# Optimal Payment Arrangements: Money vs. Reciprocal Exchange

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**Abstract** This paper investigates the interactions between two competing exchange mechanism, reciprocal exchanges and monetized trades. Three sets of conditions are developed under which agents should only exchange goods with money, engage in reciprocal exchanges of gifts, and employ both record-keeping technologies side by side, respectively. In a monetary equilibrium with reciprocal exchange where money-less consumers get to consume free gifts from producers, such presence of gift-giving decreases the optimal value of money. On the other hand, the impact of a presence of monetized trades on the optimal size of a gift is ambiguous.

"*The presence of markets and money does not necessarily affect the economic system of a primitive society - this refutes the nineteen century myth that money was an invention the appearance of which inevitably transformed a society by creating markets, forcing the pace of the division of labor, and releasing man's natural propensity to barter, truck, and exchange.*"

Karl Polanyi, 1944, "The great transformation," p. 58.

"*Most, if not all economic acts are found to belong to some chain of reciprocal gifts and countergifts, which in the long run balance, benefiting both sides equally ... The man who disobey the rulings of law in his economic dealings would soon find himself outside the social and economic order.*"

Bronislaw Malinowski, 1926, "Crime and custom in savage society," p. 40.

## 1. Introduction

Traditionally, economists emphasize the role of money as a medium of exchange in overcoming the double coincidence of wants problem, the necessary condition of barter exchange.[1] In contrast, anthropologists who have worked on investigating primitive societies suggest that "*the hardships of barter played no role in the "invention" of money.*"[2] They believe that it was the sophisticated systems of reciprocal exchange, rather than simple barter, with which money was competing in the earliest stage of use.[3]

While traditional models assume that increased specialization of production came hand in hand with introduction of money and demise of barter trade, some of the systems or reciprocal exchange, such as *Kula* in western Malanesi islands or *Potlach* in Pacific Northwest, have flourished for decades after they were first integrated into the modern society.[4,5] Indeed, gift-giving that is based on expected reciprocity has propagated even in most developed societies.[6]

The purpose of this essay is to study the interaction between monetized trade and reciprocal exchange. It incorporates a *public record* that documents past actions into a search-theoretical model of outside *money*. Since supply of money is limited and a public record may not update immediately after each transaction, both record keeping devices have their own *pros* and *cons*. When individuals trade goods for money, exchanged money immediately becomes generally acceptable piece of evidence showing that its new owner had produced and provided valuable objects to others in the past. When individuals engage in reciprocal exchange, such a piece of evidence will not become available until after the public record has been updated. Therefore, of the two, the ownership of money can serve as more valuable claim of future consumption. On the other hand, money will not be able to facilitate any indirect exchange if a buyer does not own sufficient amount to make a purchase. In this regard, the public record obviously has an advantage over money since it automatically documents all gift-giving transactions.

This essay builds on earlier studies where a combination of money and the public record was introduced.[7] The idea that money is essential only because record of past actions is imperfect goes back at least to Ostroy (1973). Aiyagari and Wallace (1991) show that when public record is lacked, there cannot exist any credit between anonymous agents. In Kocherlakota (1996), having a perfectly updated public record makes money useless. Kocherlakota and Wallace (1998) first bridge the gap between the two extremes and show that there is some role for money if the probability of updating public record approaches zero. By adding a public record that imperfectly documents past transactions, they generalize the search-theoretic model of money to permit a mix of transactions: monetized trade and reciprocal exchange of gifts. In their model, not only direct bartering is prohibited, but indirect exchanges have difficulties to execute as well. Individuals are specialized in both production and consumption, they meet privately in pairs, and they cannot pre-commit themselves to any future actions. Thus, a potential consumer must use at least one of the two record-keeping technologies to show a producer that she deserves consumption goods. We eliminate all matches without a

[1] For traditional view see for example Smith (1776) and Jevons (1875). This view is formalized in studies of exchange in environments with spatial separation of agents, see for example Diamond (1984) and Kiyotaky and Wright (1989, 1993).

[2] Polanyi (1944), p. 276.

[3] See e.g. Loeb (1968).

[4] See e.g. Ritter (1995), Rauch (2001) or Camera et al. (2003) for recent studies emphasizing this view of introduction of money into an economy.

[5] See e.g. Firth (1939), Mauss (1967) and Gregory (1987) for descriptions of Kula, Potlach and a summary of other examples.

[6] See e.g. Kranton (1996) and Offer (1997) for summary of literature related to reciprocal exchange and gift-giving.

[7] For detailed summary of earlier research in this area, see Wallace (2000).

singe-coincidence-of-wants between agents and further investigate how changes in money supply affect the roles played by money and public record.

The paper proceeds as follows. Section 2 introduces the model and defines the optimum problem. Section 3 defines an equilibrium. Section 4 concentrates on describing equilibria allocations where either trade of goods for money or reciprocal exchange of gifts maximizes social welfare. Section 5 solves for the optimum problem. Section 6 discusses interaction of trade and gift-giving in the solution of the optimum problem where both money and a public record are essential record-keeping devices. Section 7 concludes earlier analysis.

## 2. The Model

In this economy, there is a unit mass of spatially separated households. Each household maximizes the sum of lifetime utilities of its two members: a producer and a consumer. Producers are specialized in production and consumers are specialized in consumption in such a way that (i) a household cannot consume its own (autarky) production, and (ii) any two households cannot barter and consume each other's production. Therefore, gains from trade can only be realized through a sequence of indirect exchanges.

Time is divided into exchange rounds, during which consumers and producers enter the market to produce, exchange and consume perishable consumption good. Depending on the choice of the record keeping technology, goods can be exchanged through either one of the following ways: (i) they can be traded through the use of money, or (ii) they can be given based on expected future reciprocity.[8]

### 2.1 Production Technologies and Preferences

Imagine an economy with $N$ ($N \geq 3$) types of non-storable consumption goods and $N$ types of households. There is an equal mass of households of each type. A producer from type $i \in \{0,1,...N-1\}$ household can only produce type $i$ good. A consumer from type $i$ household can only consume good of type $1+i\,(\text{modulo}\,N)$.

All agents live indefinitely and have a same discount rate $R \in (0,1)$. Consumption of $q$ units of a good yields an instantaneous utility $u(q)=q$. Production of $q$ units involves instantaneous costs in disutility $c(q)=q^2$.

### 2.2 Exchange Rounds

A continuous time is divided into a sequence of instantaneous exchange rounds during which agents produce, exchange and consume. As shown in Figure 2.1, in between these exchange rounds agents rest in their own households. The arrival rate of exchange rounds is a Poisson process with a constant rate $\pi \in (0,1)$. We define $r \equiv R/\pi$ as the effective rate of time preference.

[8] In addition to these two patterns of exchange of goods, agents can also give monetary gifts to each other. I do not investigate such a pattern in this study, since it does not add to the main topic.



Exchange is not centralized. At the beginning of each exchange round, consumers seek producers of the same type of good. Once a consumer from type $i$ household finds a producer from type $1+i\,(\text{modulo}\,N)$ household, such randomly matched pair of agents enters producer's home location. There they produce, exchange and consume good of type $1+i\,(\text{modulo}\,N)$. Since the populations of consumers and producers of the same type of good are equal, all agents always form single coincidence of wants matches.

### 2.3 Record Keeping Technologies

The economy begins with a proportion $m \in (0,1]$ of consumers of each type having a single unit of money. Money is a physical object that cannot be produced, modified or split in pieces. Each agent can only hold one or zero units of it. During an exchange round, consumers can pay money to their matching producers. In between the exchange rounds, producers redistribute their money holdings to consumers from their own households. For a simplicity, we also assume that money that cannot be given to a consumer is disposed. Since there is at most one unit of it per exchange round meeting, economy is not over-supplied with money.[9]

Similar to in Kocherlakota and Wallace (1998), two agents in a matched pair are assumed to have the following information. They recognize each other's identity and the identity of each other's second household member. They observe each other's holding of money. They know the public record of all actions that occurred prior to the most recent record updating. With probability $\delta \in (0,1]$ the record is updated at the very end of an exchange round. With probability $1-\delta$, record is updated sometimes in the future.

Each agent also keeps a private record of all actions of agents from other households they learned about. At the end of an exchange round, a consumer returns back to her home to share her private record with a producer from her household.

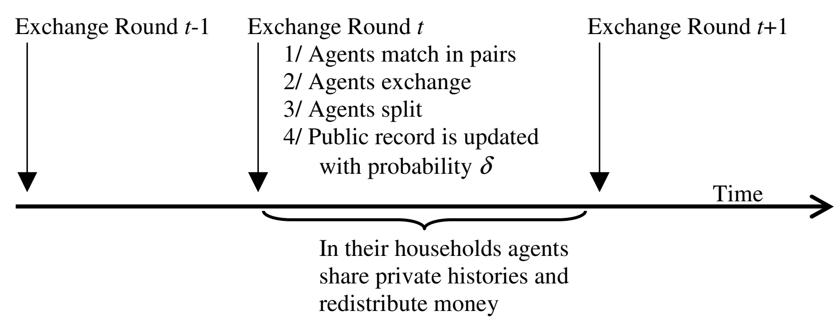


Figure 1: Timeline of Exchange Rounds

[9] In the model where agents can hold only limited amounts of money, it is possible that a buyer cannot buy, since a seller already own the maximal allowed amount of money. I do not focus on economy where the supply of money is too large to let agents trade efficiently. Thus I avoid this particular situation by imposing maximum of one unit of money per household. See Wallace (1997) for further discussion of this constraint.

This is a no-commitment environment where actions are publicly recorded by these the two imperfect record keeping technologies, and they can only be enforced by punishments imposed by agents themselves.

2.4 Exchange Mechanism

During exchange rounds, agents play a coordination game. All consumers and all producers simultaneously choose ordered tuples $a = (Q, M)$. If the tuples chosen by two agents in a pair are feasible and identical, then a producer produces $Q$ and is left with $M$ more units of money; a consumer consumes $Q$, and departs with $M$ less units of money.

Let $a_A = \{0,0\}$. Since by choosing a tuple $a_A$ agents can always avoid production and consumption and leave with the initial money holdings, they can always play sequentially and individually rational strategies.

2.5 The Optimum Problem

The optimum problem is to choose an incentive feasible allocation such that the social welfare is maximized. I restrict these allocations to those given by symmetric and stationary strategies. Let social welfare $W$ be the expected lifetime utilities of all the households at the time before the initial supply of money is distributed. That is

$$rW = m\left(q_m - q_{m^2}\right) + (1-m)\left(q - q^2\right) , \tag{1}$$

where $q_m$ and $q$ are time invariant quantities of good traded for money and of good given as reciprocal gifts respectively.

## 3. Symmetric and Stationary Equilibrium

For the described coordination game exchange mechanism with limited commitment, we focus on pure strategies that are symmetric and stationary.

**Definition.** *A symmetric and stationary equilibrium is a collection of strategies such that, if agents follow these strategies currently and in the future, then it is weekly optimal for agents to (i) play equilibrium strategy if their information set is consistent with every single agent having played equilibrium strategy in the past; (ii) play first off-equilibrium strategy if only their private information set reveals that some agent(s) did not play equilibrium strategy in the past; and (iii) play second off-equilibrium strategy if the public record reveals that some agent(s) did not play equilibrium strategy in the past.*

Equilibrium strategy is to exchange $Q = q_m$ when a good can be traded for valued money and $Q = q$ otherwise.[10] Should a deviation occur, agents play weekly optimal responses. These off-equilibrium strategies punish a deviating household, that so its members prefer to play equilibrium strategy. Since the allocation of goods across households that stay on the equilibrium path does not depend on off-equilibrium strategies, some of these allocations can be supported by more than one punishment.

[10] If money is not valued than its exchange is possible, but meaningless.



Note that the combination of a coordination game exchange mechanism and an autarky strategy $a_A$ by all agents, the largest available punishment, supports all possible equilibrium paths.[11] The advantage of such total demise of exchange by all agents over a punishment on only deviating households is that a complete autarky is robust to any deviations by matched pairs. Therefore, by focusing on these equilibria we do not omit any social welfare dominant feasible allocations of goods.

Once all agents know about a deviation, the punishment of mutual autarky results in zero gains from exchange. Since such off-equilibrium strategy punishes all households equally, those who have private knowledge of a deviation may prefer to deviate from equilibrium strategy as well.[12] Before these deviations show on the public record, all such producers would either exchange production only for money or play autarky strategy $a_A$, and all such consumers would ask for consumption goods as if the deviations had never occurred.

## 4. Equilibrium Allocations

Let $V_m$ and $V$ be the lifetime utilities of households that currently do and do not have a unit of money. In equilibrium, during an exchange meeting a consumer without money acquires utility $q$ and a consumer with money acquires utility $q_m - v_m$, where $v_m$ is the value of transferred money. At the start of every trading round a fraction $m$ of consumers have money. A producer in an exchange meeting decreases her household utility by $q^2$, when she produces a reciprocal gift, and by $v_m - q_m^2$ when she produces for money.

In equilibrium the lifetime utilities of households with and without money are:

$$rV_m = q_m - v_m + (1-m)\left(-q^2\right) + m\left(-q_m^2 + v_m\right) , \tag{2}$$

and

$$rV = q + (1-m)\left(-q^2\right) + m\left(-q_m^2 + v_m\right) . \tag{3}$$

Since the value of holding money $v_m$ equals to $V_m - V$, we can solve for equilibrium $V_m$ and $V$:

$$rV_m = m\left(q_m - q_m^2\right) + (1-m)\left(q - q^2\right) + \frac{r(1-m)(q_m - q)}{1+r} , \tag{4}$$

and

[11] This idea that indirect exchanges can be supported by a threat of autarky of all agents in the future is first seen in Kovenock and de Vries (2002).

[12] Notice that punishment of mutual autarky does not motivate agents to punish privately known deviators before such deviation becomes public. For example, an agent who was denied a gift may still prefer to buy the good using money from the same producer one round later. If only deviating households would be punished, this would not be the case. Punishment of mutual autarky assures that after a deviation appears on a public record, the deviathousehold cannot do better than autarky.

$$rV = m\left(q_m - q_m^2\right) + (1-m)\left(q - q^2\right) - \frac{rm(q_m - q)}{1+r}. \quad (5)$$

If a consumer or a producer deviates from equilibrium strategy, then the household will play first off-equilibrium strategies before the deviation appears on a public record, and second off-equilibrium strategies (autarky) forever after. We define $V_d$ as the lifetime utility of a household that knows privately about a deviation at the end of an exchange round but before the public record may get updated. After a deviation, with a probability $\delta$, public record will be updated and all households would enter autarky. With a probability $1-\delta$, the economy will continue with exchange for at least one more round.

A producer accepts money only if she believes that her household's consumer will be able to use such money in the future. When a public record is updated frequently, it is unlikely that any consumer will be able to make purchases during periods following a deviation. Thus, a producer who knows about a deviation not only refuses to provide a gift but also to trade good for money.[13] That is, the lifetime utility of her household is $V_d = V_{d1}$, where

$$rV_{d1} = (1-\delta)q - \delta V_{d1}. \quad (6)$$

On the other hand, when the probability of record updating is sufficiently small, all producers accept money before the public record is updated. Since the producers who have already known about a deviation do not provide gifts, lifetime utilities of their households without money are $V_d = V_{d2}$ and households with money are $V_{dm}$, where

$$rV_{d2} = (1-\delta)\left[q + m\left(-q_m^2 + V_{dm} - V_{d2}\right)\right] - \delta V_{d2}, \quad (7)$$

and

$$rV_{dm} = (1-\delta)\left[q_m - mq_m^2 + (1-m)(V_{d2} - V_{dm})\right] - \delta V_{dm}. \quad (8)$$

Since $V_{dm} - q_m^2 \geq V_{d2}$ and $V_{d2} > V_{d1}$ if and only if $\delta < \overline{\delta}$, with

$$\overline{\delta} = 1 - \frac{(1+r)q_m^2}{q_m - q}, \quad (9)$$

[13] When $\delta \geq \overline{\delta}$, money is no longer accepted by producers that know about a deviation, and so such private information spreads through the economy faster than the case of $\delta < \overline{\delta}$. Should the updating process of public record mirror such process, refusal of money would update public record faster and faster updating of record would encourage households to refuse money. Such a spin-over effect can result in multiple equilibria with symmetric pure strategies, so that money is accepted either by all households or by no household with private knowledge about a deviation.



the lifetime utility at the end of an exchange round of a deviating household is

$$V_d = \begin{cases} \dfrac{(1-\delta)\left[q + m\left(\dfrac{(1-\delta)(q_m - q)}{1+r} - q_m^2\right)\right]}{r+\delta} & \text{if } \delta < \overline{\delta} \\ \dfrac{(1-\delta)q}{r+\delta} & \text{otherwise} \end{cases} \quad (10)$$

A household may deviate from the equilibrium strategies by either its consumer or its producer. Clearly, since it is in the best interest of any household to receive gifts and to use money to buy consumption good, it is the producer who would deviate first. The producer will play equilibrium strategy, if and only if

$$V_m - q_m^2 \geq V_d, \quad (11)$$

and

$$V - q^2 \geq V_d. \quad (12)$$

We describe the three possible types of equilibria with exchange of goods in the following subsections.

### 4.1 Monetary Equilibrium without Reciprocal Exchange

In a monetary economy without reciprocal exchange, $q_m > q = 0$. If a household does not produce and trade production for money, then it cannot buy its consumption good and is left in autarky.[14] Additionally, the producer would deviate from exchanging $q_m$ of its production good for a unit of money, if and only if the present value of future consumption $\frac{q_m}{r+1}$ is below current production costs $q_m^2$. Therefore, the monetary equilibrium without reciprocal exchange exists given that $m > 0$ and $q_m \leq \frac{1}{r+1}$. The economy's welfare $W_m = \frac{m}{r}\left(q_m - q_m^2\right)$ is maximized at

$$q_m = \begin{cases} \dfrac{1}{r+1} & \text{if } r > 1 \\ \dfrac{1}{2} & \text{otherwise.} \end{cases} \quad (13)$$

That is,

$$W_m^{Max} = \begin{cases} \dfrac{m}{(r+1)^2} & \text{if } r > 1 \\ \dfrac{m}{4r} & \text{otherwise.} \end{cases} \quad (14)$$

[14] The punishment imposed by other agents, namely the mutual autarky, adds no additional leverage.

## 4.2 Non-monetary Equilibrium with Reciprocal Exchange

In a non-monetary economy with reciprocal gift exchange, $q_m = q > 0$. If a producer does not produce a gift for others, her household would get punished by the autarky outcome as soon as her behavior becomes a common knowledge. Each household can choose between participating on reciprocal exchange forever and receiving free gifts for a limited number of exchange rounds. Thus, by substituting $q_m = q$ into constraints (2.11) and (2.12), a household would not deviate if and only if $q_m = q \leq \frac{\delta}{r+\delta}$. The economy's welfare in the non-monetary equilibrium with reciprocal exchange $W_x = \frac{1}{r}\left(q - q^2\right)$ is maximized at

$$q_m = q = \begin{cases} \dfrac{\delta}{r+\delta} & \text{if } r > \delta \\ \dfrac{1}{2} & \text{otherwise.} \end{cases} \tag{15}$$

In other words,

$$W_x^{Max} = \begin{cases} \dfrac{\delta}{(r+\delta)^2} & \text{if } r > \delta \\ \dfrac{1}{4r} & \text{otherwise.} \end{cases} \tag{16}$$

## 4.3 Monetary Equilibrium with Reciprocal Exchange

In a monetary economy with reciprocal gift exchange, producers give gifts to money-less consumers. However, they trade goods using money as long as money is available. Thus $q_m > 0$, $q > 0$ and $q_m \neq q$. The quantities produced must satisfy constraints (11) and (12).

## 5. Solution to the Optimum Problem

All three types of equilibria described above exist for all feasible sets of model parameters $\{m, \delta, r\}$. Nonetheless, for a specific set of parameters $\{m, \delta, r\}$, there is only one equilibrium which is the solution to the optimum problem. That is, by maximizing the social welfare, we can find three regions of parameters where one type of the equilibrium generates higher social welfare than the other two. This is shown in Figure 2.2, where

$$\delta_X = \frac{r}{1+2r}, \tag{17}$$

and

$$\delta_{X2} = \frac{r(1-r)}{1+3r}. \tag{18}$$



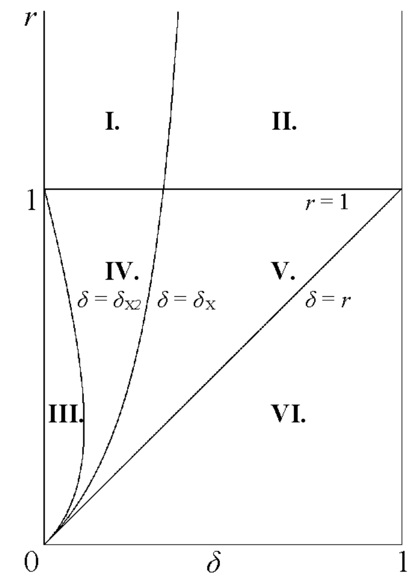


Figure 2: Social Welfare Maximizing Allocations
Monetary equilibrium without reciprocal exchange: I ($m \geq m_M$), II ($m \geq m_M$);
Non-monetary equilibrium with reciprocal exchange: II ($m \leq m_X$), V ($m \leq m_X$), VI;
Monetary equilibrium with reciprocal exchange:
I ($m < m_M$), II ($m \in (m_X, m_M)$), III, IV, V ($m > m_X$).

Monetary equilibria without reciprocal exchange are the solutions of the optimum problem only in economies where money is better record keeping device than the public record and where agents meet infrequently. If money is plentiful, this can be seen in regions I and II in Figure 2, where arrival rate of exchange rounds is small and discounting of future utilities is large. Non-monetary equilibria with reciprocal exchange solve the optimum problem if the conditions in the economy are reversed. That is if the supply of money is sufficiently small, than non-monetary equilibria with reciprocal exchange maximize the social welfare in regions II, V and VI. The monetary equilibrium with reciprocal exchange is the social optimum in the remaining part of the parameter space.

Money is never essential only in region VI, where not only the public record is updated regularly but exchanges also happen very frequently. These conditions resemble small and relatively closed tribal societies, where people interact frequently and thus deviations are quickly detected and punished. Anthropologists in general believe that the use of money in such societies lacks any economic meaning.[15] To be more specific, we quote Thurnsald (1932, p. 107.): "*if money is used, its function is quite different from that fulfilled in our civilization. …It never becomes an entirely abstract representation of value.*"[16]

In regions II and V where trade is less frequent and money is essential only when people have sufficient amount:

$$m \ge m_X = \frac{\delta - r + 2\delta r}{1 + \delta r}. \tag{19}$$

In other words, the more frequently people meet to exchange, the larger initial supply of money is needed for a monetary equilibrium to become the most efficient exchange arrangement.[17]

Finally, when the public record is updated infrequently as in regions I, III, and IV, money is always essential. Thus, it is only in region I, that even a small amount of money becomes essential and thus all transactions can be monetized. This result is consistent with Webber's (1927) view that "*the function of money as a general medium of exchange originated in foreign trade,*" where trade is infrequent and knowledge about trader's histories is limited.[18]

Figure 3 provides an example with parameters from region II, where all three types of equilibria potentially solve the optimum problem. For those parameters the non-monetary equilibrium with reciprocal exchange dominates for $m < m_X$, the monetary equilibrium without reciprocal exchange dominates for $m \ge m_M$ where

$$m_M = \frac{-B_M - \sqrt{B_M^2 - 4A_M C_M}}{2A_M}, \tag{20}$$

$A_M = -2(\delta + r)$, $B_M = (1-r)(1+2r-\delta r)$, and $C_M = (1+r)(\delta + r)$, and the monetary equilibrium with the reciprocal exchange dominates for $m \in [m_X, m_M)$.

These results are formalized in the following lemma:

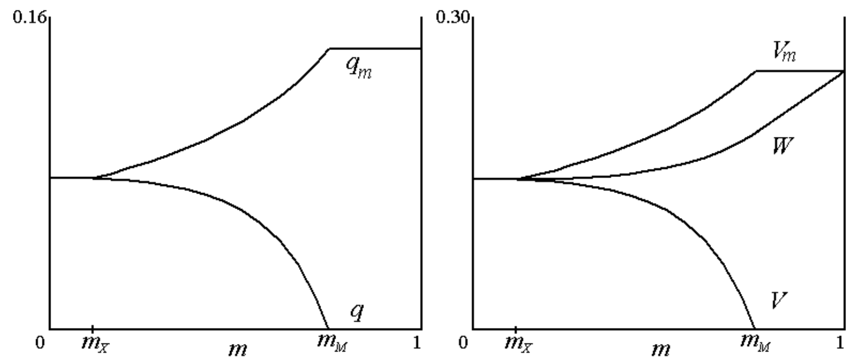


Figure 3: Solutions to the optimum problem in region II.

($r$=3, $\delta$=0.5, $m_X$=0.125 and $m_M$=0.760)

[15] See Thurnwald (1932, p. 105). In addition, Loeb (1968, p. 153) claims that "*The mere fact, that a tribe used money differentiated it very little economically from other tribes on the same cultural level, who did not.*"

[16]

[17] Note that when supply of money is small, its use can be limited to specific markets. It is known that in savage societies money often serves only exclusive set of infrequent transactions like salaries, taxes, fines or payments for brides. For salaries and taxes see Thurnwald (1932, p. 108) for brides and fines see Polanyi (1944, p. 276).

[18] Weber (1927, p. 238)



**Lemma 1.** *The solution to the optimum problem is*

*(i) a monetary equilibrium without reciprocal exchange if and only if*
$r > 1$ and $m \ge m_M$ ;

*(ii) a non-monetary equilibrium with reciprocal exchange if and only if*
$\delta > r$ or $(\delta \in [\delta_X, \min\{r,1\})$ and $m \le m_X)$;

*(iii) a monetary equilibrium with reciprocal exchange if an only if*
$[r \le 1$ and $((\delta \in (\delta_X, \delta_M)$ and $m > m_X)$ or $(\delta < \delta_X))]$ or $[r > 1$ and $((\delta \in (\delta_X, \delta_M)$ and $m \in (m_X, m_M))$ or $((\delta < \delta_X)$ and $m < m_M))]$.

**Proof:** See Appendix. ■

## 6. Interaction of Trade and Reciprocal Exchange

When a non-monetary equilibrium with reciprocal exchange solves the optimum problem, either one or both constraints (11) and (12) bind with equality. As a consequence, there are three cases that differ in their impact of trade transactions on efficiency of reciprocal exchange.

When the economy is characterized by parameters from regions I, II and IV (see for example Figure 3), gifts can be relatively large when compared to amounts of production traded for money. Consequently, when supply of money is small, the ownership of money is not valued much in optimum and thus money has only limited impact on social welfare. Only when people can trade goods for money in almost all meetings, then social welfare is maximized if money is valuable and gifts are marginalized. Since the larger the money supply, the smaller is the optimal size of a gift, an introduction of money can result in a complete demise of gift-giving.[19,20]

[19] Kranton (1996) provided model where agents can chose to participate in either trade of good for money, or bilateral reciprocal exchange, but cannot do both. As a result, participation externality provides an explanation why reciprocal exchange persists till today. Here agents can participate on both trade and reciprocal exchange simultaneously and a similar type of a participation externality also arises if a market is able to impose efficient prices. In such a case, once money is used, its use increases its value, thus attracts further inflow of money.

[20] For a supporting empirical evidence see for example Yellen (1990), where the *!Kung* tribe abandoned reciprocal exchange soon after the Botswana government encouraged monetized trade with the tribe.

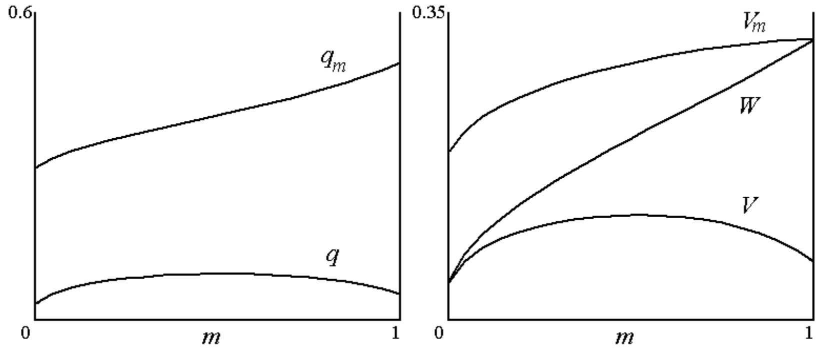


Figure 4: Solutions to the Optimum Problem in the Region III.
($r$=0.8 and $\delta$=0.02)

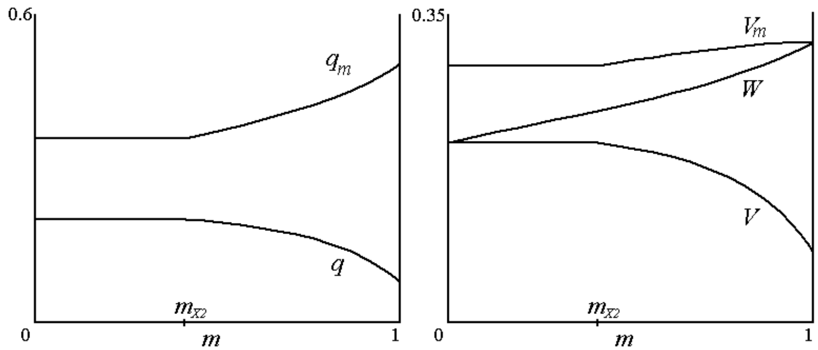


Figure 5: Solutions to the Optimum Problem in the Region IV.
($r$=0.8, $\delta$=0.2 and $m_{X2}$=0.417)

Figure 4 demonstrates that in the economy with parameters from region III an introduction of money can have a positive effect on the optimal size of a gift. In this economy, agents exchange frequently and the discounting is small, on the other hand, infrequent updating of public records allows for exchanges of only small gifts compared to the amounts of production traded for money. Since the use of money allows agents to utilize substantial gains from trade and increases their lifetime utilities, it also makes autarky a deterring punishment and thus increases the optimal size of a gift.

Finally, as it can be seen in Figure 5, for parameters from region IV, the optimal size of a gift is independent from money supply if $m \leq m_{X2}$. The following lemma summarizes the above discussion:



**Lemma 2.** *There exist sets of parameters such that the optimal size of a gift (i) increases with supply of money (regions I, II and V); (ii) decreases with supply of money (region III); (iii) does not change with supply of money (region IV).*

**Proof:** See Appendix. ■

Recall that the value of holding money is given by a difference between $V_m$ and $V$. As Figures 3, 4 and 5 show, the larger the supply of money, the larger is the value of each unit and so the possible incentive to further monetize such economy. While the dynamics of the model is left out for future research, this spin-over between optimal value and quantity of money suggests a clear explanation why monetary trade rarely co-exists with gift-giving transactions in the same markets.

## 7. Concluding Remarks

This essay investigates the interactions between two competing record-keeping technologies. It introduces money into an environment with imperfect knowledge of past histories, so that both monetized trade and reciprocal exchange transactions are allowed to exist. The equilibria in which individuals adopt symmetric and stationary strategies to achieve an incentive feasible allocation that maximizes the social welfare are identified.

The key determinant of efficient use of both record-keeping technologies is the interaction between values of exchanged gifts and money. Three sets of conditions are developed under which agents should only exchange goods with money, engage in reciprocal exchanges of gifts, and employ both record-keeping technologies side by side, respectively.

In a monetary equilibrium with reciprocal exchange where money-less consumers get to consume free gifts from producers, such presence of gift-giving decreases the optimal value of money. On the other hand, use of money can result in more efficient exchanges and thus increase agents' lifetime utilities. If the autarky becomes more harmful punishment to deviators, the economy can then support reciprocal exchanges of larger gifts. Therefore the impact of monetized trade on the optimal size of a gift is ambiguous.

## References

Camera, G., R. Reed and C. Waller, 2003 "A Jack of All Trades or a Master of One? Specialization, Trade and Money." *International Economic Review* 44, 1275–1294.

Diamond, P. A., 1984, "Money in Search Equilibrium," *Economica* 52(1), 1-20.

Firth, R., 1939, "Primitive Polynesian Economy," Routledge, London.

Gregory, C. A., 1987, "Gifts," in The new Palgrave dictionary of economics, Stockton Press, New York.

Jevons, S. W., 1875, "Money and the mechanism of exchange," H. S. King & Co., London.

Kiyotaky, N. and R. Wright, 1989, “On Money as a Medium of Exchange,” *Journal of Political Economy* 97, 927-954.

Kiyotaky, N. and R. Wright, 1993, “A Search-Theoretic Approach to Monetary Economics,” *American Economic Review,* 86(4), 63-77.

Kocherlakota, N, 1998, “Money is Memory,” *Journal of Economic Theory* 81(2), 232-251.

Kocherlakota, N. and N. Wallace, 1998, “Incomplete Record-Keeping and Optimal Payment Arrangements,” *Journal of Economic Theory*, 81(2), 272-289.

Kovenock D. and C. de Vries, 2002, “Fiat Exchange in Finite Economies,” *Economic Inquiry* 40(2), 147-157.

Kranton, R. E., 1996, “Reciprocal Exchange: A Self-Sustaining System,” *The American Economic Review,* 86(4), 830-851.

Loeb, E. M., 1968, “The Distribution and Function of Money in Early Society,” Essays in anthropology, Books for Libraries Press, Freeport, First Published in 1936.

Malinowski, B., 1926, “Crime and custom in savage society,” Harcourt, Brace & company, New York.

Mauss, M. 1967, “The gift: forms and functions of exchange in archaic societies,” Norton, New York, First Published in 1925.

Offer, A., 1997, “Between the Gift and the Market: The Economy of Regard,” *Economic History Review*, 50(3), 450-76.

Ostroy, J., 1973, “The Informational Efficiency of Monetary Exchange,” *American Economic Review* 63(4), 597-610.

Polanyi, K., 1944, “The great transformation,” Farrar & Rinehart, New York.

Ritter, J. A., 1995, “The Transition from Barter to Fiat Money,” *American Economic Review* 85, 134-149.

Rauch, B., 2001, “Money and Specialization,” manuscript.

Thurnwald, R., 1932, “Economics in Primitive Communities,” Oxford university press, London.

Wallace, N., 1997, “Short-Run and Long-Run Effects of Changes in Money in a Random-Matching Model,” *Journal of Political Economy* 105(6), 1293-1307.

Weber, M., 1927, “General Economic History,” Allen & Unwin, London.

Yellen, J. E., 1990, “The Transition of the Kalahari !Kung,” *Scientific American* 262(4), 96-105.



## Appendix

**Proof of Lemma 1.** The equilibria and corresponding regions of parameters where they exist are given by a solution of the following Lagrangian

$$L(q_m, q, \mu_m, \mu) = W - \mu_m \Delta_m - \mu \Delta$$

where $\Delta_m$ and $\Delta$ are given by constraints (2.11) and (2.12):

$$\Delta_m = q_m^2 + V_d - V_m \text{ and } \Delta = q^2 + V_d - V.$$

Lets first assume that both constraints bind with equalities, such that $\Delta_m = \Delta = 0$ and $\mu_m \geq 0$, $\mu \geq 0$. If ( $\delta < \overline{\delta}$ and $\Delta = 0$ ), then $V_{d2} + q_m^2 \leq V_{d2m}$ and $V_{d2m} < V_m$. Thus $V_m$ must be larger than $V_{d2} + q_m^2$ and so $\Delta_m < 0$. Therefore $\delta \geq \overline{\delta}$ and can substitute $V_d = V_{d1}$ to solve for $\Delta_m$ and $\Delta$:

$$r\Delta_m = \frac{r(1-\delta)q}{r+\delta} - (m+r)\left(\frac{q_m}{r+1} - q_m^2\right) - (1-m)\left(\frac{q}{r+1} - q^2\right)$$

and

$$r\Delta = \frac{r(1-\delta)q}{r+\delta} - m\left(\frac{q_m}{r+1} - q_m^2\right) - (1-m+r)\left(\frac{q}{r+1} - q^2\right).$$

Note that in equilibrium, $\mu_m \Delta_m = 0$, $\Delta_m \leq 0$, $\mu_m \geq 0$ and $\mu\Delta = 0$, $\Delta \leq 0$, $\mu \geq 0$.

I first describe the functions that define the solution and then I discuss the solution of the Lagrangian itself. Let

$$m_{X2} = \frac{-B_{X2} - \sqrt{B_{X2}^2 - 4A_{X2}C_{X2}}}{2A_{X2}}$$

where $A_{X2} = 2(\delta + r)$, $B_{X2} = 1 + \delta + r - r^2 + \frac{(r+\delta)\left[\delta + r\delta + r^2(1-\delta)\right]}{\delta - r + 2\delta r}$ and $C_{X2} = (1+r)(\delta - r)$. Note that $m_M \in [0,1]$ for $r \geq 1$, $m_X \in [0,1]$ for $\delta \geq \delta_X$, $\delta_X \in (0,1)$ always, $m_{X2} \in [0,1]$ for $\delta \in [\delta_{X2}, \delta_X]$ and $\delta_{X2} \in (0,1)$ for $r \in (0,1)$.

There are three non-empty regions of parameters, where in equilibrium both constraints (2.11) and (2.12) bind with equalities: (i) For $q_m = \frac{1}{1+r}$, $q = 0$, $\mu_m = \frac{m(1+r)(1+\delta r) - 2m^2(\delta + r)}{\delta(1+r)^2}$ and $\mu = -\mu_m + \frac{(\delta + r)[r(2m-1) - 1]}{\delta(1+r)^2}$, economy uses only money as a sufficient record of past transactions. This equilibrium exists if and only if $r \geq 1$ and $m \geq m_M$. (ii) For $q_m = q = \frac{\delta}{\delta + r}$, $\mu_m = \frac{m(1+\delta r)(r-\delta)}{\delta(1+r)(2\delta r - r + \delta)}$ and $\mu = -\mu_m + \frac{r-\delta}{\delta(1+r)}$, economy uses only public record of past transactions and money is left idle. Such equilibrium exists if and only if $\delta \in [\delta_C, r]$ and $m \le m_X$. (iii) Finally, for $q_m = \frac{(1-\delta)r}{(\delta+r)(1+r)}$, $q = \frac{\delta}{\delta+r}$, $\mu_m = \frac{2m^2(r+\delta)}{\delta(1+r)^2} - \frac{m(r^2\delta + 3r\delta^2 + 2\delta^2 + \delta - r + 2\delta r)}{\delta(1+r)(\delta - r + 2\delta r)}$ and $\mu = -\mu_m + \frac{(r-\delta s)(1+r) - 2mr(\delta+r)}{\delta(1+r)^2}$, both money and public record are used in this equilibrium if and only if $\delta \in [\delta_{X2}, \delta_X]$ and $m \le m_{X2}$. Note that in the fourth region, $q_m = q = 0$, $\mu_m = \frac{m(1-\delta(2+r))}{\delta(1+r)}$ and $\mu = -\mu_m - \frac{(\delta+r)}{\delta(1+r)}$, economy is in autarky. Since $\mu + \mu_m < 0$, therefore when money or public record is available, this region of parameters is empty and so autarky is never an equilibrium.

In one region of parameters both constraints bind with strict inequalities, such that $\Delta_m < 0$, $\Delta < 0$ and $\mu_m = \mu = 0$: If and only if $\delta < r$ there exists an equilibrium where $q_m = q = \frac{1}{2}$ and $\Delta_m = \Delta = -\frac{(1+r)(\delta - r)}{4(\delta + r)} < 0$. Therefore only public record would be used and socially optimal quantities of $q_m = q = \frac{1}{2}$ would be produced if only if $\delta \le r$.

Finally, consider the last two parameter regions left. Let region A be parameters where $[r \ge 1 \text{ and } [(\delta \in [\delta_X, 1] \text{ and } m \in (m_X, m_M)) \text{ or } (\delta \in (0, \delta_X) \text{ and } m \in (m_{X2}, m_M))]]$ or $[r < 1 \text{ and } [(\delta \in [\delta_X, r] \text{ and } m \in (m_X, 1)) \text{ or } (\delta \in (\delta_{X2}, \delta_X) \text{ and } m \in (m_{X2}, 1))]]$ and let region B parameters where $r < 1$ and $\delta \in (0, \delta_{X2})$ and $m \in (0,1)$. In both regions A and B, one constraint is binding with equality and one with inequality.

First, note that at any border between region A and of all previously identified regions where both or none constraints were binding with equality, it is that:

$$\mu_m = \mu = 0 \quad \text{for } [\delta = r \text{ or } (r = 1 \text{ and } m = m_M) \text{ or } (\delta = \delta_X \text{ and } m = m_X)],$$

$$\mu_m > \mu = 0 \quad \text{otherwise.}$$

By continuity of $\Delta_m$ and $\Delta$, since any two regions of parameters where $\Delta < 0 = \Delta_m$ and where $\Delta = 0 < \Delta_m$, would have to be separated by region where $\Delta = \Delta_m$, it is that $\mu_m \ge \mu = 0$ and $\Delta < \Delta_m = 0$. Since $\delta < \vec{\delta}$ implies $\Delta_m < 0$, so if $\Delta < \Delta_m = 0$ then $\delta \ge \vec{\delta}$ and $V_d = V_{d1}$. Also, since at any border between this region and previously identified regions, $q_m \ge q$, and since $\Delta_m > \Delta$ implies that $q_m \ne q$ inside this region, therefore $q_m > q > 0$. In equilibrium both money and public record are used.

To solve for optimal $q_m$ and $q$ in region A, use $\Delta_m = 0$ to get

$$q = \arg\max\{m(q_m(q) - q_m^2(q)) + (1-m)(q - q^2)\},$$

where $q_m(q) = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$, $A = (\delta + r)(r + m)$, $B = -(1+r)(r+\delta)(r+m)$, and $C = q[(1-\delta)(1+r) - (1-m)(r+\delta)(1-(1+r)q)]$.



At the borders of final parameter region B, if $m = 1$ then $q_m = \frac{1}{2} > q$ and $\mu = \mu_m = 0$; if $m=0$ then $q_m = \frac{(1-\delta)r}{(\delta+r)(1+r)} > q = \frac{\delta}{\delta+r}$ and $\mu > \mu_m = 0$. Since $q_m = q$ implies that $\Delta_m = \Delta$, and since in this region $\Delta = 0 > \Delta_m$, thus in this region $q_m > q$ and money are valued positively. Similarly to previous case, we can solve for optimal $q_m$ and $q$ in region B. Let $\Delta = 0$ so that

$$q(q_m) = \arg\max\{m(q_{m2}(q) - q_{m2}^2(q)) + (1-m)(q - q^2)\},$$

where $q_{m2}(q) = \frac{-B_2 - \sqrt{B_2^2 - 4A_2C_2}}{2A_2}$, $A_2 = m(1+r)(\delta+r)$, $B = -m(r+\delta)$ and $C = q|(1+r)^2(\delta - q[\delta + r]) - m(r+\delta)(1-(1+r)q)|$.

Note that unlike in the first case of monetary equilibrium with reciprocal exchange, in the two following cases the values of both gifts and monetary purchases was not constant, $q = \frac{\delta}{\delta+r}$ and $q_m = \frac{(1-\delta)r}{(\delta+r)(1+r)}$, but varied with the money supply. ■

**Proof of Lemma 2.** In regions I., II. the size of gifts is positive when money supply is small, but diminishes to zero as $m$ increases to $m_M < 1$. In region V. the size of gifts is maximized only for $m \le m_X < 1$.

In region III. the size of a gift for $m \to 0$ is strictly smaller than that for $m \to 1$ for all parameters but $\delta = \frac{r(1-r)}{3r+1}$, where the size of a gift is $q = \frac{1-r}{2(1+r)}$ for both cases. Let $\delta = \frac{r(1-r)}{3r+1}$, since $\left.\frac{\partial W(q, q_{m2}(q))}{\partial q}\right|_{q = \frac{1-r}{2(1+r)}} \ne 0$ for $m \in (0,1)$ so $q \ne \frac{1-r}{2(1+r)}$ and the optimal size of a gift must both increase and decrease on interval $m \in (0,1)$.

In region IV. $q = \frac{\delta}{\delta + r}$ for $m \le m_{X2}$. ■