsilon_d \frac{\partial Q^*}{\partial \delta} + \epsilon_c \frac{\partial Q^*}{\partial \delta} \leq 0. Rearranging terms gives that \frac{\partial D(Q^* d, Q^* c)}{\partial \delta} \leq 0 \iff \epsilon_c \frac{\partial Q^*}{\partial \delta} \leq \epsilon_d \frac{\partial Q^*}{\partial \delta}, or \frac{\epsilon_d}{\epsilon_c} \leq \frac{n+1}{n+1}. The output effect is \frac{n+1}{n+1}. Substitution yields \epsilon_c \leq \frac{n+1}{n+1}. Taking the inverse, we obtain: \frac{\partial D(Q^* d, Q^* c)}{\partial \delta} \leq 0 \iff \epsilon_d \leq \frac{1+n}{1+n}. □

**Proof of equation (35)**
The denominator of (35) is positive, so it suffices to determine the sign of the numerator: \[2 + n(1 + \delta)] [(1 + \delta \epsilon_d - (1 + n) \epsilon_c], or:

\[2(1 + \epsilon_d) + (1 + \delta)n(1 + \epsilon_d)] \epsilon_d \leq [2(1 + n) + (1 + \delta)n(1 + n)] \epsilon_c,

which subsequently simplifies to \frac{\epsilon_d}{\epsilon_c} \leq \frac{1+n}{1+n}. □

**First order derivatives**
The first order derivative of social welfare without environmental damage (27) with respect to subsidies:

\[
\frac{\partial W^*}{\partial \phi} = \frac{n(1 - \delta)(\theta_c + \theta_d) - \phi((1 - \delta) n(n + 2) + 2)}{(1 - \delta)(n(1 + \delta) + 2)^2} \leq 0
\]

\[\iff \theta_c + \theta_d \geq \frac{2\phi}{n(1 - \delta) + \phi(n + 2)}.

The first order derivative of the optimal subsidy without environmental damage (28) with respect to the degree of product differentiation:

\[
\frac{\partial \phi^*_{D=0}}{\partial \delta} = \frac{n [2n \theta_c + \theta_d(2 + n(1 + \delta))(4 + n(6 + n(1 + \delta)))]}{[2 + n(1 + \delta)]^2} > 0.
\]
References


