Lateral Boundary Conditions

Introduction

For any non-global numerical simulation, the simulation domain is finite. Consequently, some means of handling the outermost extent of the simulation domain – its *lateral boundaries* – must be employed. There are three general classes of lateral boundary conditions:

- **Specified**: With specified lateral boundary conditions, time-varying values of all model dependent variables are prescribed along each lateral, or side, of the simulation domain. Specified lateral boundary conditions are widely used in real-data model simulations.

- **Periodic**: With periodic lateral boundary conditions, information that exits the simulation domain on one end returns into the simulation domain on the opposite end. Periodic lateral boundary conditions are often used in idealized model simulations but never used in real-data model simulations.

- **Rigid**: With rigid lateral boundary conditions, one or more laterals are represented by impervious walls through which nothing may pass. Unless addressed through some means such as damping, phenomena that impinge upon a rigid boundary are reflected away from the boundary (i.e., back into the model domain). For numerical weather prediction, rigid lateral boundary conditions are used only in idealized model simulations on laterals where all simulated phenomena travel tangent to the rigid boundary. These are referred to as *channel model simulations*.

Specified lateral boundary conditions may be obtained from one of two sources. In the first, data at the analysis and all forecast times from a *previously run* larger-area model simulation is used to specify the lateral boundary conditions. This is known as a *one-way* nest, where no interaction is permitted between the limited-area model simulation and the model simulation that provides the lateral boundary conditions. Single-domain limited-area model simulations and the outermost domain of multiple-domain limited-area model simulations are examples of one-way nests.

In the second, data at the analysis and all forecast times from a model simulation *run concurrently* with the limited-area model simulation that is used to specify the lateral boundary conditions. This is known as a *two-way* nest, wherein interaction between the outer and inner simulation domains – both on the lateral boundaries and on the interior of the inner simulation domain – is permitted during the simulation. Two-way interactive nesting is typically, though not always, utilized with the inner domain(s) of a multiple-domain limited-area model simulation.

Fig. 1 provides an illustration of how specified lateral boundary conditions are implemented with the WRF-ARW model; it is generally representative of lateral boundary formulations in grid-based models. Model dependent variables are specified by the lateral boundary conditions at each time step on the outermost grid points of the simulation domain. As the data used to prescribe these values typically have coarser spatial and temporal resolution than does the limited-area model simulation, interpolation is utilized to interpolate to the grid spacing (in three dimensions) and time step of the limited-area model simulation.

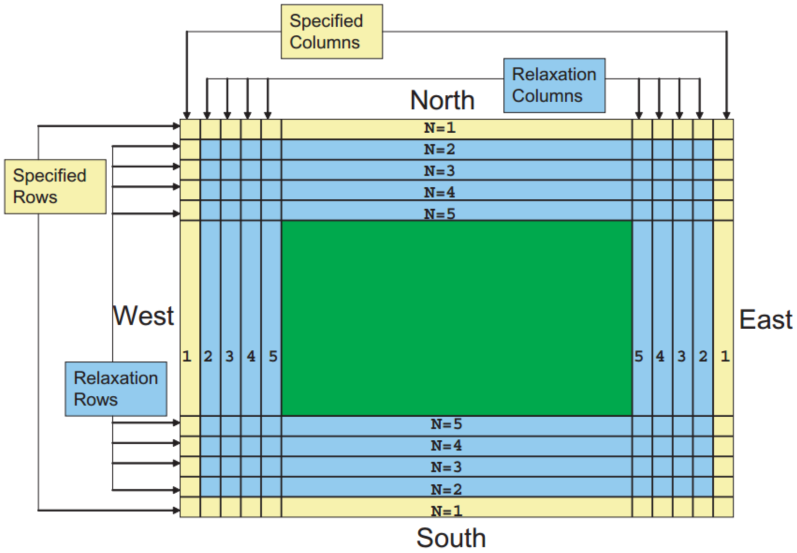

Figure 1. Illustration of specified lateral boundary conditions (yellow), including the model grid points (blue) over which the prescribed values provided by the lateral boundary conditions are relaxed to their values within the interior of the limited-area simulation domain (green), as used within real-data numerical simulations conducted using the WRF-ARW model. Figure reproduced from Skamarock et al. (2019), their Fig. 6.1.

As the prescribed values of model variables on the lateral boundaries can differ from those on the interior of the limited-area model simulation, particularly at later forecast times and for one-way nested simulations, a *relaxation zone* is employed near the specified lateral boundaries. The width of this relaxation zone is often a user-specifiable parameter. The purpose of the relaxation zone is to mitigate sharp gradients (short wavelength features) that may exist along the lateral boundaries due to the specified lateral boundary data differing from the interior of the simulation domain.

There exist multiple ways by which this relaxation zone may be specified. One that we consider here, as manifest within the WRF-ARW model, was formulated by Davies and Turner (1977, *Quart. J. Roy. Meteor. Soc.*) and is described in Skamarock et al. (2008). For any prognostic model variable f, its time tendency within the relaxation zone is given by:

$$\frac{\partial f}{\partial t} = F_1\left(f_{LBC} - f\right) - F_2\Delta^2\left(f_{LBC} - f\right)$$

In the above, f is predicted by the model, f_{LBC} is the value of f specified on the lateral boundary, and Δ^2 represents a five-point horizontal smoother applied on model coordinate surfaces. F_1 and F_2 are weighting coefficients, given by:

$$F_1 = \frac{1}{10\Delta t}\frac{n_{spec} + n_{relax} - m}{n_{relax} - 1}$$

$$F_2 = \frac{1}{50\Delta t}\frac{n_{spec} + n_{relax} - m}{n_{relax} - 1}$$

In the above, n_{spec} is the number of rows or columns of grid points over which specified lateral boundary conditions are applied, n_{relax} is the number of the row or column of grid points at which the relaxation zone ends, and m is the number of rows or columns in from the lateral boundary. Note that m has a minimum value of $n_{spec} + 1$ and a maximum value of $n_{spec} + n_{relax} – 1$. It is also possible for these weighting coefficients to be multiplied by an exponential function to make smoother the transition from the lateral boundaries to the interior for large relaxation zones. The weighting coefficients thus relax, or nudge, the values of model dependent variables specified on the lateral boundaries to their values on the interior of the model domain.

The relaxation zone formulation described above can be viewed as one means of damping short wavelength variability that may exist between the specified lateral boundary conditions and the model solution on the interior of the simulation domain. As described in the lecture on diffusion, one may also apply a low-order damping operator along the lateral boundaries to smooth out the sharp gradients in model dependent variables that may exist between the specified lateral boundary conditions and the model solution on the interior of the simulation domain.

Not all model dependent variables have their values specified from another data set on the lateral boundaries. For instance, in WRF-ARW, vertical velocity, all microphysical species except water vapor, and all non-conserved scalars for one-way nests have their values specified by other means. For vertical velocity, a zero-*gradient* boundary condition over the specified zone is applied. For the other variables, this zero-*gradient* boundary condition is applied when exiting the domain and a zero-*valued* boundary condition is applied when entering the domain.

Desired Characteristics of and Sources of Error with Lateral Boundary Conditions

There exist several desirable characteristics for any specified lateral boundary formulation, some of which are hinted at above. In particular, meteorological phenomena moving into or out of simulation domains that use specified lateral boundary conditions should do so without significant distortion. For those phenomena moving *into* a nested simulation domain, both interpolation from the coarser (in time and space) lateral boundary conditions and the probable existence of short wavelength variability between the lateral boundaries and the interior of the simulation domain should not negatively impact simulation quality. Likewise, phenomena should be allowed to move *out of* a nested simulation domain; in other words, they should not be reflected back into or trapped within the domain. Of particular importance in this regard are inertia-gravity waves, particularly those associated with geostrophic adjustment (i.e., response in either the mass or kinematic fields to a change in the kinematic or mass fields so as to maintain or restore geostrophic balance). In essence, the use of lateral boundary conditions should not negatively impact forecast performance.

As denoted within the course text, there exist six primary potential sources of error associated with specified lateral boundary conditions:

- **Coarse resolution of the specified lateral boundary conditions**. As noted above, data used for specified lateral boundary conditions is typically available at a coarser resolution in both time and space than the time step and horizontal grid spacing of the simulation domain on which the lateral boundary conditions are specified. As a result, smaller-scale variability cannot be accurately captured by the lateral boundary conditions, which may impact forecast quality at and near the lateral boundaries. Inherent to this premise is the necessity for the model to continuously spin-up smaller-scale phenomena near the lateral boundaries.

- **Group and phase speed errors**. As the simulation domain utilizes finer resolution than the specified lateral boundary conditions, what is coarsely resolved by the specified lateral boundary conditions may be better resolved within the simulation domain (and vice versa). For example, consider a simulation domain with Δx = 15 km and lateral boundary conditions with Δx = 45 km. A feature of wavelength 200 km will be crudely resolved by the lateral boundary conditions (less than $5\Delta x$) but will be well-resolved within the simulation domain (greater than $13\Delta x$). This discrepancy, which is a particular challenge for features passing from a fine inner domain to a coarse outer domain, impacts the extent to which truncation error, numerical diffusion, numerical dispersion, and aliasing influence the model solution.

- **Errors in the specified lateral boundary conditions**. Lateral boundary conditions can be from numerical model analyses or forecasts, both of which are imperfect approximations to the real atmosphere. Consequently, the specified lateral boundary conditions are to some extent erroneous, particularly those that are drawn from numerical model forecasts, and to some extent this error will impact the quality of the solution in the inner simulation domain.

- **Lack of feedback between the simulation domain and the specified lateral boundary conditions**. Unless a two-way nest is used, what happens on the interior of the simulation domain cannot influence the values of the specified lateral boundary conditions. The resultant short wavelength variability between the lateral boundaries and the interior of the simulation domain can influence forecast accuracy.

- **Dynamical balance between specified lateral boundary conditions and the solution on the interior of the simulation domain**. Discrepancies between the specified lateral boundary conditions and the interior of the simulation domain can result in imbalanced kinematic and mass fields near the lateral boundaries. To attempt to restore balance, the model will generate non-physical inertia-gravity waves. The extent to which these waves influence the model solution depends upon how close the primary feature of interest is to the lateral boundaries and how large of a difference is seen between the specified lateral boundary conditions and the model solution on the interior of the simulation domain.

- **Inconsistencies between physical parameterizations used to generate the specified lateral boundary conditions and within the simulation domain**. In all likelihood, the model configuration used to generate the lateral boundary conditions will not be identical to that used for the simulation domain on which the lateral boundary conditions are to be applied. One way in which this is manifest is by the choice of physical parameterizations. For example, vertical mixing within the planetary boundary layer differs in how it is handled between planetary boundary layer parameterizations. This impacts near-surface temperature, moisture, and wind fields. This discrepancy can impact simulation quality, particularly near the lateral boundaries where these fields most closely resemble the lateral boundary conditions. Such inconsistencies most commonly exist for one-way nests; however, where parameterized convection is used on an outer nest but not an inner nest, such inconsistencies may also exist for two-way nested simulations.

The existence of such sources of error motivates a series of practical recommendations for simulations in which specified lateral boundary conditions are utilized. These are discussed in more detail at the end of this set of lecture notes.

The Influence of Lateral Boundary Conditions upon Limited-Area Model Simulations

In the following, we describe *possible* influences of specified lateral boundary conditions upon limited-area model simulations. It should be emphasized that the specifics of these influences are *not* necessarily generalizable to a wide range of model applications. As a result, we focus upon the *methods* that can be used to examine the influence of specified lateral boundary conditions upon limited-area model simulations more than the findings obtained from such methods. Furthermore, note that variants of these methods can and do exist. The intention here is not to describe every possible method by which the influence of specified lateral boundary conditions can be identified but rather to describe a subset of these methods and their applications.

One way to examine the influence of specified lateral boundary conditions on limited-area model simulations is to conduct a series of simulations in which the lateral boundaries are placed at increasingly large distances from an area or feature of interest. Output from these simulations is then verified over the area covered by the smallest simulation domain. This allows for the influence of the proximity to the lateral boundaries upon forecast quality to be quantified, at least in part.

An example is provided by the Treadon and Petersen (1993) study cited in the course text. Therein, they found that forecast error – defined in their study by the domain-averaged root-mean squared error of 500 hPa geopotential height – increased as the distance between the verification area and the lateral boundaries decreased. Their study also depicts the constraining influence on the model solution near lateral boundaries due to specified lateral boundary conditions (Fig. 2). A limited-area model solution near the lateral boundaries will closely resemble that provided by the specified lateral boundary conditions. As the specified lateral boundary conditions are typically coarser than the limited-area model, features will also appear with less short wavelength variability near to the lateral boundaries.

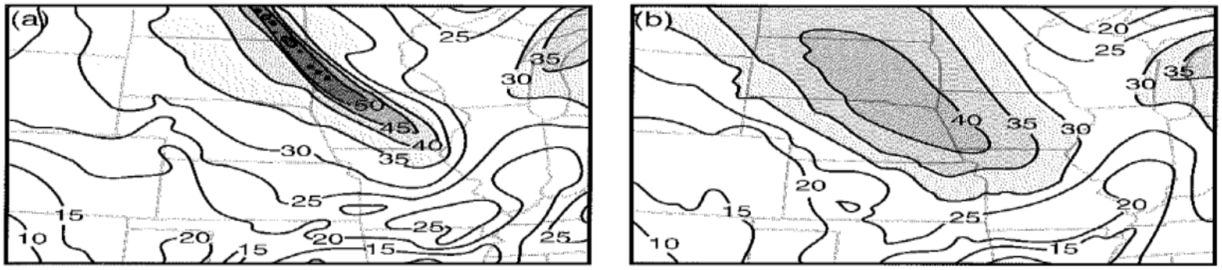

Figure 2. 250 hPa isotachs (m s^{-1}) from two 12-h Eta model simulations, each with Δx = 40 km, initialized at 1200 UTC 3 August 1992. In panel (a), the lateral boundaries are placed approximately 2,000 km away from the edges of the plotted domain. In panel (b), the lateral boundaries are placed at the outermost periphery of the plotted domain. Figure reproduced from Warner (2011), their Fig. 3.44.

One can also examine the influence of specified lateral boundary conditions by conducting a simulation in which the lateral boundary conditions are provided at identical grid spacing to that utilized by the simulation itself. If one verifies this simulation and that from which the specified lateral boundary conditions were drawn, differences in the verification metric between the two simulations provides quantification of the influence of the lateral boundary conditions upon the forecast. Such differences may result from the lateral boundary condition formulation itself, temporal interpolation between times at which the lateral boundary conditions are specified, and/or variability in model configuration, particularly that associated with the choices of physical parameterizations. A variant of this experiment is the so-called “Big Brother-Little Brother” experiment, as briefly described in both Sections 3.5 and 10.4 of the course text.

As forecast lead time increases, and particularly over relatively small simulation domains, the lateral boundary conditions exert an increasingly large control upon the subsequent forecast relative to the initial conditions. There exist multiple methods by which the sensitivity of some element of the model forecast to the specified lateral boundary conditions may be examined. These methods do not directly quantify forecast error due to the specified lateral boundary conditions but provide a means of assessing the influence of the specified lateral boundary conditions upon the model forecast. One means by which this influence can be illuminated is though adjoint methods, as illustrated in Fig. 3. The course text reports on the works of Errico et al. (1993) and Warner et al. (1997) that utilized adjoint methods in this way.

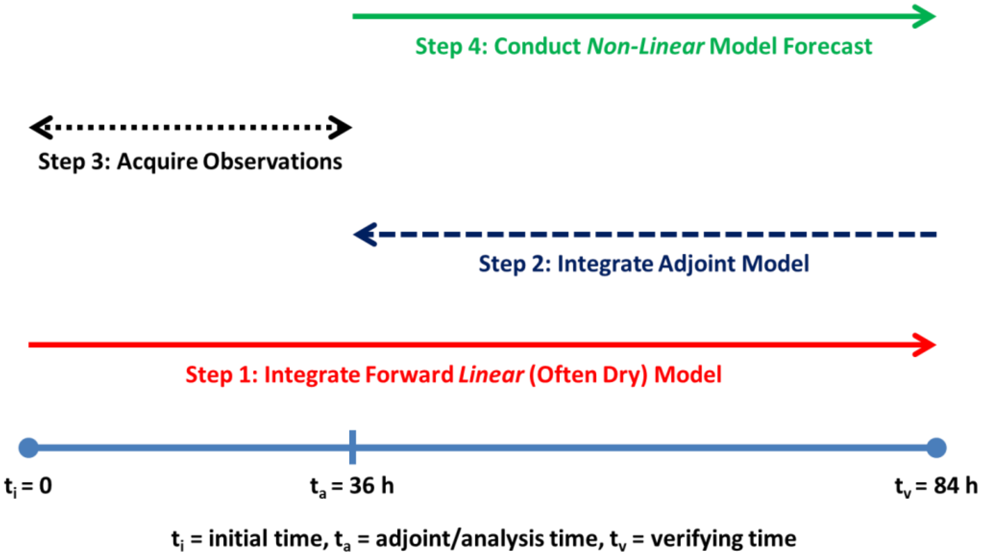

Figure 3. Schematic representation of adjoint methods. At some initial time, the linear version of a numerical model is integrated forward in time. At some future time, a forecast metric of interest is identified. Next, the adjoint (or inverse linear) version of the numerical model is integrated backward in time from the future time to a given time of interest at or after the initial time. The resulting analysis provides data from which sensitivity in the forecast metric to the values of model dependent variables at the earlier time can be determined. Adjoint methods are often used with observation targeting and data assimilation, wherein the sensitivity metric can provide a means of identifying where to collect observations and what type of observations to collect so as to improve a subsequent non-linear model forecast. Applied to lateral boundary condition sensitivity studies, adjoint methods can be used to identify to where along the lateral boundaries the forecast is most sensitive.

The influence of specified lateral boundary conditions upon the model forecast can also be illuminated by conducting two nearly identical model simulations where only lateral boundary conditions are varied. For instance, consider the case where the differences between the two sets of lateral boundary conditions are of similar magnitude to that of observational uncertainty. Any differences between the two simulations are entirely attributable to the specified lateral boundary conditions. Such studies allow for the illumination of how lateral boundary condition information spreads inward from the lateral boundaries with time.

An example is provided by the Vukicevic and Errico (1990) study cited in the course text. In their study, two limited-area model simulations encompassing much of Europe and north Africa were conducted. The simulations were identical expect for the specified lateral boundary conditions, which varied as described in the paragraph above. Integrating the model forward for six hours, differences in the 500 hPa geopotential height of in excess of 5 m are evident across much of the simulation domain, even well-removed from the lateral boundaries (Fig. 4). As the advective velocity is far too small to account for such differences on the interior of the simulation domain, this example highlights how inertia-gravity waves generated along the lateral boundaries can spread lateral boundary information over a large area in a short amount of time.

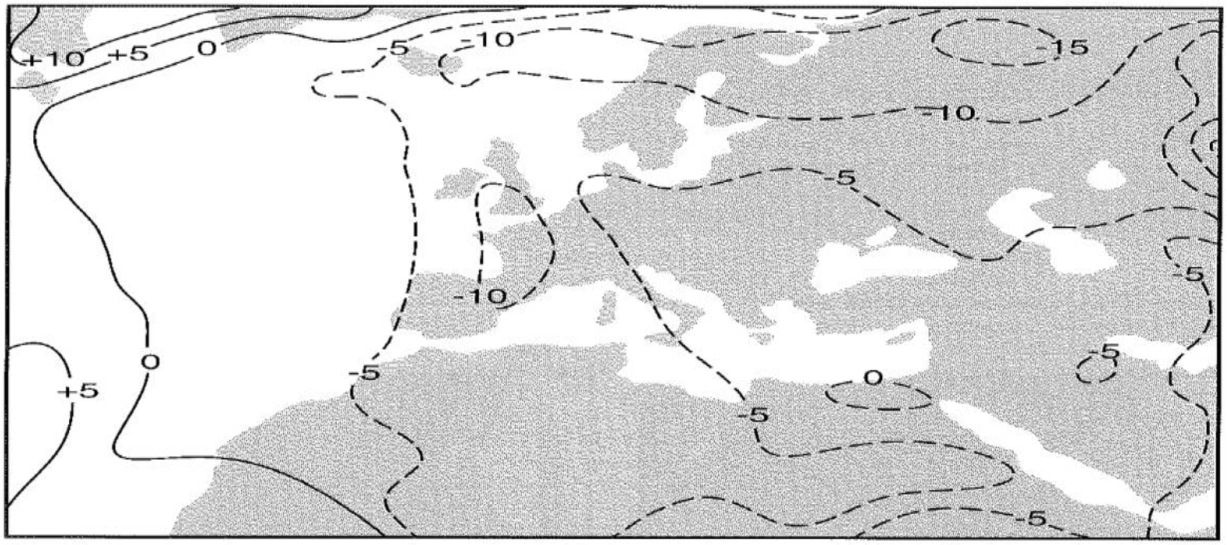

Figure 4. Difference in 500 hPa geopotential height (m) between two nearly identical model simulations conducted using a limited-area model, one using unperturbed and one using perturbed lateral boundary conditions, at $t = 6$ h. Figure reproduced from Warner (2011), their Fig. 3.46.

Practical Recommendations for Lateral Boundary Conditions

To mitigate negative lateral boundary conditions influences on model forecasts, the course text outlines a series of practical recommendations. These include:

- **The use of a buffer zone between the primary area or phenomenon of interest within the simulation domain and the lateral boundaries**. To first order (i.e., excluding any influence of inertia-gravity waves), the time it takes for lateral boundary information to reach a given grid point on the interior of the simulation domain is controlled by the advective velocity U and the distance of that grid point from the lateral boundaries. If the lateral boundary conditions are desired to have as small of an impact as possible upon the forecast within an area of interest, the area of interest should be placed sufficiently far away from the lateral boundaries and the model integrated for as short of a time as possible to encompass the desired period or event. In practice, however, the influence of lateral boundary conditions upon the model solution is at least to some extent unavoidable.

- **Minimize interpolation error with the specified lateral boundary conditions**. So that small-scale variability (in time and space) is not altogether missed by the lateral boundary conditions, and thus likely not captured within the interior of the simulation domain, the specified lateral boundary conditions should be updated frequently and be provided at a grid spacing as close to that of the simulation domain as is possible.

- **Maintain as much consistency as possible between the model configuration used to provide lateral boundary conditions and to integrate the model**. Inconsistency between physical parameterizations used to generate lateral boundary condition data and within the simulation domain upon the model forecast can influence forecast quality. As a result, it is desirable to utilize similar (if not identical) physical parameterizations for the simulation domain as were used to generate the lateral boundary conditions.

- **Utilize well-tested and effective lateral boundary condition formulations**. To large extent, lateral boundary condition formulations available within modern numerical weather prediction models are well-tested and effective. Of course, "well-tested and effective" does not mean free from error, but rather associated with comparatively small error.

- **Avoid strong forcing of either meteorological or geophysical origin near the lateral boundaries**. Strong forcing associated with meteorological and/or geophysical features – particularly sloped terrain and ocean currents – typically results in sharp gradients in model dependent variables. As noted above, so too do the lateral boundaries. Superposing the two with each other can result in a large accumulation of wave energy at short wavelengths that may compromise the model solution, particularly along and near the lateral boundaries.

- **Where possible, use two-way nests rather than one-way nests**. One-way nests are unavoidable for limited-area model domains on the outermost simulation domain. Two-way nests for inner simulation domains allow for their lateral boundary conditions to be updated in light of what occurs within the inner simulation domain. Further, the lateral boundary conditions are typically updated more frequently – once every time step on the outer simulation domain rather than once every history output time – using two-way nests than if they are specified using a one-way nest.

- **Evaluate the extent to which the chosen lateral boundary configuration influences the model solution**. The methods introduced above provide illustrative examples of how lateral boundary condition influences upon numerical model forecasts can be quantified. In as much as is possible, one or more of these or other related methods should be used to inform lateral boundary condition formulation and placement. In practice, however, this criterion is not always adhered to, sometimes to the detriment the one who conducted the simulations!