Synoptic Meteorology I: Other Force Balances

For Further Reading

Section 2.1.3 of *Mid-Latitude Atmospheric Dynamics* by J. Martin provides a discussion of the frictional force and considerations related to its numerical representation within the equations of motion. Chapter 6 of *Meteorology: Understanding the Atmosphere* (4^{th} Ed.) by S. Ackerman and J. Knox describes the effects of friction on the near-surface winds in some detail. Wind balances in natural coordinates is presented in Section 4.4 of *Mid-Latitude Atmospheric Dynamics*.

The Effects of Friction on Geostrophic Balance

In our scale analysis of the horizontal equations of motion in the previous lecture, we found that the magnitude of the frictional terms was approximately 10^{-12} m s^{-1}, many orders of magnitude smaller than the magnitude of the Coriolis and horizontal pressure gradient terms (10^{-3} m s^{-2}). This led us to neglect friction-related terms at that time. For atmospheric motions occurring more than 1-2 km above the Earth's surface, one can nearly always neglect friction. However, for motions occurring nearer to the surface, in what is known as the *planetary boundary layer*, one cannot neglect friction.

What, therefore, is friction? Let us consider a familiar example. Imagine that you have a ball and, with your hand, you apply a force to cause it to begin to roll away from you along a smooth horizontal surface. After some time, the precise amount of which depends on how forcefully you caused the ball to roll to begin with, the ball stops rolling. The surface along which the ball rolled applied a force, manifest as the frictional force, that continually slowed the ball's motion until it came to a stop. Of course, not all surfaces apply friction equally; if you applied the same force to a ball on a level carpeted surface, it will stop sooner because carpet is a relatively rough surface.

The above examples consider friction in light of two solid objects – a ball and some underlying surface – in contact with each other. To a very basic first approximation, we can consider friction in the atmosphere similarly: the air particles that comprise the atmosphere, which move with the wind, in contact with Earth's surface. Friction has the same effect upon the wind as it did in our example above: it slows it down without *itself* causing a change in direction. (An important caveat to this statement will be introduced shortly.) Faster wind is more greatly impacted by friction than is slower wind. Rougher surfaces, such as jagged terrain and dense forests, exert a greater frictional force upon the wind than do smoother surfaces such as bodies of water.

Friction's effects upon the wind are primarily communicated upward from Earth's surface by what are known as *turbulent eddies*. Their uppermost extent, typically 1-2 km above Earth's surface, is what defines the top of the planetary boundary layer. A full treatment of turbulence, turbulent eddies, and internal friction associated with molecular interactions is beyond the scope of this course, however. We thus proceed with two guiding points:

- Friction is very important at and near Earth's surface.
- Friction acts in the opposite direction of the wind, acting to slow it down.

Recall that geostrophic balance represents a balance between the horizontal pressure gradient and Coriolis forces. The horizontal pressure gradient force depends exclusively upon the magnitude of the horizontal pressure gradient, which is not impacted by friction. The Coriolis force depends upon latitude, which is not impacted by friction, and wind speed, which *is* impacted by friction. Since friction reduces the wind speed, it also reduces the magnitude of the Coriolis force. Thus, there no longer exists a balance between the horizontal pressure gradient and Coriolis forces; the latter is larger than the former, and friction *indirectly* cause a change in direction due to its impact on the Coriolis force.

Instead, there exists a new balance, known as *Guldberg-Mohn balance*, between the horizontal pressure gradient, Coriolis, and frictional forces. This is depicted in Figure 1 below. This force balance resulting in air near the surface no longer flowing parallel to isobars or isohypses, but rather across the isobars or isohypses from high pressure or geopotential height to low pressure or geopotential height. This leads us to a couple of guiding principles:

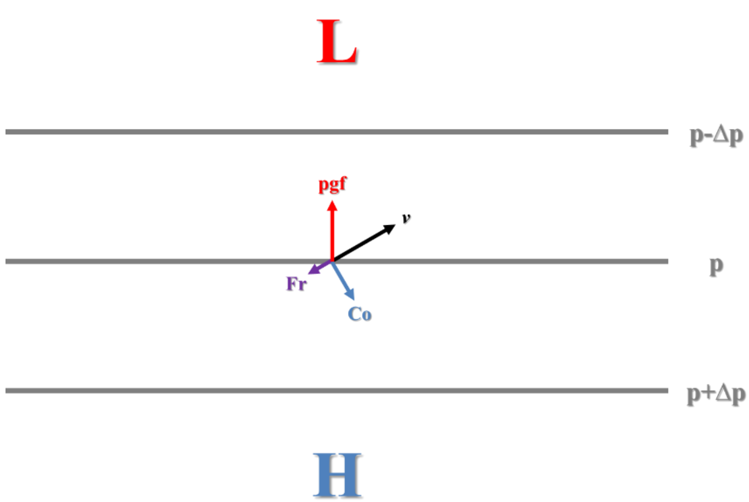

Figure 1. Depiction of Guldberg-Mohn balance for a hypothetical air parcel near Earth's surface. The red arrow labeled 'pgf' indicates the horizontal pressure gradient force, which is directed from high toward low pressure and is the longest of the three arrows that depict forces. The purple arrow labeled 'Fr' indicates the frictional force, which is directed in the opposite direction of the wind and is the shortest of the three arrows that depict forces. The blue arrow labeled 'Co' indicates the Coriolis force, which is directed 90° to the right of the wind in this Northern Hemisphere example. The wind, given by the black arrow labeled '*v*', is directed at an angle from high toward low pressure, as described above.

- Highs “empty” and lows “fill.” In other words, air flows (or *diverges*) out of areas of high pressure or high geopotential height near the surface and *converges* into areas of low pressure or low geopotential height near the surface. As we will soon establish, this has important implications for vertical motions within the troposphere.

- Because of friction, low pressure will not be found 90° to your left with the wind at your back, as we stated that it would under the construct of geostrophic balance. Instead, it will be found at a smaller (~60°) angle to your left with the wind at your back.

The Equations of Motion in Natural Coordinates

In order to consider other atmospheric force balances applicable to the horizontal wind, it is helpful to recast the equations of motion from Cartesian or spherical coordinates into a natural coordinate system. Presented without derivation, and neglecting friction and terms related to the curvature of the Earth, the equations of motion applicable for horizontal motions on isobaric surfaces – hereafter referred to as the horizontal momentum equations – are given by:

$$\frac{D\vec{v}}{Dt} = -\nabla_p \Phi - f\hat{\mathbf{k}} \times \vec{v} \qquad (1)$$

The terms of (1), from left to right, are the total derivative of the horizontal wind $\vec{v}$, the horizontal gradient of geopotential height, and the Coriolis terms. These terms are associated with parcel accelerations (following the motion), the horizontal pressure gradient force, and the Coriolis force, respectively. The total derivative on the left-hand side of (1) has the general form:

$$\frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + u\frac{\partial(\ )}{\partial x}\hat{\mathbf{i}} + v\frac{\partial(\ )}{\partial y}\hat{\mathbf{j}} + \omega\frac{\partial(\ )}{\partial p}\hat{\mathbf{k}} \qquad (2)$$

The partial derivative with respect to time on the right-hand side of (2) refers to the local change of ( ) with time. The remaining three terms on the right-hand side of (2) are advection terms. In (2), ω is pressure vertical motion (Pa s^{-1}), representing the vertical motion of air between isobaric surfaces.

Recall that a natural coordinate system is one defined not based upon the geography or geometry of the Earth but, rather, based upon the local wind. In a natural coordinate system, there exist two horizontal coordinates: *s*, following the flow (**streamwise**), and *n*, perpendicular (**normal**) to the flow. The positive *s*-axis is defined in the direction to which the wind is blowing, while the positive *n* axis is defined 90° to the left of the positive *s*-axis. An illustrative example of a natural coordinate system is given by Figure 6 within the “Map Projections and Coordinate Systems” lecture notes.

Let us now consider each of the terms of (1) and how they might be represented in natural coordinates. We begin with the total derivative on the left-hand side of (1). In natural coordinates, the horizontal velocity vector $\vec{v}$ is equal to $V\hat{\mathbf{s}}$, where V is the magnitude of the velocity vector (i.e., the wind speed) and $\hat{\mathbf{s}}$ is a unit vector along the wind in the downstream direction. Substituting, we obtain:

$$\frac{D\vec{v}}{Dt} = \frac{D(V\hat{\mathbf{s}})}{Dt} = V\frac{D\hat{\mathbf{s}}}{Dt} + \hat{\mathbf{s}}\frac{DV}{Dt} \tag{3}$$

Through a geometric consideration of how the axes of the natural coordinate system change following the flow, it can be shown that:

$$V\frac{D\hat{\mathbf{s}}}{Dt} + \hat{\mathbf{s}}\frac{DV}{Dt} = \hat{\mathbf{n}}\frac{V^2}{R} + \hat{\mathbf{s}}\frac{DV}{Dt} \tag{4}$$

In (4), R is equal to the radius of curvature, or the distance of the air parcel being considered from the center of what it is rotating (or curving) around. It is positive for counter-clockwise rotation and negative for clockwise rotation. Equation (4) enables us to describe the physical processes that result in air parcel acceleration following the motion. The first term on the right-hand side of (4) represents centripetal acceleration arising from curvature in the flow – i.e., the centrifugal force! The second term is associated with wind speed changes following the motion.

Next, we consider the horizontal geopotential height gradient term on the right-hand side of (1), describing the horizontal pressure gradient force. Most generally, geopotential height can vary both in the streamwise (*along-flow*) and normal (*across-flow*) directions. Thus, the horizontal geopotential height gradient term can simply be recast as:

$$-\nabla_p \Phi = -\left(\frac{\partial \Phi}{\partial s}\hat{\mathbf{s}} + \frac{\partial \Phi}{\partial n}\hat{\mathbf{n}}\right) \tag{5}$$

In the case where the full wind equals the geostrophic wind, geopotential height does not change in the streamwise direction; this is because the geostrophic wind blows parallel to isohypses, such that it maintains a constant value of geopotential height following the motion.

Finally, we consider the Coriolis term on the right-hand side of (1), related to the Coriolis force. If we expand the cross-product that defines this term in natural coordinates, making use of the definition $\vec{v} = V\hat{\mathbf{s}}$ from above, we obtain:

$$-f\hat{\mathbf{k}} \times \vec{v} = \begin{vmatrix} \hat{\mathbf{s}} & \hat{\mathbf{n}} & \hat{\mathbf{k}} \\ 0 & 0 & -f \\ V & 0 & 0 \end{vmatrix} \tag{6}$$

Expanding (6), we find that $-f\hat{\mathbf{k}}\times\vec{v} = -(fV)\hat{\mathbf{n}}$. Thus, if we combine (4), (5), and (6), we obtain:

$$\hat{\mathbf{s}}\frac{DV}{Dt} + \hat{\mathbf{n}}\frac{V^2}{R} = -\left(\frac{\partial\Phi}{\partial s}\hat{\mathbf{s}} + \frac{\partial\Phi}{\partial n}\hat{\mathbf{n}}\right) - (fV)\hat{\mathbf{n}} \tag{7}$$

Bringing the Coriolis term in (7) to the left-hand side of the equation and separating the full equation into its along-flow ($\hat{\mathbf{s}}$) and across-flow ($\hat{\mathbf{n}}$) components, we obtain:

$$\frac{DV}{Dt} = -\frac{\partial\Phi}{\partial s} \tag{8a}$$

$$\frac{V^2}{R} + fV = -\frac{\partial\Phi}{\partial n} \tag{8b}$$

Equation (8a) is the equation of motion in the along-flow direction, while equation (8b) is the equation of motion in the across-flow direction. If we now make the assumption that air flows parallel to geopotential height contours, then the right-hand side of (8a) is equal to zero. This means that we are assuming that the wind speed following the motion is constant. This leaves us with one equation, (8b), that we now wish to examine further in the light of atmospheric balance.

Geostrophic Balance Expressed in Natural Coordinates

Geostrophic balance is obtained from a scale analysis of the equations of motion for synoptic-scale motions. In that analysis, the total derivative term describing changes in the wind follow the motion (or accelerations) is neglected, such that we implicitly assume that air parcels do not accelerate in geostrophic balance. This allows us to neglect (8a) in its entirety. However, acceleration can also be associated with a change in direction (e.g., a deceleration in one direction and an acceleration in another direction as air rotates around some axis). Under the assumption that this does not occur, then the flow is said to be straight (or uncurved) and the radius of curvature *R* approaches infinity. This allows us to neglect the first left-hand-side term of (8b), such that we are left with:

$$fV = -\frac{\partial\Phi}{\partial n} \tag{9}$$

Equation (9) describes geostrophic balance in the natural coordinate system. The geostrophic wind can be defined from (9) as:

$$V_g = -\frac{1}{f}\frac{\partial\Phi}{\partial n} \tag{10}$$

The geostrophic wind *speed* can be evaluated from observations using (10), an example of which is drawn from Figure 2, which is the same example as in the previous lecture.

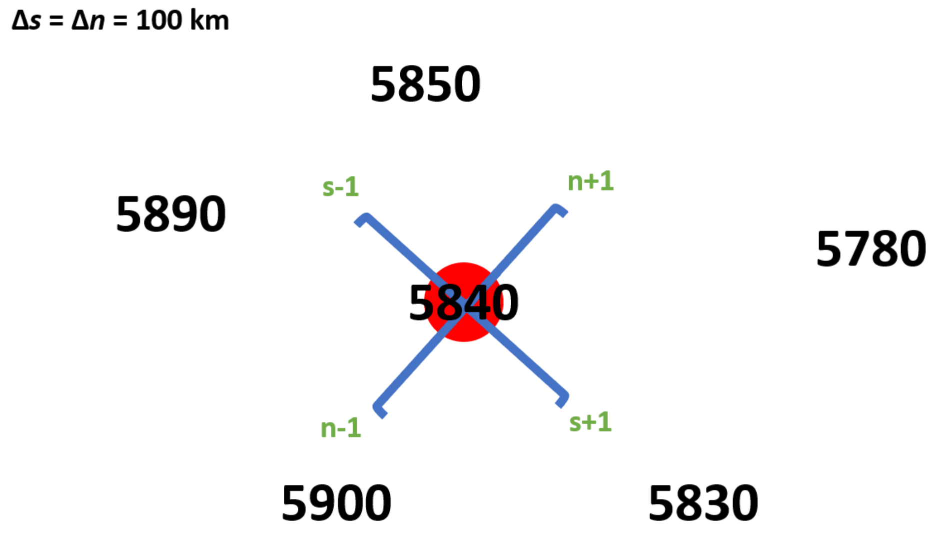

Figure 2. Hypothetical 500 hPa geopotential height (*z*; units: m) observations.

Since we must know the geostrophic wind *direction* to establish the streamwise and normal axes, an isohypse analysis must be conducted (preferred) or the geostrophic wind must be computed in Cartesian coordinates to determine the geostrophic wind direction. We take the latter approach in this example since we used these data in the previous lecture to find the geostrophic wind direction.

Next, we interpolate between observations to obtain the geopotential heights at *n*+1 and *n*-1. Doing so, we obtain 5880 m at *n*-1 and 5815 m at *n*+1. Assuming that we are at 38°N, where $f = 8.98 \times 10^{-5}\ s^{-1}$, we obtain the geostrophic wind *speed* as follows:

$$V_g = -\frac{1}{f}\frac{\partial \Phi}{\partial n} = -\frac{g}{f}\frac{z_{n+1} - z_{n-1}}{2\Delta n} = -\frac{9.81 m s^{-2}}{8.98 x 10^{-5} s^{-1}}\left(\frac{5815m - 5880m}{2\left(100000m\right)}\right) = 35.50 \text{ m s}^{-1}$$

This is close to the geostrophic wind speed obtained in Cartesian coordinates in a previous lecture (32.89 m s^{-1}), with differences between the two exclusively resulting from inherent uncertainties in interpolating data from a sparse analysis or set of observations.

The Centrifugal Force

Geostrophic balance represents the balance between the horizontal pressure gradient and Coriolis forces. However, there is another force, the centrifugal force, which is important for atmospheric motions. The centrifugal force is an *apparent* force associated with *rotational flow on the Earth*. Thus, the centrifugal force is important only for rotating, or curved, flow. By contrast, the Coriolis force is an apparent force associated with the rotation of the Earth. The centrifugal force is always directed *outward*, no matter which direction the flow (or some object) is rotating, and the change in direction associated with rotating/curved flow is known as *centripetal acceleration*. Thus, the centrifugal force is associated with a rotation-driven acceleration.

Geostrophic balance is often a good approximation on the synoptic-scale because the synoptic-scale flow is minimally curved. However, the synoptic-scale flow is typically always curved to some extent, and there are many situations in which the synoptic-scale flow is highly curved. These considerations motivate us to consider balances that incorporate the centrifugal force so as to be able to best understand situations in which geostrophic balance is ill-suited. In the process of doing so, we first recast our equations of motion in natural coordinates.

Cyclostrophic Wind Balance

Cyclostrophic balance is a balance applicable to rapidly rotating phenomena, or those for which the rotation rate greatly exceeds that of the Earth. In this sense, the Coriolis force is negligible, and the resulting force balance is between the horizontal pressure gradient and centrifugal forces.

If we neglect the Coriolis term in (8b), we obtain:

$$\frac{V^2}{R} = -\frac{\partial \Phi}{\partial n} \quad (11)$$

If we solve (11) for the wind speed *V*, we obtain:

$$V = \sqrt{-R\frac{\partial \Phi}{\partial n}} \quad (12)$$

The wind speed *V* defined by (12) is known as the *cyclostrophic wind*. For *V* to be physically realistic (i.e., real), either *R* or $\frac{\partial \Phi}{\partial n}$, but not both, must be negative.

Recall the directions of the horizontal pressure gradient and centrifugal forces:

- The horizontal pressure gradient force always points from high to low pressure (i.e., from high to low geopotential height on an isobaric surface).
- The centrifugal force always points outward from a rotating air parcel.

Under the condition of cyclostrophic balance, these two forces must be of equal magnitude but opposite direction. This only holds true when the horizontal pressure gradient force is directed inward of a rotating air parcel, i.e., for an area of low pressure or geopotential height. Thus, cyclostrophic balance only holds for lows, not for highs. However, under cyclostrophic balance, the wind may rotate either cyclonically (as we expect) or anticyclonically (!) around an area of low pressure or geopotential height. This is illustrated in Figure 3.

Earlier, we defined the Rossby number as the ratio of the scales of the acceleration and Coriolis terms. Here, we have stated that the Coriolis term is negligibly small – explicitly, at least one order of magnitude smaller – compared to the centripetal acceleration term. Thus, for cyclostrophic balance to hold, we would expect the Rossby number to be large: 10 or greater. The most telling example of such conditions is of a tornado. For a radius of curvature of 1,000 m (1 km), a wind speed of 50 m s^{-1} (roughly 111 MPH), and a latitude of 40°N (f = 9.37 x 10^{-5} s^{-1}), the Rossby number is equal to approximately 533, easily meeting cyclostrophic balance criteria.

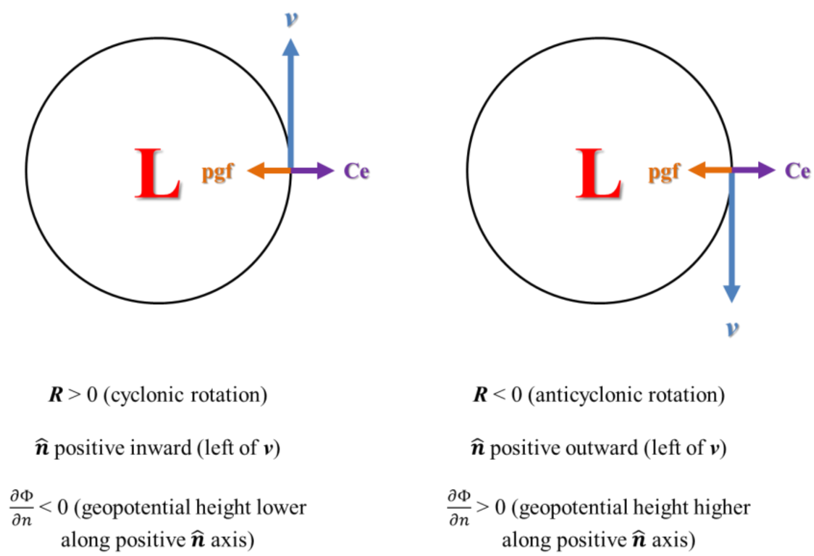

Figure 3. Cyclostrophic force balance diagram for cyclonic rotation (left) and anticyclonic rotation (right) around an area of low pressure or low geopotential height. Note that the force balance is identical but that the signs of R and $\frac{\partial \Phi}{\partial n}$ differ between the two examples. This can help explain, for instance, the existence of anticyclonic-rotating tornadoes.

Gradient Wind Balance

The Gradient Wind Equation and its Solutions

Belying its discussion at the end of this set of lecture notes, gradient wind balance is the most general of the force balances that we have considered to date. It represents the balance between the horizontal pressure gradient, Coriolis, *and* centrifugal forces, as represented by (8b) above. This equation represents a quadratic equation for V, where $a = 1/R$, $b = f$, and $c = \frac{\partial \Phi}{\partial n}$.

If we solve this equation for V, we obtain:

$$V = \frac{-f \pm \sqrt{f^2 - \frac{4}{R}\frac{\partial \Phi}{\partial n}}}{\frac{2}{R}} \tag{13}$$

Or, equivalently,

$$V = \frac{-fR \pm \sqrt{(fR)^2 - 4R\frac{\partial \Phi}{\partial n}}}{2} \tag{14}$$

This equation represents the *gradient wind equation*, where we refer to V as the gradient wind. There are a total of *eight* possible solutions to (14), as controlled by the sign of R, the sign of $\partial\Phi/\partial n$, and the leading $\pm$ on the radical. We now wish to examine these solutions to determine which are physically realistic and the gradient wind that they describe.

Case 1: $R > 0$, $\partial\Phi/\partial n > 0$

Because both R and $\partial\Phi/\partial n$ are positive, the second term under the radical in (14) is negative. This means that the entire term under the radical will be less than $|fR|$. No matter whether you add this to $-fR$ or subtract it from $-fR$, because the leading $-fR$ term is negative for $R > 0$, $V < 0$. As a result, this case provides two physically unrealistic solutions and is not considered further.

Case 2: $R < 0$, $\partial\Phi/\partial n > 0$

Because R is negative but $\partial\Phi/\partial n$ is positive, the second term under the radical in (14) is positive. This means that the entire term under the radical will be greater than $|fR|$. Here, the leading $-fR$ term is positive for $R < 0$. Thus, the negative sign leading the radical results in $V < 0$, a physically unrealistic solution. Conversely, the positive sign leading the radical results in $V > 0$, a physically realistic solution.

Let us consider this physically realistic solution further. Recall that, in the Northern Hemisphere, $R < 0$ implies anticyclonic curvature (or flow). By definition, the positive n-axis is defined as perpendicular and to the left of the flow. Thus, for $R < 0$, it points outward. Because $\partial\Phi/\partial n$ is positive, higher geopotential heights are found outward (along the positive n-axis) and lower geopotential heights are found inward of the flow.

This gives us a paradox: anticyclonic flow around an area of lower geopotential heights! This solution represents what is known as the **anomalous low**, the force balance for which is depicted in Figure 4a. While a physically realistic solution, it is also one that is rarely observed – and typically never observed on the synoptic- and larger scales.

Case 3: $R > 0$, $\partial\Phi/\partial n < 0$

Because R is positive but $\partial\Phi/\partial n$ is negative, the second term under the radical in (14) is positive. This means that the entire term under the radical in (14) is greater than $|fR|$. Here, the leading $-fR$ term is negative given $R > 0$. The negative sign leading the radical gives $V < 0$, a physically unrealistic solution. However, the positive sign leading the radical gives $V > 0$, a physically realistic solution.

As before, let us consider this physically realistic solution further. In the Northern Hemisphere, $R > 0$ implies cyclonic curvature (or flow). By definition, the positive n-axis is defined as perpendicular and to the left of the flow. Thus, for $R > 0$, it points inward. Because $\partial\Phi/\partial n$ is negative, higher geopotential heights are found along the negative n-axis – or outward – and lower geopotential heights are found along the positive n-axis, or inward of the flow.

Thus, this case represents cyclonic curvature with lower geopotential heights inward of the flow. This solution represents what is known as a **regular low**, the force balance for which is depicted in Figure 4b.

Case 4: $R < 0$, $\partial\Phi/\partial n < 0$

Because both R and $\partial\Phi/\partial n$ are negative, the second term under the radical in (14) is negative. As a result, this case only produces a physically realistic solution if $(fR)^2$ is larger than is $4R\frac{\partial\Phi}{\partial n}$. When this is true, the entire term under the radical in (14) is smaller than $|fR|$. Since the leading $-fR$ term is positive given $R < 0$, both the positive and negative signs leading the radical give $V > 0$, a physically realistic solution. Thus, this case provides two physically realistic solutions to the gradient wind equation.

Let us consider general aspects of both solutions first. In the Northern Hemisphere, $R < 0$ implies anticyclonic curvature or flow. By definition, for $R < 0$, the positive n-axis thus points outward. Because $\partial\Phi/\partial n$ is negative, higher geopotential heights are found along the negative n-axis – or inward – and lower geopotential heights are found along the positive n-axis, or outward. Thus, both solutions represent anticyclonic curvature with higher geopotential heights inward of the flow, or what we might otherwise call a traditional *anticyclone*.

Let us now dig a little deeper into each of these two solutions. The solution with the positive sign leading the radical means that $V > -fR/2$. If we square both sides of this expression, we obtain:

$$V^2 > \frac{f^2R^2}{4} \tag{15}$$

Dividing both sides of (15) by R, we obtain:

$$\frac{V^2}{R} > \frac{f^2 R}{4} \tag{16}$$

If we substitute V for $-fR/2$ on the right-hand side of (16), noting that V actually is greater than $-fR/2$ (after all, that's what we started with!), we obtain:

$$\frac{V^2}{R} > -\frac{fV}{2} \tag{17}$$

Now, let us consider the solution with the negative sign leading the radical. Here, $V < -fR/2$, such that (17) becomes:

$$\frac{V^2}{R} < -\frac{fV}{2} \tag{18}$$

For anticyclonic flow in the Northern Hemisphere, the Coriolis force is directed inward and the horizontal pressure gradient force is directed outward. The centrifugal force is always directed outward. Thus, the Coriolis force balances the net of the horizontal pressure gradient and centrifugal forces.

The inequality in (17) states that the centrifugal force is greater than half of the Coriolis force. Thus, the centrifugal force for this solution must be larger than the horizontal pressure gradient force. We denote this solution to represent the **anomalous high**, characterized by large Coriolis and centrifugal forces but a small horizontal pressure gradient force, as depicted in Figure 4c.

Conversely, the inequality in (18) states that the centrifugal force is less than half of the Coriolis force. Thus, the centrifugal force for this solution must be smaller than the horizontal pressure gradient force. We denote this solution to represent the **regular high**, characterized by large Coriolis and horizontal pressure gradient forces (e.g., as in geostrophic balance) but a small centrifugal force, as depicted in Figure 4d.

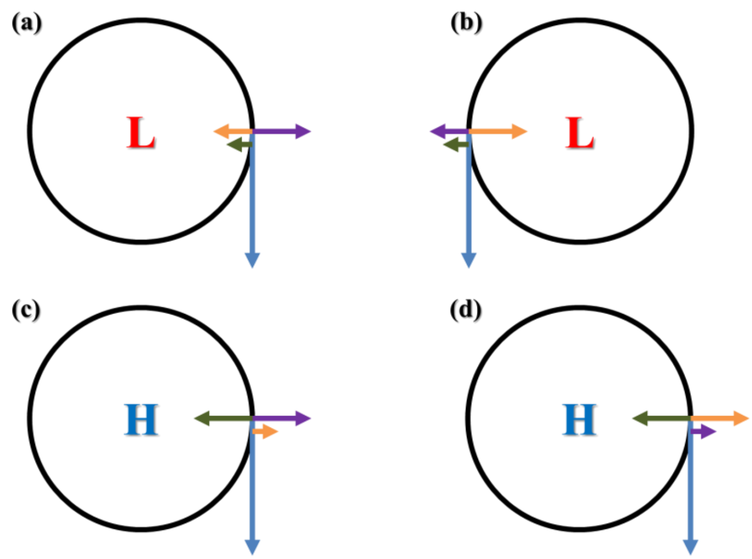

Figure 4. Force balance diagrams for the (a) anomalous low, (b) regular low, (c) anomalous high, and (d) regular high. In each panel, the blue arrow denotes the gradient wind, the purple arrow denotes the centrifugal force (always directed outward), the orange arrow denotes the horizontal pressure gradient force (always directed from high toward low), and the green arrow denotes the Coriolis force (always directed perpendicular and to the right of the wind). All arrows are scaled to the approximate magnitude of each force in each panel.

The Cyclone-Anticyclone Dichotomy

Recall that the anomalous high and regular high solutions to the gradient wind equation are only possible when $(fR)^2$ is larger than $4R\frac{\partial \Phi}{\partial n}$. If we solve this inequality for the horizontal pressure gradient force term, we obtain:

$$\frac{\partial \Phi}{\partial n} < \frac{f^2 R}{4} \quad (19)$$

Equation (19) tells us that the magnitude of the horizontal geopotential height gradient around an area of high pressure is constrained by latitude (as manifest through the Coriolis parameter *f*) and, more importantly, the radius of curvature $|R|$. As one approaches the center of an area of high pressure, where $|R|$ approaches zero, the horizontal geopotential height gradient must approach zero. Consequently, the horizontal geopotential height gradient is generally smaller for areas of high pressure than it is for areas of low pressure, for which this constraint does not exist. This constraint also means that winds are light – and, at the center of a high, very nearly zero – with areas of high pressure. One can confirm both inferences from any meteorological analysis, whether at the surface or aloft.

Subgeostrophic and Supergeostrophic Flow

If we solve (10) for V_g and substitute into (8b), we obtain:

$$fV_g = \frac{V^2}{R} + fV \qquad (20a)$$

In the absence of curvature, (20a) simplifies to the geostrophic wind. Thus, the first right-hand-side term of (20a) can be thought of as the gradient wind "correction" to the geostrophic wind, accounting for the effects of curvature on the flow.

Letting $V = V_{gr}$ (for the gradient wind) and dividing through by fV_{gr}, we obtain:

$$\frac{V_g}{V_{gr}} = \frac{V_{gr}}{fR} + 1 \qquad (20b)$$

The left-hand-side of (20b) represents the ratio of the geostrophic to the gradient wind speed. When it is greater than 1, the geostrophic wind speed is larger than the gradient wind speed, denoting a situation where the gradient (or, to close approximation, full) wind is *subgeostrophic*. Conversely, when the left-hand-side of (20b) is less than 1, the geostrophic wind speed is smaller than the gradient wind speed, denoting situation where the gradient wind is *supergeostrophic*.

For cyclonic flow, where $R > 0$, the right-hand-side of (20b) is greater than 1, indicating that the true wind is *subgeostrophic* around areas of low pressure. It is only slightly subgeostrophic where the flow is weakly cyclonically curved; it is more subgeostrophic where the flow is strongly cyclonically curved. Because of this, the true wind is weaker than inferred from a given horizontal gradient of pressure (on a constant height surface) or geopotential height (on an isobaric surface).

For anticyclonic flow, where $R < 0$, the right-hand side of (20b) is less than 1, indicating that the true wind is *supergeostrophic* around areas of high pressure. It is only slightly supergeostrophic where the flow is weakly anticyclonically curved; it is more supergeostrophic where the flow is strongly anticyclonically curved. Because of this, the true wind is stronger than inferred from a given horizontal gradient of pressure (on a constant height surface) or geopotential height (on an isobaric surface).