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Trading horizons and the value of money

Gabriele Camera[a,*], Filip Vesely[b]

[a] Purdue University, USA
[b] University of Wisconsin-Milwaukee, USA

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Abstract

This paper shows that fiat money can be feasible and essential even if the trading horizon is finite and deterministic. The result hinges on two features of our model. First, individual actions can affect the future availability of productive resources. So, agents may be willing to sell for money, even if on that date they have no reason to accept it. This makes monetary trade feasible in all preceding dates. Second, agents are anonymous and direct their search for partners. So, gift-giving arrangements may be prevented because agents can misrepresent their consumption needs. This makes money essential in exploiting any gains from specialization and trade.

JEL classification: C70; E40

Keywords: Monetary exchange; Finite horizon; Limited enforcement; Specialization

1. Introduction

A basic idea in economics is that allocations can be improved by specialization and trade and can be further expanded by innovations in the trading technology. Fiat money is such an innovation. Indeed, several observers have indicated that the use of ‘‘barren’’ tokens facilitates beneficial trades in markets that are subject to a variety of frictions (e.g., see Ostroy and Starr, 1990).

Interestingly, virtually all fiat monetary models consider infinite trading sequences. The reason is that for intrinsically useless tokens to have value in equilibrium, agents must

*Corresponding author. Tel.: +1 765 494 4456.
E-mail address: gcamera@mgmt.purdue.edu (G. Camera).

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expect that someone will want to trade consumption for money at some future date (Cass and Shell, 1980). One may thus infer that money necessarily loses its beneficial role in environments where the trade sequence is finite and deterministic (e.g., see Kocherlakota, 1998, p. 244). In this study we explain why this would be a rushed conclusion, by using a model in which the actions of a single agent can reduce aggregate productive capacity.

The economy is populated by finitely many dynastic (altruistic) agents from two overlapping generations. The initial old have one token each, the young are productive and society can benefit from specialization and trade. This simply means that the young should produce a high-value good for the old, avoiding other alternatives. We add a reason to worry about consumption of the old, relative to the usual (e.g., Antinolfi et al., 2001), by letting a dynasty's survival depend on the old's consumption. Hence, the current allocation of output can affect the future availability of productive resources and there is a consumption externality. Another main feature is that matching is endogenous, i.e., agents can direct their search in order to meet a trading partner of their liking. Finally, the model displays a set of frictions that preclude history-dependent credit trades and give money a central role. The dynastic formulation naturally motivates the existence of difficulties in maintaining long-term relationships and in conditioning current actions on observation of past play. We also assume anonymity and limited communication/commitment; only money holdings of an individual are observable, but not his identity and characteristics, trades are unobservable to third parties, and agents can choose autarky at any point in time. These frictions are common in the "foundations of money" literature.

A key departure from the typical monetary model, besides consumption externalities, is that the trading sequence is finite. Specifically, we impose a publicly known and deterministic end on the economy's life. For this environment, we prove that multiple Pareto-ranked equilibria are possible, and in some of these equilibria high-value trades take place only if they are supported by the exchange of tokens. In these outcomes, monetary transactions occur until a date sufficiently ahead of the economy's end and, above all, tokens are exchanged in trade because their use allows society to avoid an inferior equilibrium. That is to say, we demonstrate that, for certain parameters, fiat monetary exchange is not only *feasible* when the horizon is finite, but it is also *essential* (e.g., Huggett and Krasa, 1996; Kocherlakota, 1998). What generates this result?

Clearly, monetary exchange is feasible with infinite horizons, since it is always possible to trade in the future any money that is accepted today. In contrast, with a finite horizon, the feasibility of monetary exchange rests on the assumption that individual actions are strategically non-negligible and have permanent aggregate consequences. This implies that, on money's last trading date, making unilateral transfers to the old is necessary to preserve the stock of productive resources. So, agents may be willing to produce for someone who holds money, even if on that date they have no reason to accept it. Incentives to do so exist when the population is small enough, which is when the future benefit from high productive capacity offsets the current production disutility, i.e., consumption externalities are sufficiently large. This makes monetary trade feasible in all preceding dates, by backward induction.

The essentiality of monetary exchange, instead, is due to anonymity and endogenous matching. These features make "gift-giving" trading arrangements unsustainable in large economies because in such settings consumption externalities are small. So, the young would want to direct their search in a socially undesirable manner, misrepresenting their consumption needs. That is, they would attempt to consume, instead of producing, by pretending to be old. In cases such as these, a monetary trading arrangement can sustain an allocation that is socially preferred. Indeed, if only the old are endowed with money, then the type of undesirable behavior described above can be deterred by making matching and production contingent on money holdings.

This last result is more delicate since it is not robust to variations in the key features of the environment. For example, without anonymity search could be directed based on individual features. This is likely to make money inessential since agents can condition production on age or other observable elements. The same could occur in economies where matching is exogenous, especially if it is random, since this would prevent a socially desirable selection of partners, and hence an efficient allocation of consumption.

Our study makes several contributions to monetary theory. First, we add to a literature concerned with the purpose of monetary exchange in environments with finite populations. Prior research (e.g., see Kandori, 1992; Araujo, 2004) has shown that money does not play an essential role when infinitely-lived agents are sufficiently patient, since social norms can sustain beneficial trades. If trading horizons are finite and publicly known, however, social norms may break down. In this case, we prove that incorporating money in the exchange process is welfare improving as it prevents agents from taking undesirable actions.

Second, we complement work on the existence of monetary equilibria in economies characterized by finite trade sequences, in which money has either an implicit role or the final date is uncertain.[1] We contribute to this research by proving that fiat money has a fundamental allocative function in environments where, more generally, money has an *explicit* medium-of-exchange role and the trading horizon is also *publicly* known.

Finally, we contribute to a literature on payments systems in the context of models characterized by commitment and enforcement limitations (e.g., see Mills, 2004). In economies with these types of frictions, the incentive-compatibility of debt repayment generally rests on the possibility to either directly reward desirable behavior, currently or in the future, or to collateralize loans, or to punish undesirable behavior, for instance by identifying and excluding defaulters from future credit. We extend this literature by studying anonymous trading environments where these types of incentives cannot be offered and, yet, an agent will still prefer to voluntarily reduce his current payoff (he will repay his debt) because this is necessary to avert an adverse aggregate state.

The study is organized as follows. Section 2 describes the economic environment. Section 3 discusses the dynamic game and the equilibrium concept adopted. Section 4 presents the main result and establishes the key requirements for feasibility and essentiality of monetary exchange. Finally, Section 5 concludes.

2. Environment

We propose a conceptually simple model where the available productive resources depend on past allocations of consumption, in order to bring to light the workings of an anonymous trading framework where specialization and trade are beneficial to society. Along the way, we will motivate its central features and its simplifying assumptions.

[1] See Faust (1989); Kultti (1995); Kovenock and de Vries (2002). The continuous-time model of Faust has a terminal date but an infinite number of trade rounds. The models of Kultti and Kovenock and de Vries share one or more of these features: (i) Agents are asymmetrically informed on either the last trading date or their position at that date (buyer or seller) or (ii) money is a unit of account.

Time is discrete, $t = 0, 1, \ldots, J < \infty$. In $t = 0$ there are $2N \in \mathbb{N}_+$ spatially separated dynastic agents divided in equal proportion between young and old members of two-period-lived overlapping generations. The agents are altruistic in that their objective is the maximization of their dynasty's lifetime utility from consumption of either of two types of non-storable goods, denoted "market" and "home". The key difference from similar models of dynastic agents (e.g., Fuster et al., 2003) is that consumption of some good is a necessary input for the preservation of one's dynasty. Specifically, an old agent generates an offspring only if he consumes; otherwise, the dynasty dies out.[2] As in other overlapping generations models, we make productive resources available to the young so the size of the new generation is a key aggregate state variable. These features capture—simply and intuitively—the notion that the current allocation of consumption may affect the future availability of productive resources.

Endowments are as follows. Each initial old has an indivisible unit of fiat money. Each young has a production opportunity that can be used in one of two ways. He can use it to produce alone one indivisible market good by supplying effort that generates disutility $e \in (0, \beta u)$. Here, $u > 0$ is the period utility from consumption of a market good and $\beta \in (0, 1)$ is the discount factor. To motivate the need for trade, we assume that the young do not derive utility from their own market production (e.g., Diamond, 1982). The alternative use of the production opportunity is as follows. The young can team-up with someone else—young or old—in order to costlessly produce a home good that can be only split in half and then consumed.[3] Each partner is assumed to derive αu period utility from such team activity, with $\alpha \in (0, \frac{1}{2})$. Thus, although home goods are least preferred, team activity allows agents to improve over autarky in the stage game. Indeed, we set the agents free to choose autarky at every point in time and the possibility of home production will allow us to sustain active trade equilibria when horizons are finite.

On each date agents select independently and simultaneously either autarky—which generates zero current utility—or anonymous trade. By this we mean that the agent elects to match to someone else—whose identity and history cannot be observed—for the purpose of trading. We assume that, although the initial population is known, the number of traders cannot be directly observed. Traders' interaction occurs according to a directed matching process whose operation is detailed in the next section. Here, we simply note that this process is essentially an assignment rule that exhausts all mutually desirable pairings selecting traders at random. As is standard in many models of money, we assume that traders can hold only one unit of money, cannot communicate across matches, are anonymous and are unable to enforce or commit to an action. Finally, we assume that paired agents can only observe their respective actions and money holdings and that their exchange is based on a direct trading mechanism that is taken as given.

In summary, in this model trading arrangements are endogenous and the current allocation of consumption affects the future availability of productive resources. Money may have a fundamental role since the owners of productive resources cannot access trade

[2]Hence, the model can be interpreted as a simpler version of one in which either survival probabilities or the discount factor depend explicitly on the allocation of consumption, approaches seen in the literature of life-cycle consumption choices (e.g., Ray and Streufert, 1993 or Shi and Epstein, 1993). The simplicity of our formulation enhances the transparency of the analysis.

[3]This technological assumption simplifies the exposition and creates incentives to find a partner. Making the home good divisible and introducing bargaining leaves the main results unaltered as long as no party earns an excessive share.

histories. The dynastic agents formulation not only highlights the link between current consumption and future productive capacity, but it naturally motivates the absence of repeated interaction and especially the unobservability of trading histories. Indeed, old agents cannot share trade histories with their offspring, and the offspring cannot directly observe past play. These features allow us to make explicit a set of informational frictions that preclude credit-type trades.

3. Symmetric equilibria

Agents play a game of imperfect information that, on every date, has two stages. At the beginning of each period, the agent must choose whether to trade and the partner's desired features. Then, pairwise matches are formed among those who have selected to trade. In the second stage, each matched agent proposes a trading plan that is implemented only if the proposals are consistent. At each stage agents can select autarky.

Specifically, denote the agent's state at the beginning of t by $z(t) = (m, a)$ where $m = 0, 1$ is money holdings and $a = 0, 1$ is the age, young or old. The agent must choose whether and with whom to trade, taking as given the choices of others. He can choose either autarky, denoted 0, or a trading position denoted $\tau = b, s$, i.e., trade as a buyer or seller (b or s). He also chooses whom to match with by selecting the partner's desired characteristics: Money holdings m' and trading position τ'. We focus on pure strategies and denote by $\omega_z(t)$ these beginning-of-period t choices of a representative agent, so

$$\omega_z(t) = \begin{cases} 0 & \text{if autarky is selected,} \\ (\tau, \tau', m') & \text{if trade is selected.} \end{cases}$$

For instance, if $\omega_{1,1}(t) = (b, s, 0)$, then we are dealing with an old agent with money, since $z = (1, 1)$. This agent has chosen to trade as a buyer, since $\tau = b$, and to match to a seller without money, i.e., $\tau' = s$ and $m' = 0$.

Given these choices, pairings take place in a manner that reflects the traders' selections. This can be thought of as a directed matching process that—without going into unnecessary details—has the following key characteristics. First, two traders can match only if their choices ω are mutually consistent. For example, consider two agents, say, 1 and 2, on some date t. Considering agent $j = 1, 2$, suppose he is characterized by the state $z_j = (m_j, a_j)$ and his desire to trade is specified by a matching choice $\omega_j = (\tau_j, \tau'_j, m'_j)$. Let $-j$ denote the other agent when j is fixed. The agents' matching choices are consistent if we have $\tau_j = \tau'_{-j}$ and $m'_j = m_{-j}$ for $j = 1, 2$. That is agent 1 wants to match with someone who fits the profile of agent 2, and vice versa.

Second, matching must be feasible, a notion formalized by assuming that the probability of the desired matching depends on the number of buyers B and sellers S. In particular, if sellers want to match with buyers—and vice versa—every trader is matched as desired only if $B = S$. If $B \neq S$, then the representative trader on the "long" side of the market matches with someone with probability $\min(S, B)/\max(S, B)$ and remains unmatched otherwise. Matched traders are anonymous.[4]

Unmatched traders do nothing while paired agents interact via a direct trading mechanism. Each trader can take a single action that may depend only on z, t and the

[4]For another interpretation of anonymity in matching models see Aliprantis et al. (2006).

partner's money holdings m'. Assuming the action induces a single outcome, we can think of traders as playing a coordination game simultaneously proposing a feasible transfer of goods (home or market) and tokens, denoted

$$x_z(t, m') = (h, g, d).$$

Here $d, g \in \{-1, 0, 1\}$ and $h \in \{\frac{-1}{2}, 0, \frac{1}{2}\}$ denote the proposed transfer (−) or request (+) of h home goods, g market goods and d money. Consistent proposals are implemented, else autarky results. Actions and outcomes are unobserved by others and the match breaks at the period's end.

We model limited commitment by requiring that the trading mechanism satisfies sequential rationality, following Kocherlakota (1998). Without introducing additional notation, this simply means that the agent's pure strategy must be a mapping from all possible information sets into actions and it specifies a weakly optimal action at each information set, given that all others follow their strategies in the current and all future information sets. In particular, we note that since an agent can always choose autarky in (or outside) a match, for any action taken by anyone else, then it must be the case that equilibrium actions be compatible with individual incentives.

In constructing an equilibrium we focus on symmetric (pure) strategies, checking only unilateral one-period deviations (the unimprovability principle allows us to do so), restricting attention to environments where multiple deviations are not allowed (e.g., matched agents cannot both deviate). To sum up, we adopt the following equilibrium concept.

Definition 1. An equilibrium is a list of pure strategies $\{\omega_z(t), x_z(t, m')\}_{t=0}^{J}$ that are sequentially rational and are identical for agents in an identical state.

In what follows, we will say that on date t there is *monetary trade* if sellers of market goods require a token in exchange for their production, i.e., if $d = 1$. Throughout the discussion, we retain the following assumption:

$$\frac{-e + \beta u}{1 + \beta} \leqslant \alpha u < \frac{u - e}{2}. \tag{1}$$

Simply put, the second inequality implies that market production and trade generates the greatest surplus in the economy. Thus, a planner who treats agents identically, would choose to have the young produce a market good for the old, on each date. The first inequality, however, implies that a young would rather avoid this plan and would prefer to engage in home production on each date. Clearly, with an infinite horizon, monetary exchange can support production and trade of market goods. The question is whether this is also possible when the trading horizon is finite, which is what we study next.

4. Trading in a finite horizon

This section shows how, despite the impossibility to pass on money ad infinitum, society can exploit the availability of tokens to maximize the gains from specialization in production and trade.[5] The main result is summarized as follows.

[5]See Camera et al. (2003) for an infinite-horizon model in which the use of money allows greater production specialization but this also lowers societal welfare.

Proposition 1. *If agents are sufficiently patient, then there exist finite-horizon economies in which fiat monetary exchange is not only feasible but it is also essential.*

The remainder of this section is devoted to proving this statement. To do so, we will take several steps, formalized into distinct Lemmas. First, we show that fiat monetary exchange can be feasible on a set of initial dates, even if the economy has a finite and deterministic life. This amounts to demonstrating that on money's last trading date young agents are willing to produce for someone who holds a token, despite the fact that they have no reason to accept it. By backward induction, monetary trade is feasible in all preceding dates. Second, we establish conditions such that gift-giving arrangements cannot implement the allocation that is supported by fiat monetary exchange. That is, fiat money is essential. Finally, we present conditions, in terms of the parameter of the economy, such that monetary trade is both feasible and essential.

4.1. Patterns of matching and exchange

We start our analysis by formalizing two basic matching and trading strategies, which, for simplicity, we call monetary and non-monetary.

We say that on some date t agents adopt a *non-monetary* pattern of exchange if on that date they engage in home production. That is, each young chooses to match to an old, and vice versa, in order to produce and share consumption of a home good. This strategy is independent of money holdings. Formally, on date t, if we call the young a seller and the old a buyer (without loss in generality), then for all m and all m' we have

$$\omega_z(t) = \omega_z^{**} = \begin{cases} (b, s, \cdot) & \text{if } z = (m, 1), \\ (s, b, \cdot) & \text{if } z = (m, 0), \end{cases}$$
$$x_z(t, m') = x_z^{**} = \begin{cases} (\frac{1}{2}, 0, 0) & \text{if } z = (m, 1), \\ (\frac{-1}{2}, 0, 0) & \text{if } z = (m, 0), \end{cases} \tag{2}$$

where, slightly abusing notation, we set $m' = \cdot$ in ω_z^{**} to emphasize that matching does not depend on money holdings.

This pattern of matching and exchange on date t implies that there are N young-old matches where the home good is produced and consumed in equal amounts. It is immediate that non-monetary trade for all dates is an equilibrium because it is an equilibrium of the stage game. To see why, consider a representative date $t = 0, 1, \ldots, J$. Notice that no-one would want to deviate from equilibrium play because home production is costless and any deviation implies zero consumption. Indeed, if the agent deviates from the matching choice ω_z^{**}, then he cannot find a partner. If he defects from the trading proposal x_z^{**}, then he cannot consume. Either way, a defection generates zero payoff, instead of αu.

Now, instead, suppose that on date t old agents choose to buy market goods in exchange for money, while the young choose to sell market goods for money, which is what we have earlier called monetary trade. Formally, we have

$$\omega_z(t) = \omega_z^{*} = \begin{cases} (b, s, 0) & \text{if } z = (1, 1), \\ (s, b, 1) & \text{if } z = (0, 0), \end{cases}$$

$$x_z(t,m') = x_z^*(m') = \begin{cases} (0,1,-1) & \text{if } z=(1,1) \text{ and } m'=0, \\ (0,-1,1) & \text{if } z=(0,0) \text{ and } m'=1. \end{cases} \tag{3}$$

If (3) is the pattern of exchange adopted on date t, then there are N buyer–seller pairs in the economy, but only the old consume. Clearly, this cannot be an equilibrium of the stage game, because production is costly. Therefore, it cannot be a stationary pattern of exchange for a finite horizon economy; no one would produce for a token on date J. However, we can conjecture an equilibrium with a simple ''regime switch'' occurring on some date $T \in \{1,\dots,J\}$ that separates two different, time-invariant, trading regimes.

Specifically, we will proceed by conjecturing that monetary trade can be sustained only until a date $T-1$, which we suppose to be an odd number, without loss in generality. Then, on date T tokens lose value and subsequently agents engage in home production. We call this a *monetary equilibrium* (slightly abusing the language) since when market goods are produced, they are only sold for money. Such a trading arrangement is characterized by strategies that are time-invariant in the subintervals $[0, T-1]$ and $[T, J]$.

Formally, along the equilibrium path we have

$$\omega_z(t) = \begin{cases} \omega_z^* & \text{if } t=0,\dots,T-1, \\ \omega_z^{**} & \text{if } t=T,\dots,J, \end{cases}$$
$$x_z(t,m') = \begin{cases} x_z^*(m') & \text{if } t=0,\dots,T-1, \\ x^{**} & \text{if } t=T,\dots,J, \end{cases} \tag{4}$$

which is a combination of (2) and (3). Therefore, the allocation associated to (4) is characterized by a deterministic consumption sequence, although on T agents switch to a less rewarding consumption. Note that, in formalizing the equilibrium strategy, it is unnecessary to specify the actions taken, off equilibrium, when $z=(1,0)$ or $z=(0,1)$ on dates $t=0,\dots,T-1$. Indeed, a decision node in which some young has money at the start of life, i.e., $z=(1,0)$, cannot be reached as a result of one-shot deviations from equilibrium play, by a single agent. A node in which an old is without money, i.e., $z=(0,1)$, can be reached as a result of a deviation. However, the actions of this agent affect neither his payoff nor the payoff of others.

To see why $z=(1,0)$ cannot arise on $t \leqslant T-1$, notice that a young starts life with money, off equilibrium, only if he has a bequest from his predecessor. No deviation can lead to this, since the old must consume to have an offspring. Due to the direct trading mechanism adopted, consumption requires the exchange of money. Thus, if an old is left with money, then he must have not consumed and, especially, he must have not consumed a home good (this requires two deviators). Suppose, instead, that some agent x is old without money, i.e., $z=(0,1)$. This node is reached on t only if on $t-1$ an old does not buy, or a young does not sell. Either way, some old does not consume on $t-1$ and his dynasty dies out. The deviation is known only to agent x, since the number of traders is unobservable, and on t there are $N-1$ young sellers and $N-1$ old buyers with money, who play equilibrium. Thus, old agent x can neither match nor consume, no matter how he behaves; his dynasty dies out and from $t+1$ there are $N-1$

old buyers and $N-1$ young sellers. Thus, the actions of x on date t affect neither his nor the payoff of others.

4.2. Payoffs

Given (4), we can define equilibrium payoffs or present discounted value of expected lifetime utilities. Let $V_{\mathrm{s}}(t)$ denote the equilibrium payoff to a young agent, a seller, at the start of date $t=0,\ldots,J$. Similarly, if the agent is old, i.e., he is a buyer, we denote his payoff by $V_{\mathrm{b}}(t)$. Recursive formulation of payoffs yields

$$V_{\mathrm{s}}(t)=\begin{cases}-e+\beta V_{\mathrm{b}}(t+1) & \text{if } t=0,\ldots,T-1,\\ v_c(J-t+1) & \text{if } t=T,\ldots,J,\end{cases}$$
$$V_{\mathrm{b}}(t)=\begin{cases}u+\beta V_{\mathrm{s}}(t+1) & \text{if } t=0,\ldots,T-1,\\ v_c(J-t+1) & \text{if } t=T,\ldots,J,\end{cases} \tag{5}$$

where for $t=T,\ldots,J$ we define

$$v_c(J-t+1)=\frac{(1-\beta^{J-t+1})\alpha u}{1-\beta}. \tag{6}$$

On date $t=T$, the equilibrium payoff $v_c(J-T+1)$ depends on the trade rounds that separate the end of monetary trade T from the terminal date J. Of course, payoffs increase with J and as we move to an infinite horizon we have $v_c(J-t+1)\to\alpha u/(1-\beta)$.

Now, consider equilibrium payoffs on a date $t=0,1,\ldots,T-1$, recalling that T is an even number. Suppose that t is also even (including zero). Hence, a young seller on date t expects his dynasty to accomplish $(T-t)/2$ monetary trade cycles, selling and buying a market good. Each cycle generates utility $-e+\beta u$. On date T his descendant is old and engages in home production. Similar considerations can be made for an old buyer, with the difference that each monetary trading cycle generates $u-\beta e$ utility. Now, instead, suppose that t is an odd period. Again, consider a young seller on date t. He suffers disutility $-e$ and expects his dynasty will subsequently accomplish $[T-(t+1)]/2$ monetary trade cycles, buying and selling a market good earning utility $u-\beta e$. For a buyer, instead, each subsequent monetary cycle generates utility $-e+\beta u$.

Thus, if we consider a date $t=0,\ldots,T-1$ and let n denote the remaining monetary trading cycles, then the equilibrium payoff satisfies:

$$V_{\mathrm{b}}(t)=\begin{cases}\sum_{n=1}^{\frac{T-t}{2}}\beta^{2(n-1)}(u-\beta e)+\beta^{T-t}v_c(J-t+1) & \text{if } t \text{ is even},\\ -e+\beta V_{\mathrm{s}}(t+1) & \text{if } t \text{ is odd},\end{cases}$$
$$V_{\mathrm{s}}(t)=\begin{cases}\sum_{n=1}^{\frac{T-t}{2}}\beta^{2(n-1)}(-e+\beta u)+\beta^{T-t}v_c(J-t+1) & \text{if } t \text{ is even},\\ u+\beta V_{\mathrm{b}}(t+1) & \text{if } t \text{ is odd},\end{cases}$$

Having defined payoffs, we can easily demonstrate that society can benefit from specialization in production and trade. To do so, we rank allocations based on their social welfare, which we calculate using the standard measure of average payoff (e.g., see Diamond, 1982). We find the following.

Lemma 1. *Fix some date $T \leqslant J$. The allocation associated to the trading pattern in* (4) *generates higher social welfare than permanent home production.*

Proof. Notice that on any date t monetary trade maximizes aggregate surplus because $u - e > 2\alpha u$ from (1). Clearly, the allocation generated by trading as in (3) on all dates, is the unique efficient allocation, but it is unattainable since there is neither commitment nor enforcement. Thus, suppose that the allocation associated to (4) can be attained. Recalling that $1 + \beta^2 + \beta^4 + \cdots + \beta^{T-2} = \{1 - \beta^{2[(T-2)/2+1]}\}/(1 - \beta^2)$ for T even, we have

$$V_{\mathrm{s}}(0) = \frac{1-\beta^T}{1-\beta^2}(-e + \beta u) + \beta^T v_c(J - T + 1),$$

$$V_{\mathrm{b}}(0) = \frac{1-\beta^T}{1-\beta^2}(u - \beta e) + \beta^T v_c(J - T + 1).$$

To measure social welfare, consider a representative buyer and a seller. Permanent home production generates payoff $v_c(J+1) = (1 - \beta^{J+1})\alpha u/(1-\beta)$, while the allocation associated to (4) generates average payoff

$$\frac{V_{\mathrm{s}}(0) + V_{\mathrm{b}}(0)}{2} = \frac{1-\beta^T}{1-\beta}\left(\frac{u-e}{2}\right) + \beta^T \frac{(1-\beta^{J-T+1})\alpha u}{1-\beta}.$$

It is immediate that $v_c(J+1) < [V_{\mathrm{s}}(0) + V_{\mathrm{b}}(0)]/2$ can be rearranged as $\alpha u < (u-e)/2$. Hence, the monetary allocation generates higher social welfare, due to assumption (1). Social welfare increases with the rounds of monetary trade, since $[V_{\mathrm{s}}(0) + V_{\mathrm{b}}(0)]/2$ is increasing in T. □

In our model, society benefits from specialization and trade. The young should produce market goods and the old should consume them. This pattern of exchange is socially beneficial because it maximizes both the number of matches and the surplus generated in each match. Consequently, two natural questions arise. First, is monetary exchange feasible? Second, can we sustain the monetary allocation by simply devising a self-enforcing plan of unilateral transfers? We proceed by providing an answer to the first question.

4.3. Feasibility of monetary exchange

To address the feasibility of monetary exchange, we must verify that certain individual rationality and incentive compatibility requirements are satisfied.

To start, participation in trade must be individually rational, i.e., on each date agents must weakly prefer trade to autarky. Clearly $\omega_z(t) \neq 0$ for all t if $V_{\mathrm{b}}(t) \geqslant 0$ and $V_{\mathrm{s}}(t) \geqslant 0$. Since we have assumed $-e + \beta u > 0$, equilibrium payoffs are always positive as long as

$$V_{\mathrm{s}}(T-1) = -e + \beta v_c(J - T + 1) \geqslant 0,$$

i.e., on money's last trading date a seller's payoff must be positive.

A trader's actions must also be incentive compatible. This amounts to verifying that the agent does not strictly prefer to match or trade differently than what is prescribed by the monetary equilibrium strategy. Of course, the feasibility of fiat monetary exchange hinges on the seller's behavior on date $T-1$, since he knows that money has no future value. Intuitively, on this date there is no reason to accept money and so selling amounts to offering a gift to someone who will never be met again. Below, we provide a condition sufficient to ensure the feasibility of monetary exchange.

Lemma 2. *Fix a date* $T \leqslant J$. *Conjecture a monetary equilibrium as in* (4). *If we let*

$$\bar{N} = \frac{\beta \alpha u(1-\beta^{J-T+1})}{e(1-\beta)},$$

then for each $N \leqslant \bar{N}$ *the monetary equilibrium strategies are individually optimal.*

Proof. Fix a date $T \leqslant J$. Conjecture a monetary equilibrium as in (4). Our previous discussion indicates that these strategies are individually optimal in all dates $t = T, \ldots, J$. Hence, we must check for deviations only on $t < T$. Clearly, the old prefer to buy since any deviation prevents consumption and generates zero payoff (the agent and his dynasty leave the economy). Thus, focus on a young on date $t < T$, in equilibrium.

The young can only meet a buyer of market goods on $t < T$ and, since no one desires to meet a home producer, deviating by attempting home production is suboptimal. If a young deviates trying to buy, then he would not match because he has no money; this is because in equilibrium every seller desires to meet only someone who has money. Choosing to meet a buyer, is weakly preferred since the young can always refuse to trade. Thus, focus on a seller–buyer (young–old) match, in equilibrium, when the key deviation is to refuse to sell. Incentives to deviate exist only on date $T-1$, since money exhausts its record-keeping role. To see why this is so, let $\hat{V}_s(t)$ denote the deviator's payoff on date $t < T$. Selling is incentive compatible if $V_s(t) \geqslant \hat{V}_s(t)$, which holds on any date $t \leqslant T-2$ because $\hat{V}_s(t) = 0$. Intuitively, a young who does not sell does not earn money, hence does not consume when old. So, we have $x_{(0,0)}(1) = (0, -1, -1)$ on any $t \leqslant T-2$.

Now focus on $T-1$. There is a reason to deviate since production is costly and money loses value in T. Here, trading for money amounts to making a unilateral transfer since the seller neither receives current or future rewards, nor faces a threat of punishment. If the young seller deviates, then his discounted continuation payoff is $\beta \hat{v}_c(J-t+1)$ with

$$\hat{v}_c(J-T+1) = \frac{N-1}{N}[\alpha u + \beta v_c(J-T)].$$

To derive $\hat{v}_c$, note that a deviation on $T-1$ lowers to $N-1$ the young on T (an old does not consume on $T-1$). The deviation is undetected so, on T there are $N-1$ young sellers and N old buyers of home production, among whom is the deviator. Due to rationing, on T one old does not consume, so on $T+1$ there are $N-1$ sellers and $N-1$ buyers of home production, getting the equilibrium payoff $v_c(J-T)$. The payoff $\hat{v}_c(J-T+1)$ indicates that on T the deviator matches with probability $(N-1)/N$, and otherwise his dynasty exits the economy. This risk is exactly what may induce a young to sell for money on date $T-1$.

To see it, note that selling is incentive compatible on that date whenever

$$-e + \beta v_c(J-T+1) \geqslant \beta \hat{v}_c(J-T+1).$$

Given v_c and $\hat{v}_c$ we rearrange the above as $N \leqslant \bar{N}$, with $\bar{N}$ defined earlier. If $N \leqslant \bar{N}$, then the strategy in (4) is an equilibrium.[6] We have $\partial\bar{N}/\partial\beta > 0$, $\lim_{\beta\to 0}\bar{N} = 0$ and, using l'Hospital rule, $\lim_{\beta\to 1}\bar{N} = \alpha u(J - T + 1)/e$. Also, $\partial\bar{N}/\partial(J - T) > 0$, $\lim_{e\to 0}\bar{N} = \infty$, and $\partial\bar{N}/\partial e < 0$. $\square$

In our economy there is neither enforcement nor commitment, and the overlapping generations structure prevents contagious punishment schemes (see Kandori, 1992). So, the feasibility of monetary exchange hinges on the value of non-monetary trade after date $T - 1$, and rests on the feature that productive capacity can be preserved only if the old consume. Since in equilibrium a young can only meet old buyers (who would not deviate), the key defection is to refuse selling on the date before money loses value. The defector's payoff depends on production disutility, impatience but also market size. Indeed, refusing to sell on $T - 1$ does not affect the young's wealth (money loses value anyway) but it spawns consumption risk since it permanently reduces future productive resources. This decline is significant only if the population is small enough, i.e., when the consumption externality is sufficiently large, which is why the future benefit from full productive capacity offsets current production disutility only if $N \leqslant \bar{N}$. Naturally, $\bar{N}$ grows with greater patience and smaller production costs, as these reduce the incentive to defect. Also, $\bar{N}$ increases in $J - T$ since greater spells of home consumption—from longer horizons or shorter monetary trading—raise $T - 1$ payoffs. The question that remains is whether money is at all necessary to achieve the allocation associated to (4).

4.4. *Essentiality of monetary exchange*

We now present a condition under which monetary trade improves upon non-monetary trading arrangements, thus establishing a key requisite for the essentiality of money. For this purpose, in what follows we let T be associated to the longest feasible monetary trade pattern. That is, T will denote the greatest element of the set $\{1, 2, \ldots, J\}$ that is consistent with the feasibility requisite $N \leqslant \bar{N}$ of Lemma 2. Since society benefits from specialization and trade, then this is the best that society can do with money.

Lemma 3. *Fix a date $T \leqslant J$. If we let*

$$\underline{N} = \frac{2}{u + e}\left[\frac{\alpha u\beta(1 - \beta^{J-T+1})}{1 - \beta} + \frac{u - e}{2}\right],$$

then for each $N > \underline{N}$ the monetary equilibrium allocation cannot be attained with "gift-giving" trading arrangements.

Proof. Fix a date $T \leqslant J$ and recall that this denotes the greatest T that satisfies $N \leqslant \bar{N}$. Conjecture a "gift-giving" equilibrium that replicates the allocation given by (4), by which we mean that trade of market goods is not conditional on holdings and transfers of tokens. On the first T dates the young transfer a market good to anyone who asks for it, while the

[6] We do not need to check for one-shot deviations off the equilibrium path. The only off equilibrium node that can be reached following a deviation is such that a single agent is in state $z = (0, 1)$, i.e., old and without money. Our previous discussion indicates that actions of this agent have no effect on payoffs.

old demand it from anyone who wish to supply it. This is the only way to achieve the monetary allocation. Notice also that there is no gift-giving scheme that can improve over the monetary allocation.[7]

Formally, the equilibrium strategies on $t = T, \ldots, J$ are as in (4), and on $t = 0, \ldots, T-1$ they are independent of money, i.e., for any m and m' we have $\omega_z(t) = \omega_z$ and $x_z(t, m') = x_z$ where

$$\omega_z = \begin{cases} (b, s, \cdot) & \text{if } z = (m, 1), \\ (s, b, \cdot) & \text{if } z = (m, 0), \end{cases}$$
$$x_z = \begin{cases} (0, 1, 0) & \text{if } z = (m, 1), \\ (0, -1, 0) & \text{if } z = (m, 0). \end{cases}$$

Again, we set $m' = \cdot$ in ω_z to emphasize that matching is independent of money holdings.

Due to anonymity, matching and transfers cannot be based on the partner's age, productivity or trade history. The key deviation, therefore, is for a young to ask for a transfer on t, instead of making one, reverting to equilibrium (consuming) on $t+1$, when old. From Lemma 2 we know that the key date is $T-1$. To prove that money is essential, we must thus show that, without tokens, on date $T-1$ a young would misrepresent his needs by choosing to claim consumption instead of offering it. This deviation implies that on $T-1$ there are $N+1$ buyers and $N-1$ sellers, out of equilibrium, so the deviator consumes with probability $(N-1)/(N+1)$ and does nothing otherwise. Specifically:

1. On date $T-1$ the young deviator is unmatched with probability $2/(N+1)$. In that case denote the deviant's date T payoff by $\hat{v}_c(J-T+1)$, i.e., the value from starting T as an old who unsuccessfully defected on $T-1$. To determine $\hat{v}_c$, note that some old cannot consume on $T-1$, due to the deviation. This goes undetected since this agent and his dynasty dies out and the number of traders is unobservable. Hence, on T there are $N-1$ young sellers and N old buyers of home goods. Again, an old cannot match so $T+1$ starts with $N-1$ of old and $N-1$ young, who play equilibrium. Thus, on T the old deviator consumes only with probability $(N-1)/N$, so
$$\hat{v}_c(J-T+1) = \frac{N-1}{N}[\alpha u + \beta v_c(J-T)].$$

2. On date $T-1$ the young deviator matches to a young seller with probability $(N-1)/(N+1)$. In this case, two old cannot consume and their dynasties die out. The deviator's date $T-1$ payoff is $u + \beta \check{v}_c(J-T+1)$ where $\check{v}_c$ is the payoff from starting T as an old who successfully defected on $T-1$. This is undetected, so on T there are

[7]Doing so would require exchange of gifts for at least one date beyond $T-1$. However, T is the maximal element of the set $\{1, 2, \ldots, J\}$ that is consistent with the requisite $N \leqslant \bar{N}$ in Lemma 2. This means that the young would refuse to make unilateral transfers on any date $t > T-1$. So, gift-giving can only last up to date $T-1$.

$N-2$ young sellers and N old buyers of home goods. Again, on T two old cannot match, their dynasties die out and so $T+1$ starts with $N-2$ young and $N-2$ old, who play equilibrium. On T the old deviator consumes with probability $(N-2)/N$, so

$$\check{v}_c(J-T+1) = \frac{N-2}{N}[\alpha u + \beta v_c(J-T)].$$

Given the above, the start-of-date $T-1$ payoff to a young defector is denoted

$$\hat{v}_s = \frac{N-1}{N+1}[u + \beta\check{v}_c(J-T+1)] + \frac{2}{N+1}\beta\hat{v}_c(J-T+1).$$

On $T-1$ a defection takes place if $-e+\beta v_c(J-T+1) < \hat{v}_s$, rearranged as $N > \underline{N}$, where $\underline{N}$ is defined above. Note that $\partial\underline{N}/\partial e < 0 < \partial\underline{N}/\partial\beta$ and $\lim_{\beta\to 1}\underline{N} = [2(J-T+1)+u-e]/(u+e)$. □

The essentiality of monetary exchange rests on the features of anonymity and endogenous matching. Alternating production when young to consumption when old requires a precise pattern of matching and trade, which cannot be based on individual features such as age, productivity, or trade history. Hence, without money, a young might want to pass himself as buyer on date $T-1$, claiming a market good, instead of offering it. Knowledge of the deviation cannot be made public since the old leaves the economy and the number of traders is unobservable, which is good for the deviator. The downside is a permanent drop in productive capacity, since the defection deters consumption of at least one old. In a large market this has a small negative impact, which is why gift-giving *cannot* be sustained when $N \geqslant \underline{N}$. Naturally, greater production costs, more impatience, or shorter horizons raise the incentive to deviate, so $\underline{N}$ falls.

Selling for money is an obvious remedy to these incentive problems, since the initial old are endowed with it. Conditioning matching and trade on the partner's money holdings allows society to ensure that market goods are consumed only by the old and that a verifiable trade record can be passed on to future generations. Hence, we say that money is essential if the economy is sufficiently large. Given our finding in Lemma 2, however, monetary exchange may be unfeasible in such economies. Hence, we need one last step to complete the proof of Proposition 1.

4.5. Existence of monetary equilibrium

We now show that parameterizations exist, which satisfy both feasibility and essentiality.

Lemma 4. *Fix a date $T \leqslant J$. There exists a value $\bar{J} \in \mathbb{N}_+$ and an associated value $\bar{\beta} \in (0,1)$ such that for each $\beta \in (\bar{\beta}, 1)$ and $J \geqslant \bar{J}$ we have some $N \in \mathbb{N}_+$ that satisfies $\underline{N} < N < \bar{N}$. Given such a value N, we have that*:

(i) *there exists a monetary equilibrium as defined in* (4);
(ii) *the equilibrium allocation cannot be sustained without money.*

Proof. Fix a date $T \leqslant J$. Conjecture that (4) is a monetary equilibrium. The equilibrium is feasible if $N \leqslant \bar{N}$, by Lemma 2. Money is essential if $N > \underline{N}$, by Lemma 3. Note that $\lim_{\beta\to 0}(\bar{N} - \underline{N}) < 0$. In addition, it is easy to demonstrate that $\lim_{\beta\to 1}(\bar{N} - \underline{N}) > 0$ if J is sufficiently large. To see it, rearrange $\bar{N} - \underline{N}$ as follows:

$$\bar{N} - \underline{N} = \frac{u-e}{u+e}\left[\frac{\beta(1-\beta^{J-T+1})\alpha u}{1-\beta}\frac{1}{e} - 1\right].$$

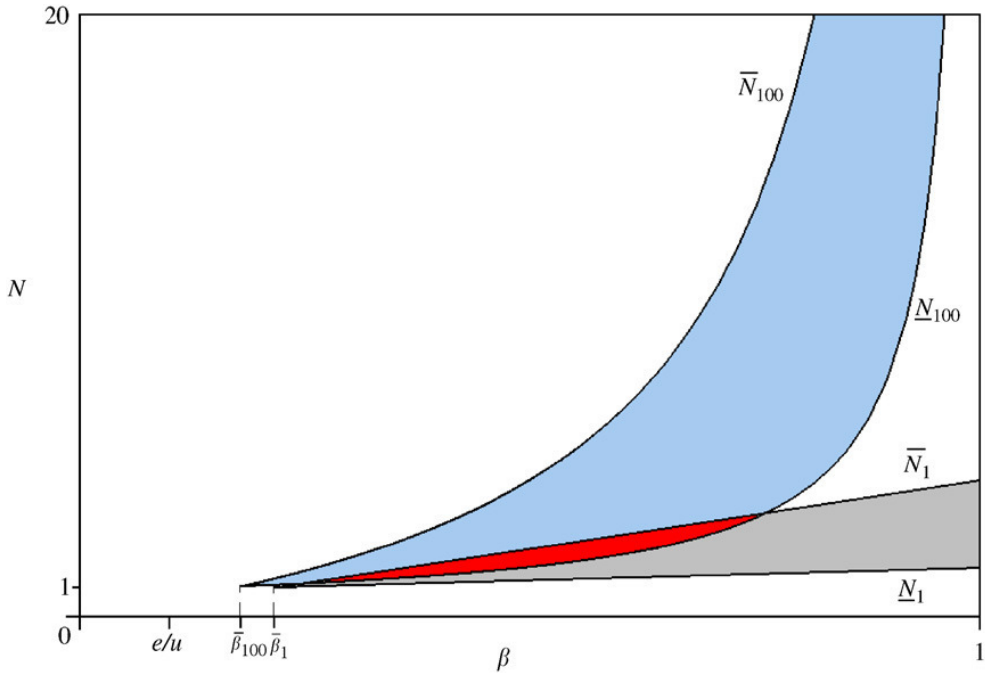
Notice that $\lim_{\beta\to 1}(1-\beta^{J-T+1})/(1-\beta) = J - T + 1$, and so $\lim_{\beta\to 1}(\bar{N} - \underline{N}) > 0$ if J is sufficiently large, say $J \geqslant \bar{J} \in \mathbb{N}_+$. Clearly, we can choose $\bar{J}$ large enough so that $\lim_{\beta\to 1}(\bar{N} - \underline{N})$ contains at least one positive integer (we need $N \in \mathbb{N}_+$). Thus, fix $J \geqslant \bar{J}$. Since $\bar{N} - \underline{N}$ is continuous in β then, by the intermediate value theorem, there exists a value $\bar{\beta} \in (0, 1)$ that satisfies $\bar{N} - \underline{N} = 0$. This $\bar{\beta}$ is unique since $\partial(\bar{N} - \underline{N})/\partial\beta > 0$. If $\beta \in (\bar{\beta}, 1)$ and $J \geqslant \bar{J}$ then we can choose some $N \in \mathbb{N}_+$ such that $\underline{N} < N < \bar{N}$. For these parameters, (4) defines an equilibrium that cannot be sustained without money. □

The lemma completes the proof of Proposition 1 and establishes three key requisites for feasibility and essentiality of monetary exchange. First, the economy's size must be moderate. This ensures that failure to sell on $T - 1$ hurts the deviator's payoff, or else no one would sell for money. It also prevents gift-giving arrangements, or else tokens would play no allocative role. The other two requisites are patient agents and enough dates after monetary trade stops. This ensures that production losses are offset by future consumption rewards. Intuitively, monetary trade of market goods is feasible in the early life of the economy only because some (less valuable) trading arrangement can be sustained after money loses value. The production loss on $T - 1$ must thus be compensated by a sufficiently long spell of trade and consumption of home goods. As the horizon shortens, the duration of monetary exchange shrinks and goes to zero if β is too low.

Lemma 4 also sheds light on the double role played by consumption externalities.[8] On one hand, consumption externalities can make monetary trade feasible even if trading horizons are finite. Money allows sellers to identify those who are in most need to consume and the presence of strong consumption externalities provides incentives to make unilateral transfers to these needy agents. As noted in Lemma 2, this occurs when the population is small, $N < \bar{N}$. On the other hand, consumption externalities facilitate gift-giving and so conspire against the essentiality of money. Now, for gift-giving to be unsustainable there must be strong incentives to misreport own consumption needs, i.e., consumption externalities must be sufficiently weak. Hence, Lemma 3 states that the population cannot be too small, $N > \underline{N}$. Finally, Lemma 4 ensures that there are parameters that satisfy both requirements, i.e., $\underline{N} < N < \bar{N}$ or, equivalently, that consumption externalities are moderate.

As an illustration we report results of numerical simulations involving two types of economies, one of which is longer lived than the other, i.e., for some given T we have $J - T + 1 = 100$ and $J - T + 1 = 1$. This distinction is reflected in the subindices of the thresholds $\underline{N}$ and $\bar{N}$.

[8] We thank a referee for suggesting this interpretation.



Each type of economy is characterized by different initial populations $2N$, ranging from 2 to 40, and different discount factors β.[9] In the area below $\bar{N}$ the monetary equilibrium is feasible, while above $\underline{N}$ monetary exchange is essential. Hence, the shaded areas identify finite-horizon economies in which monetary exchange is feasible *and* essential. Greater patience and longer horizons both expand the set of economies in which monetary exchange is feasible; the curves $\bar{N}$ are upward sloping and shift up with J. However, greater patience and longer horizons make money essential only in larger economies. Indeed, continuation payoffs increase with β and J, so it is easier to sustain gift-giving arrangements, which is why $\underline{N}$ is upward sloping and $\underline{N}_{100} > \underline{N}_1$. This explains the limited overlap of parameters consistent with feasible and essential monetary exchange in the two types of economies (the dark shaded area).

5. Concluding remarks

Our analysis has demonstrated that fiat monetary exchange can expand the set of allocations even if trading horizons are finite and deterministic. The result hinges on two features of the model. First, individual actions can affect the future availability of productive resources. So, agents may be willing to sell for money, even if on that date they have no reason to accept it. This makes monetary trade feasible in all preceding dates. Second, agents are anonymous and direct their search for partners. So, gift-giving arrangements may be prevented because agents can misrepresent their consumption needs. This makes money essential in exploiting any gains from specialization and trade.

It must be emphasized that the essentiality result is not robust to alteration of anonymity or matching protocol. Indeed, though anonymity is standard in the foundations of money literature, it may be an unreasonable feature of ‘‘small’’ economies. One may also suppose that agents are unable to direct their search for partners, in ‘‘large’’ economies. Essentiality

[9] The parameters are: $u = 10$, $e = 0.99$, $\alpha = 0.45$. This ensures that (1) is satisfied.

would suffer with either variation. For instance, gift-giving arrangements could be sustained if matching and trade is based on observable characteristics, such as age. This is also possible with anonymous random matching, since agents could end up being old and without money simply due to the randomness in meetings. In this case, it may be in the seller's best interest to produce for anyone and not only for those who hold money. In this sense, one can view our model as an extreme case of a more general environment which displays varying degrees of both randomness in matching as well as anonymity in trade meetings.

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