

## Mesoscale Meteorology: Radar Fundamentals

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### *Introduction*

A weather radar emits electromagnetic waves in pulses. The wavelengths of these pulses are in the microwave portion of the spectrum; for NWS WSR-88D radars,  $\lambda = 10.7$  cm. These waves travel at the speed of light, or  $3.0 \times 10^8$  m s<sup>-1</sup>. Since frequency is equal to the speed of light divided by wavelength, the frequency of the WSR-88D is on the order of 3 GHz. The total energy emitted by a radar beam is on the order of 100-1000 kW. Upon interacting with atmospheric phenomena, some of the waves' energy is absorbed, scattered, or transmitted. It is the small portion of order  $10^{-8}$  to  $10^{-16}$  kW that is backscattered (directed back toward the radar) in which we are interested.

The length of time that a radar transmits energy is known as the *pulse duration*, which for NWS WSR-88D radars is on the order of  $10^{-6}$  s. The *pulse repetition frequency* (PRF) is the number of pulses transmitted per second by the radar and is on the order of  $10^3$  pulses s<sup>-1</sup>. The *pulse repetition time* (PRT) is the elapsed time between pulses, or the inverse of the PRF (on the order of  $10^{-3}$  s). The importance of the PRF and PRT to the maximum detectable distance and unambiguous velocity will be demonstrated shortly.

A radar beam's *angular width* determined by the width of the region of transmitted energy bounded by half of the maximum power. In general, this is less than 1° in azimuth. As the angular width is constant with distance, the *physical width* of the beam increases with distance from the radar. The radial data resolution is a function of the pulse duration and the speed of light (e.g., distance that a single pulse can cover), while the azimuthal data resolution is a function of the angular width and distance from the radar. For current-generation NWS WSR-88D radars, the radial data resolution is 250 m and the azimuthal data resolution is 0.5°. The physical azimuthal data resolution can be obtained at any distance from the radar by dividing the circumference of the circle cut out by the range ring at that distance by the number of discrete azimuthal bins (for 0.5°, this is 720).

Radars typically scan at multiple elevation angles, with the lowest just above the horizon and the highest at ~20° above the horizon. Assuming that a radar moves through all elevation angles before repeating a given elevation angle scan, the updating frequency at any given elevation angle is inversely proportional to the number of elevation angles. Scanning strategies that return to lower elevation angles before completing a full volume scan have recently been developed; the SAILS algorithms recently implemented by the NWS are one such example. Generally, low altitudes can only be sensed by radar at short distances from the radar.

### *Radar Scanning and Returned Energy*

The *pulse volume* defines the three-dimensional area that a single pulse covers. As physical width increases away from the radar, the pulse volume increases with distance from the radar. The *pulse energy* over the pulse volume is constant, however, which results in *power density* (energy divided by volume) decreasing with distance from the radar. The power density determines the amount of

energy that a radar target can intercept and backscatter toward the radar; thus, targets closer to the radar will have greater power density.

The amount of energy backscattered by a target to a radar is a function of the target's size, shape, state (e.g., ice vs. liquid), and concentration. Larger-sized targets will backscatter more energy to the radar, albeit subject to the scattering caveat noted below. Because radar pulses have horizontal polarization, objects that are wider – such as raindrops – will backscatter more energy to the radar. Small ice particles will generally backscatter less energy, while large ice particles will generally backscatter more energy, to the radar. Water has a larger dielectric constant than does ice and thus backscatters more energy to the radar. Finally, when particle concentration within the pulse volume is large, so too is the backscattered energy to the radar.

There are two types of scattering of particular importance to radars: *Rayleigh scattering* and *Mie scattering*. In the former, target diameter (or size) is much less than the radar beam's wavelength. In the latter, target diameter is approximately equal to the radar beam's wavelength. The assumed scattering type determines how returned radar energy is related to, for instance, target reflectivity. For liquid species such as raindrops, target diameter is much less than radar beam wavelength, and Rayleigh scattering is assumed. This assumption is violated for most hail stones, however. Since hail is not the primary feature we wish to detect with radar, we assume Rayleigh scattering and take care when interpreting radar echoes where hail may be present.

The amount of energy backscattered to the radar is given by the Probert-Jones equation:

$$P_r = \frac{P_t G^2 \theta^2 H \pi^3 K^2 L_a}{709.78 \lambda^2} \frac{z}{R^2}$$

Here,  $P_r$  is the power returned to the radar and  $P_t$  is the peak transmitted power. Both are in W.  $G$  is the antenna gain, related to the ability to focus outgoing energy into a concentrated beam.  $H$  is the pulse length, here given by the radial data resolution defined previously.  $K$  is a physical constant describing the physical properties of a given target; for the WSR-88D,  $K^2$  is set to 0.93 assuming only liquid targets.  $z$  is the target reflectivity.  $R$  is the target range; note its presence in the denominator to account for decreasing power density with increasing distance from the radar.  $\theta$  is the angular width.  $L_a$  is a measure of signal loss factors associated with, e.g., attenuation.  $\lambda$  is the wavelength (10.7 cm for the WSR-88D).

For a given radar, such as the WSR-88D, the only variables that are not known constants are  $P_r$ ,  $z$ ,  $L_a$ , and  $R$ . This allows the above equation to be simplified as:

$$P_r = \frac{C_r z L_a}{R^2}, \text{ or } z = \frac{C_r P_r L_a}{R^2}$$

Here,  $C_r$  is known as the *radar constant*. Because of the definition of  $K$ , this  $Z$  defines an *equivalent reflectivity* assuming only liquid targets. It can be viewed as a range-corrected measure of returned power. Reflectivity values can range over many orders of magnitude; the *logarithmic reflectivity factor* provides a measure of rectifying this:

$$Z = 10 \log \left( \frac{z}{1 \text{mm}^6 \text{m}^{-3}} \right)$$

It is this *logarithmic reflectivity factor* that we commonly refer to as *reflectivity*, with units of dBZ.

### *Radar Principles: Complicating Factors*

When a target fills the entire beam, it will have equal reflectivity no matter its distance from the radar due to the  $R^{-2}$  factor in the definition of  $Z$  above. More commonly, however, a target will not fill the entire beam. Other targets, or the lack thereof, within the beam will return different (often reduced) power to the radar. Thus, for two equivalent targets, the one closer to (further from) the radar will typically have higher (lower) reflectivity.

As a radar beam traverses the atmosphere and encounters various targets – some meteorological, others not – some of the transmitted energy is absorbed or scattered in other directions. The loss of energy along the path of the radar beam is known as *attenuation*. Since the absorptivity of water vapor generally decreases with increasing wavelength through the microwave portion of the electromagnetic spectrum, attenuation is inversely related to wavelength. Longer wavelength radars such as the WSR-88D are less impacted by attenuation than are shorter wavelength radars. Heavy precipitation can attenuate sufficient energy such that more distant targets are represented as having lower reflectivity than if the nearest targets were not present.

Earlier, we said that radars emit energy in narrow conical-shaped beams of electromagnetic pulses. We used information about how the beam's power decreases away from the center of the beam to define the angular width of the radar beam. This does not include all power emitted by the beam, however; some of this power is emitted on its adjacent sides, here termed *sidelobes*. Radars assume that all backscattered energy comes from the main lobe, which is a reasonable assumption in most cases. *Sidelobe contamination* occurs when some backscattered energy is associated with targets intercepted by the sidelobes. This is most common with strong temperature inversions and with intense convection in close proximity to the radar.

It may go without saying that the targets intercepted by a radar beam need not be meteorological in nature. Buildings, topographic features, and biological scatterers such as bugs and chaff can also be detected by radar. Stationarity and permanence is a hallmark of buildings and topography. Non-stationary biological scatterers are often detected by other means, including pattern recognition and advanced dual-polarization technology (discussed later).

### *Radar Principles: Distance, Direction, and Elevation*

The distance of a target from the radar is related to the time that it takes for a portion of the energy emitted by the radar to be returned to it after scattering by the target:

$$r = \frac{ct}{2}$$

where  $c$  is the speed of light and  $t$  is time. The product is divided by 2 because  $t$  measures the time for the emitted energy to travel from the radar to the target **and** from the target back to the radar. Only the one-way travel time is needed to infer distance. The direction and altitude of a target are functions of the angle of the radar beam from due north and from the horizon, respectively. These three pieces of information enable target location identification.

Determination of a target's altitude must also consider the *Earth's curvature*, which results in the radar beam targeting progressively higher altitudes with increasing distance from the radar, and *refraction* via Snell's law, which results from atmospheric density being non-constant with height. As density typically decreases with increasing altitude, refraction generally results in the radar beam targeting progressively lower altitudes with increasing distance from the radar.

From the ideal gas law, recall that density is directly (inversely) proportional to pressure (virtual temperature). In estimating target altitude, algorithms typically assume a standard atmosphere with an approximately moist adiabatic lapse rate to an altitude of  $\sim 11$  km above ground level. When the lapse rate is closer to dry adiabatic, density decreases less rapidly with height than in the standard atmosphere. This defines *subrefraction*, leading to underestimated target altitudes. When the lapse rate is less than moist adiabatic, as with an inversion, density decreases more rapidly with height than in the standard atmosphere. This defines *superrefraction* and overestimated target altitudes.

### *The Doppler Effect*

Targets moving toward (away from) the radar result in shorter (longer) distances successive pulses must travel before intercepting the target. This changes the wave phase: backscattered energy has reduced (increased) wavelength than it had when transmitted. This phase shift can be used to infer the target's radial velocity. The phase shift itself is given by:

$$\Delta\phi = \frac{4\pi\Delta r}{\lambda}$$

$\Delta r$  is the distance traveled between pulses (positive if the target is in the positive radial direction, or away from the radar),  $\lambda$  is the radar's wavelength, and  $\Delta\phi$  is the phase shift in radians. Radial velocity  $v_r$  is defined as the change in radial position over the time between pulses (the PRT), i.e.,

$$v_r = \frac{\Delta r}{\Delta t} = \frac{\lambda}{4\pi} \frac{\Delta\phi}{PRT}$$

Thus, with knowledge of the PRT and the phase shift, the radial velocity can be obtained. As with distance, positive radial velocity indicates a target moving away from the radar. This velocity is the average radial velocity of all targets within a given radar beam.

Note that there is no change of phase associated with a target moving perpendicular to the radar beam (i.e., at constant range). Consequently, only the radial velocity can be measured by a single Doppler radar. Further, the increasingly large elevation to the radar beam with distance from the

radar results in a single elevation angle scan depicting winds at multiple altitudes. These factors impact how radar velocity signatures are interpreted, as demonstrated in accompanying examples.

### *Range and Velocity Folding and the Doppler Dilemma*

Since radars transmit pulses discretely, rather than continuously, the listening time between pulses (or the PRT) defines to the maximum distance at which objects can be detected. As pulses travel at the speed of light, the maximum distance at which objects can be detected is given by:

$$r_{\max} = \frac{cPRT}{2} = \frac{c}{2PRF}$$

While a target at a distance greater than  $r_{\max}$  can backscatter energy to the radar, the next pulse of energy will have already been transmitted, and the radar will be awaiting backscattered energy from that pulse. Thus, backscattered energy from targets further than  $r_{\max}$  from the radar will be interpreted as coming from a later pulse. This gives the appearance of false targets at distances less than  $r_{\max}$  from the radar: a target that lies just beyond  $r_{\max}$  will appear close to the radar, a target that lies halfway between  $r_{\max}$  and  $2r_{\max}$  will appear halfway between the radar and  $r_{\max}$ , and a target that lies just inside of  $2r_{\max}$  will appear just inside of  $r_{\max}$ . This describes what is known as *range folding*. Range folded echoes tend to be elongated because of the large angular width of the radar beam at such large radii; they also tend to be weak due to the low power density of the radar beam at such large radii.

Similarly, if the PRT is too large, or PRF too small, then the distance that a target travels between pulses may result in a phase shift that is greater than that which can be unambiguously detected. This most commonly results in a positive radial velocity being aliased or folded to a negative radial velocity, or vice versa. A half-wavelength shift in either direction can be unambiguously detected if the pulse did not have to return to the radar; because it does, then this shift is halved. Thus, the maximum unambiguous velocity, or *Nyquist velocity*, that may be detected is given by:

$$v_{\max} = \pm \frac{\lambda}{4PRT} = \pm \frac{\lambda PRF}{4}$$

Note, however, that both  $r_{\max}$  and  $v_{\max}$  are related to the PRT or PRF. If we solve each for the PRT and set the resulting expressions equal to each other, we obtain:

$$\frac{2r_{\max}}{c} = \pm \frac{\lambda}{4v_{\max}}, \text{ such that } r_{\max} v_{\max} = \pm \frac{c\lambda}{8}$$

Because  $c$  and  $\lambda$  are fixed, as  $r_{\max}$  increases,  $v_{\max}$  must decrease, or vice versa. Stated differently, high PRT/low PRF allows large  $r_{\max}$ , while low PRT/high PRF allows large  $v_{\max}$ . Consequently, radars have a trade-off between range and maximum detectable velocity: you can't have your cake and eat it too! This is known as the *Doppler dilemma*. Due to the reduced absorptivity and scattering by water molecules at longer wavelengths, radar effectiveness decreases as  $\lambda$  increases, negating the added range and/or maximum detectable velocity that would result.

For the WSR-88D, with  $\lambda = 10.7$  cm, the product of  $r_{max}$  and  $v_{max}$  can have a magnitude as large as 4,012,500. For  $r = 25$  km,  $v_{max} = 160.5$  m s<sup>-1</sup>. For  $r = 50$  km,  $v_{max} = 80.25$  m s<sup>-1</sup>. For  $r = 100$  km,  $v_{max} = 40.125$  m s<sup>-1</sup>. Figure A.7 in the course text provides a graphical illustration of this relation.

### *Accumulation Precipitation Estimation*

Both reflectivity and rain rate are functions of the drop size distribution: reflectivity is related to the sixth power of the drop diameter, while rain rate is related to drop volume (the third power of the drop diameter). We generally do not know the drop size distribution to any reasonable degree of accuracy. We know conditions under when drop sizes are generally large (e.g., melting solids) and small (e.g., maritime liquid droplets) but need other tools – disdrometers, dual-polarization radar, etc. – to be able to measure or infer observed drop sizes.

However, the desire to estimate precipitation using radar has led to the development of reflectivity-rain rate, or  $Z$ - $R$ , relationships. The general form of a  $Z$ - $R$  relationship is given by a power law:

$$Z = AR^B$$

where  $A$  and  $B$  are specified constants that are related to the drop size distribution. Generally, these are determined empirically from limited observations in particular meteorological conditions. For example, the Marshall-Palmer drop size distribution applicable for general stratiform precipitation has  $A = 200$  and  $B = 1.6$ , whereas the standard WSR-88D summertime deep, moist non-tropical convection distribution has  $A = 300$  and  $B = 1.4$ . Many other such relationships exist. For all such relationships, however, rain rate  $R$  increases as reflectivity  $Z$  increases.

### *Dual-Polarization Theory and Applications*

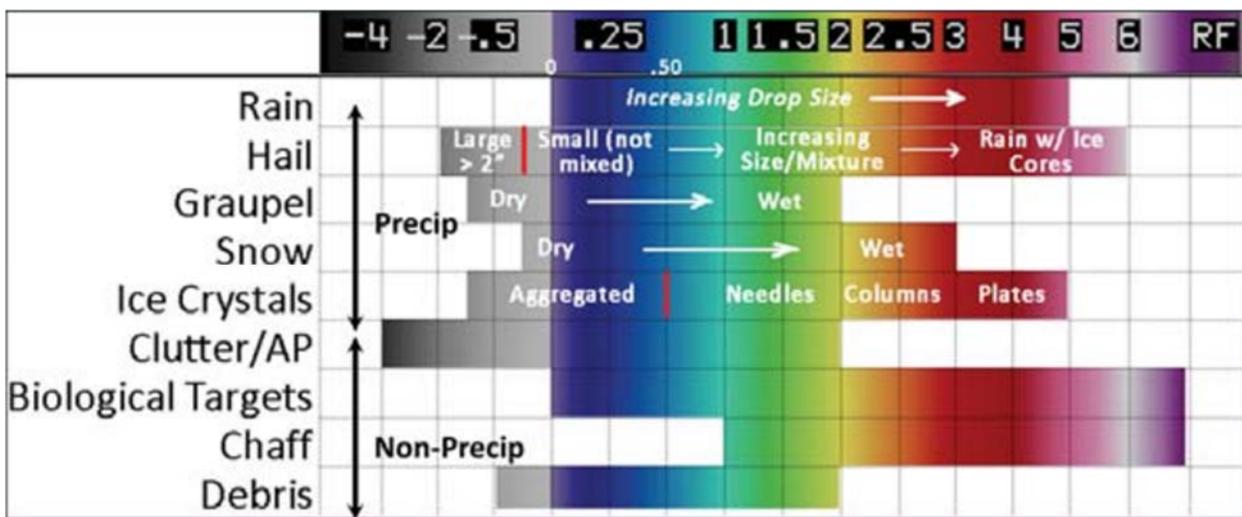
Conventional weather radars emit electromagnetic waves with only horizontal polarization; i.e., the electric field wave crests lie in the horizontal. *Dual-polarization* radars, however, transmit and receive electromagnetic waves with both horizontal and vertical polarization. In this nomenclature, the reflectivity defined above is known as  $Z_{hh}$ , whereas that returned for the vertical polarization is known as  $Z_{vv}$ . The first subscript denotes the polarization of the *transmitted* pulse, whereas the second subscript denotes the polarization of the *received* pulse. One can also define reflectivities for opposing transmitted and received pulses ( $Z_{hv}$  for horizontally transmitted-vertically received and  $Z_{vh}$  for vertically transmitted-horizontally received).

These measurements can be used to, for instance, infer characteristics such as hydrometeor/target type and drop size distribution. Common dual-polarization fields include *differential reflectivity* ( $Z_{DR}$ ), *linear deposition ratio* ( $LDR$ ), *specific differential phase* ( $K_{DP}$ ), and *co-polar correlation coefficient* ( $\rho_{hv}$ ). Of these, the most commonly used are  $Z_{DR}$  and  $\rho_{hv}$ ; consequently, we will focus on these two fields in this class.

Differential reflectivity is defined as the difference in reflectivity between the horizontally- and vertically-polarized pulses, i.e.,

$$Z_{DR} = Z_{hh} - Z_{vv}$$

Differential reflectivity is commonly used to infer target shapes and, consequently, type. Targets that are nearly spherical (e.g., hail, drizzle) backscatter the horizontally- and vertically-polarized pulses roughly equally, such that  $Z_{hh} \sim Z_{vv}$  and  $Z_{DR} \sim 0$ . Most meteorological targets are wider than they are tall, such that  $Z_{hh} > Z_{vv}$  and  $Z_{DR} > 0$ . Due to gravity, larger rain drops are flatter, such that  $Z_{DR}$  is directly proportional to rain drop size. Wetter and/or more densely-packed frozen species also tend to be flatter, leading to another direct relationship with  $Z_{DR}$ . Less commonly, targets such as vertically-oriented ice crystals may be taller than they are wide, such that  $Z_{hh} < Z_{vv}$  and  $Z_{DR} < 0$ . Violation of the Rayleigh scattering assumption, such as with large hail or debris, can result in negative  $Z_{DR}$ . Units of  $Z_{DR}$  are dB. The following table summarizes typical  $Z_{DR}$  values for the most common targets sampled by radar:



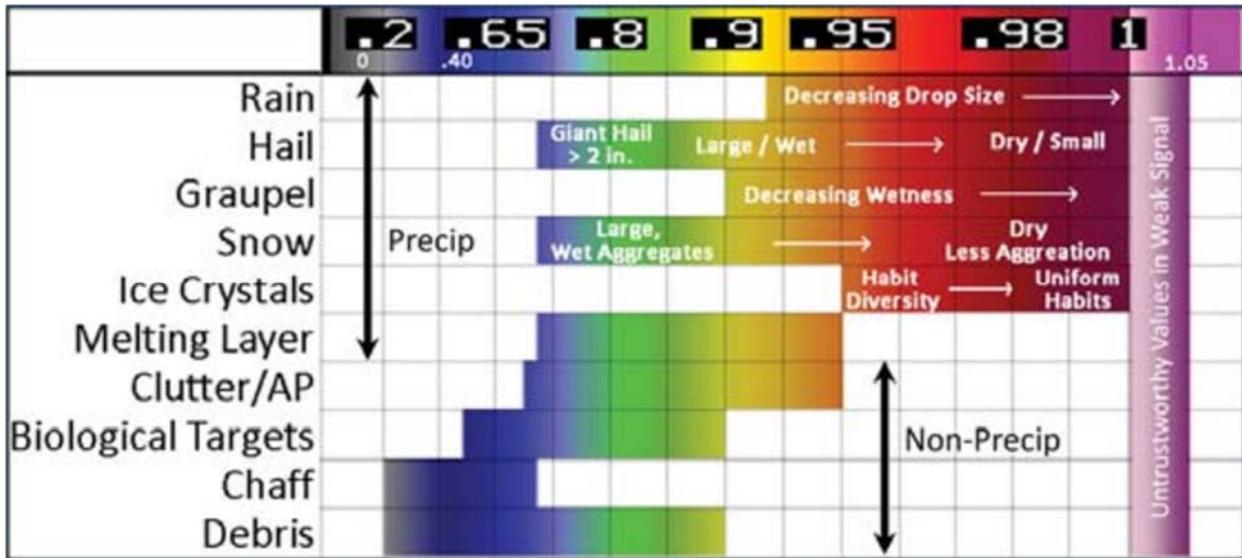
**Fig. 1.** Typical  $Z_{DR}$  values for common targets sampled by dual-polarization meteorological radars. Taken from NWS WDTD Radar and Applications Course  $Z_{DR}$  training material.

Common applications of  $Z_{DR}$  data include, but are not limited to,

- Identifying large hail: low to negative  $Z_{DR}$  collocated with large  $Z_{hh}$ .
- Identifying intense updrafts: column-like regions with large  $Z_{DR}$  (large rain drops lofted to high altitudes by the updraft) collocated with large  $Z_{hh}$ .
- Identifying debris: after evaluation of reflectivity, velocity, and correlation coefficient data, regions of low to negative  $Z_{DR}$  collocated with rotation.
- Identifying the height of the melting layer: large  $Z_{DR}$  (with wet and/or melting particles) surrounded by low to moderate  $Z_{DR}$  (with drier frozen and melted liquid particles).
- Identifying the rain/snow transition line: smaller  $Z_{DR}$  (with snow) adjacent to larger  $Z_{DR}$  (with raindrops).

Co-polar correlation coefficient quantifies the correlation in the returned power and phase between the horizontally- and vertically- polarized pulses from pulse to pulse within a given scan volume. If the backscattered power and phase are nearly uniform between the horizontal and vertical pulses,

targets in the sample volume have similar shape and size and the correlation coefficient (or CC) is approximately 1. If the scatterers are more diverse within a scan volume (e.g., as a function of non-meteorological phenomena, mixed precipitation, and/or varied in size or shape), then the returned power and phase will be less uniform between the horizontal and vertical pulses and  $CC < 1$ . The following table summarizes typical CC values for the most common targets sampled by radar:



**Fig. 2.** Typical CC values for common targets sampled by dual-polarization meteorological radars. Taken from NWS WDTD Radar and Applications Course CC training material.

With the exception of large hail, wet snow, and melting precipitation, which all tend to be oblong in shape with many irregular protuberances, meteorological phenomena typically have  $CC > 0.9$ - $0.95$ . Non-meteorological phenomena typically have  $CC < 0.8$ , with particularly low values for chaff (emitted by military aircraft) and debris (e.g., with tornadoes).  $CC > 1$  indicates a low signal-to-noise ratio (e.g., along the fringes of non-zero reflectivity regions) where CC should be ignored.

Common applications of CC data include, but are not limited to,

- Identifying non-meteorological echoes and tornado debris: low CC.
- Identifying the height of the melting layer: commonly, a ring of moderate ( $\sim 0.8$ - $0.9$ ) CC surrounded on both sides by high CC. Gradients in CC can also be used to identify the rain-snow transition line.
- Identifying the presence of large hail: moderate CC collocated with high  $Z_{hh}$ .

You will gain experience in applying concepts of both single- and dual-polarization radar to mesoscale meteorological phenomena identification in the first course assignment later this week.