Synoptic Meteorology I: Thermal Wind Balance

For Further Reading

Sections 1.4.1 and 1.4.2 of *Midlatitude Synoptic Meteorology* by G. Lackmann derives the thermal wind relationship and relates the thermal wind to the mean temperature advection in a given vertical layer. Section 4.3 of *Mid-Latitude Atmospheric Dynamics* by J. Martin provides a derivation and discussion of thermal wind balance and is the primary source for the applications of thermal wind balance discussed herein.

Deriving the Thermal Wind Relationship

Recall that the geostrophic relationship applicable on isobaric surfaces is given by:

$$fv = \frac{\partial \Phi}{\partial x} \tag{1a}$$

$$fu = -\frac{\partial \Phi}{\partial y} \tag{1b}$$

If we substitute v_g for v and u_g for u and divide both sides of (1) by f, we obtain:

$$v_g = \frac{1}{f}\frac{\partial \Phi}{\partial x} \tag{2a}$$

$$u_g = -\frac{1}{f}\frac{\partial \Phi}{\partial y} \tag{2b}$$

The *thermal wind* is defined as the vector difference in the geostrophic wind between two pressure levels p_1 and p_0, where p_0 is closer to the surface (and thus $p_0 > p_1$). It is not an actual wind, but it is a useful construct that allows us to link the geostrophic wind (a kinematic field) to temperature (a mass field).

To obtain an expression for the thermal wind, we differentiate (2) with respect to p ($\partial/\partial p$) to obtain:

$$\frac{\partial v_g}{\partial p} = \frac{1}{f}\frac{\partial}{\partial p}\left[\frac{\partial \Phi}{\partial x}\right] \tag{3a}$$

$$\frac{\partial u_g}{\partial p} = -\frac{1}{f}\frac{\partial}{\partial p}\left[\frac{\partial \Phi}{\partial y}\right] \tag{3b}$$

If we commute the order of the partial derivatives on the right-hand sides of (3), then both equations contain a common $\partial\Phi/\partial p$ term. The hydrostatic equation states that:

$$\frac{\partial p}{\partial z} = -\rho g \rightarrow \frac{\partial p}{\rho} = -g\partial z$$

If we plug in to the hydrostatic equation with the definition of the geopotential Φ ($\partial\Phi = g\ \partial z$) and the ideal gas law ($\rho = p/R_d T_v$), we obtain the following expression:

$$\frac{\partial\Phi}{\partial p} = -\frac{R_d T_v}{p}$$

Substituting this into (3), we obtain:

$$\frac{\partial v_g}{\partial p} = \frac{1}{f}\frac{\partial}{\partial x}\left[-\frac{R_d T_v}{p}\right] \tag{4a}$$

$$\frac{\partial u_g}{\partial p} = -\frac{1}{f}\frac{\partial}{\partial y}\left[-\frac{R_d T_v}{p}\right] \tag{4b}$$

We can simplify (4) by taking R_d out of the derivatives as a constant. Furthermore, because we are using the form of the geostrophic wind applicable on isobaric surfaces, p is constant with respect to both x and y. We can thus extract p and put it into ∂p, where by definition, $\partial p/p = \partial(\ln p)$ and thus $p/\partial p = 1/\partial(\ln p)$. Doing so, we obtain:

$$\frac{\partial v_g}{\partial\ln p} = -\frac{R_d}{f}\frac{\partial T_v}{\partial x} \tag{5a}$$

$$\frac{\partial u_g}{\partial\ln p} = \frac{R_d}{f}\frac{\partial T_v}{\partial y} \tag{5b}$$

Equation (5) gives us expressions for the vertical wind shear of the geostrophic wind. If we multiply both sides of (5) by $\partial\ln p$ and integrate from p_0 to p_1, we obtain:

$$\int_{p_0}^{p_1}\partial v_g = \int_{p_0}^{p_1} -\frac{R_d}{f}\frac{\partial T_v}{\partial x}\partial\ln p \tag{6a}$$

$$\int_{p_0}^{p_1}\partial u_g = \int_{p_0}^{p_1}\frac{R_d}{f}\frac{\partial T_v}{\partial y}\partial\ln p \tag{6b}$$

We note that R_d and f are both constant with respect to p and can thus be pulled out of the integrals. Likewise, if we substitute $\overline{T_v}$ for T_v, where $\overline{T_v}$ is a layer-mean (or vertically averaged) temperature, then the derivative terms with respect to $\overline{T_v}$ can be pulled out of the integrals. This is similar to what we did to obtain the hypsometric equation. Doing so, (6) becomes:

$$v_T \equiv v_g(p_1) - v_g(p_0) = -\frac{R_d}{f}\frac{\partial \overline{T_v}}{\partial x}\left(\ln(p_1) - \ln(p_0)\right) = \frac{R_d}{f}\frac{\partial \overline{T_v}}{\partial x}\ln\left(\frac{p_0}{p_1}\right) \quad (7a)$$

$$u_T \equiv u_g(p_1) - u_g(p_0) = \frac{R_d}{f}\frac{\partial \overline{T_v}}{\partial y}\left(\ln(p_1) - \ln(p_0)\right) = -\frac{R_d}{f}\frac{\partial \overline{T_v}}{\partial y}\ln\left(\frac{p_0}{p_1}\right) \quad (7b)$$

Equation (7) defines the *thermal wind*. It states that the change in geostrophic wind between two isobaric levels p_1 and p_0 is directly related to the horizontal layer-mean (between p_1 and p_0) virtual temperature gradient. As the layer-mean virtual temperature is directly proportional to thickness, we can thus state that *the thermal wind is directly related to the horizontal thickness gradient*.

Inspecting (7), we see that the meridional component of the thermal wind is a function of the zonal gradient in the layer-mean virtual temperature. Likewise, the zonal component of the thermal wind is a function of the meridional gradient in the layer-mean virtual temperature. Compare this to (2): the meridional component of the geostrophic wind is a function of the zonal geopotential height gradient, whereas the zonal component of the geostrophic wind is a function of the meridional geopotential height gradient.

Before, we stated that the geostrophic wind blows parallel to the isobars or isohypses. Likewise, we can state that *the thermal wind "blows" parallel to the isotherms* (of the layer-mean virtual temperature). Equivalently, we can state that the thermal wind "blows" parallel to lines of constant thickness. This means that if we know the thermal wind, we know how the isotherms and lines of constant thickness are oriented spatially.

In the Northern Hemisphere, warm air is found 90° to the *right* of an observer facing downstream of the thermal wind. We can prove this to ourselves using (7). Consider a thermal wind "blowing" from west to east ($u_T > 0$). On the right-hand side of (7b), R_d, f, and $\ln(p_0/p_1)$ are all positive. As a result, $\frac{\partial \overline{T_v}}{\partial y}$ must be negative, meaning that $\overline{T_v}$ is warmer along the negative y-axis (or to the south). This is 90° to the right of an observer facing downstream (here, to the east) of the thermal wind. Similar arguments can be made for any thermal wind orientation, independent of hemisphere.

Application to Horizontal Temperature Advection

If we know the geostrophic wind at two pressure levels, we can determine the thermal wind over the vertical layer between those two levels using vector subtraction. Given the thermal wind, we know how the isotherms are oriented horizontally, as described above. From this information, we can analyze how the geostrophic wind at each pressure level blows with respect to the isotherms to qualitatively assess the mean horizontal temperature advection within the vertical layer. These principles are illustrated graphically below in Figure 1.
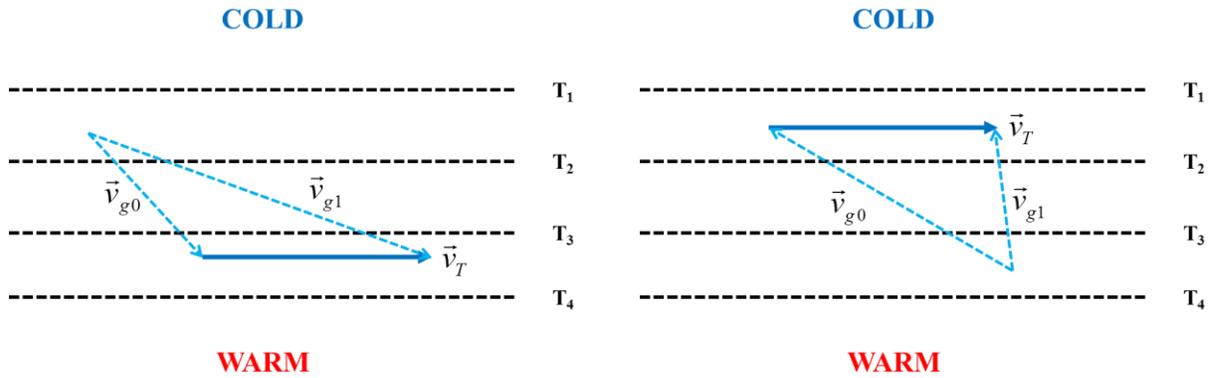
Figure 1. (left) A thermal wind v_T that is associated with cold air advection. (right) A thermal wind v_T that is associated with warm air advection. For simplicity, layer-mean temperature is depicted in place of layer-mean virtual temperature; the qualitative interpretation is unchanged between the two quantities, and the quantitative interpretation is only minimally affected. Note how the thermal wind is of identical direction and magnitude between the panels but that the sign of the horizontal temperature advection is different. This is due to how the geostrophic wind within the layer defined by the thermal wind is oriented with respect to the isotherms of layer-mean temperature.

In the left-most panel of Figure 1, the geostrophic wind $\vec{v}_{g0}$ at p_0 (the lower level) is from the northwest. The geostrophic wind $\vec{v}_{g1}$ at p_1 (the upper level) is from the west-northwest. The geostrophic wind is turning counterclockwise with increasing distance above the ground, which we refer to as *backing* winds.

At this point, it is helpful to recall basic tenets of vector addition and subtraction. Two vectors $\vec{A}$ and $\vec{B}$ can be added together by placing the *beginning* of $\vec{B}$ at the *end* of $\vec{A}$ and drawing a vector from the beginning of $\vec{A}$ to the end of $\vec{B}$. The difference $\vec{A} - \vec{B}$ may viewed as $\vec{A} + \left(-\vec{B}\right)$, where $\left(-\vec{B}\right)$ is simply $\vec{B}$ rotated by 180°. Equivalently, $\vec{A} - \vec{B}$ may be obtained by placing the *beginning* of $\vec{B}$ at the *beginning* of $\vec{A}$ and drawing a vector from the end of $\vec{B}$ to the end of $\vec{A}$.

We now return to our examples in Figure 1. The vector difference $\vec{v}_{g1} - \vec{v}_{g0}$, defining the thermal wind, is obtained by placing the two vectors at a common origin, then drawing a vector from the end of $\vec{v}_{g0}$ to the end of $\vec{v}_{g1}$. By definition, the layer-mean isotherms are parallel to this vector with warm air found to the right (here, the south). Because the geostrophic wind at both p_0 and p_1 blows from cold toward warm air, *backing of the geostrophic wind with increasing distance above the ground is associated with cold air advection.*

Similarly, in the right-most panel of Figure 1, the geostrophic wind $\vec{v}_{g0}$ at p_0 is from the southeast. The geostrophic wind $\vec{v}_{g1}$ at p_1 is from the south-southeast. The geostrophic wind is turning clockwise with increasing distance above the ground, which we refer to as *veering* winds. The vector difference between the two is again obtained by placing the two vectors at a common origin and drawing a vector from the end of $\vec{v}_{g0}$ to the end of $\vec{v}_{g1}$. By definition, the layer-mean isotherms are parallel to this vector with warm air found to the right (again, the south). Since the geostrophic wind at both p_0 and p_1 blows from warm toward cold air, *veering of the geostrophic wind with increasing distance above the ground is associated with warm air advection.*

A great way to put these principles into practice is to regularly look at skew-T/ln p charts from one or more location(s). If you approximate the geostrophic wind by the total wind, how it changes with height will give you an approximate idea of whether cold or warm air advection is ongoing within a given layer at a given location without needing other data. This principle is demonstrated in the accompanying "Thermal Wind Application" notes.

There is an important caveat to keep in mind when applying these principles: the thermal wind is based upon geostrophic balance, which does not hold near the ground (because of friction) and in strongly curved flows (with features where acceleration is important; i.e., hurricanes or tornadoes). The extent to which the flow departs from geostrophic balance depends on the extent to which the flow is curved (more curvature = greater departure) or affected by friction (closer to the ground or over a rougher surface = greater departure). In these situations, insights from the thermal wind are at best only qualitatively accurate and (rarely) are at worst altogether incorrect. A simple way of estimating the impact is to use a series of isohypse analyses on the isobaric levels in question to qualitatively estimate the geostrophic wind direction (if not also speed) and compare the resulting insight to that obtained using the full wind (as is often done in practice).

Further Insights from the Thermal Wind

Under the constraint of thermal wind balance, as manifest by (7), the following tenets are true:

- Where there is a change in geostrophic wind (speed and/or direction) with height, there must be a horizontal gradient in layer-mean temperature.

- Where there is a horizontal gradient in layer-mean temperature, there must be a change in geostrophic wind (speed and/or direction) with height.

The term *vertical wind shear* is commonly used to refer to changes in wind speed and/or direction with height. Recall from our discussion of (5) that the thermal wind can be viewed as the vertical wind shear of the geostrophic wind. Because of (7), thermal wind balance can also be viewed as the balance between vertical wind shear and horizontal gradients of layer-mean virtual temperature

or thickness. In general, if one were to analyze synoptic-scale observations, one would find that this balance holds quite frequently in the midlatitudes.

We can also apply thermal-wind–related concepts to understanding Earth's general circulation, particularly in the midlatitudes. Consider that, in an annual average, latitudes closer to the Equator receive more insolation than do latitudes closer to the poles. As a result, again in an annual average, it is warmer throughout the troposphere closer to the Equator – say, at 30°N – than it is closer to the poles – say, at 60°N. This means that there exists a horizontal layer-mean temperature gradient. From the hypsometric equation, it also means that the thickness between two isobaric surfaces located near the surface (e.g., 1000 hPa) and the tropopause (e.g., 200 hPa) is greater near the Equator than near the poles. This is graphically depicted in Figure 2.
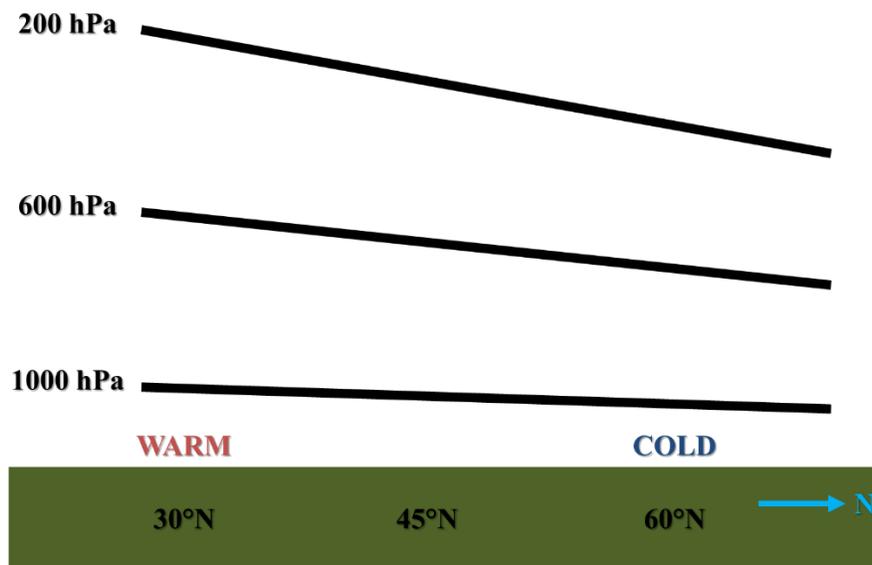

Figure 2. Annually averaged vertical cross-section between 30°N and 60°N of three selected isobaric surfaces (200, 600, and 1000 hPa; black lines) and the relative layer-mean temperature between the 1000 hPa and 200 hPa isobaric surfaces at 30°N (**WARM**) and 60°N (**COLD**).

The north-south gradient in thickness – i.e., layer-mean virtual temperature, or to an approximation layer-mean temperature – means that the thermal wind for this layer is directed from west to east, such that warm air is found 90° to the right of an observer with the thermal wind at their back. This defines the vector difference in the geostrophic wind within the layer being considered. Note that a non-zero thermal wind defines what is known as a *baroclinic* atmosphere.

We can go a step further, however. Recall that geostrophic balance is defined as the balance between the horizontal pressure gradient force and the Coriolis force. In Figure 2, the horizontal (here, north-south) pressure gradient is greater at higher altitudes than it is closer to the surface. This implies that the horizontal pressure gradient force is greater at higher altitudes, thus requiring a greater Coriolis force to maintain geostrophic balance. Because the magnitude of the Coriolis

force is directly proportional to the horizontal wind speed, this implies a faster horizontal wind speed at higher altitudes than closer to the surface.

Furthermore, the horizontal pressure gradient is directed from north to south at all altitudes, such that the horizontal pressure gradient force is directed from south to north. The Coriolis force, under the constraint of geostrophic balance, is thus directed from north to south. Because we know that the Coriolis force is directed 90° to the right of the wind, we know that the geostrophic wind is directed from west to east at all altitudes. Because the horizontal wind speed – or magnitude of the geostrophic wind vector – is larger at higher altitudes, this implies westerly geostrophic winds that increase with increasing distance above ground. This is a defining characteristic of the midlatitudes and is frequently observed on the synoptic- to planetary-scales.