**Cumulus Parameterization**

**Reference Materials**


**Overview**

There are two types of cumulus clouds that parameterizations represent: shallow and deep. Both act to vertically heat, moisture, and momentum; deep cumulus produces rainfall, whereas shallow cumulus typically does not produce precipitation except for drizzle. A cumulus parameterization may parameterize both or only one of shallow and deep cumulus, and most only parameterize their effects on heat and moisture and not momentum. Cumulus parameterizations predict the effects of sub-grid-scale convection – whether comprising one or more clouds within a grid box – on the modeled atmosphere in terms of resolved-scale variables; i.e., they use resolved-scale variables to diagnose whether convection exists and, if so, diagnose its impacts on resolved-scale variables. Cumulus parameterizations do not require saturation on the resolved-scale in order to activate; microphysical parameterizations do, however, yet both can act concurrently.

Shallow cumulus includes the fair-weather cumulus common during the warm season and oceanic cumulus beneath trade wind inversions in the tropics and subtropics. It is associated with turbulent vertical mixing that destabilizes the lower troposphere by cooling and moistening atop the cloud while warming and drying below the cloud. Deep cumulus, including thunderstorms, stabilizes the environment by warming and drying the middle to upper troposphere and cooling and moistening the lower troposphere. It typically forms in environments of deep conditional instability and large-scale lower tropospheric convergence, though meso-to-microscale triggers are also often required.

Precipitation in association with deep convection can also be classified as convective or stratiform. Convective precipitation results from individual cells, organized or isolated, with localized regions of intense vertical motions (both upward and downward) over a deep layer. Stratiform precipitation is associated primarily with older, often decaying convective cells over larger areas with weaker vertical motions in a shallower layer. Organized convective systems such as mesoscale convective systems may have both convective and stratiform precipitation. Examples are given in Fig. 1.

Latent warming in convective precipitation is found over a deep vertical layer, whereas that with stratiform precipitation is confined to the mid-troposphere, with lower tropospheric latent cooling due to evaporation within a subsaturated environment found beneath. Convective cells dry over a deep vertical layer, while stratiform precipitation dries the middle to upper troposphere and moistens the lower troposphere (again by evaporation). These traits, as derived from field program datasets and budget analyses, are illustrated in Fig. 2. Importantly, data such as in Fig. 2 provide benchmarks against which parameterization output can be evaluated: does the parameterization correctly represent cumulus’ impact to the resolved-scale temperature and moisture fields?
Figure 1. Base reflectivity radar mosaics (a) over the central Plains at 1155 UTC 8 May 2009 and (b) over the northern Plains at 1955 UTC 8 May 2009. In (a), an organized mesoscale convective system with well-defined leading convective band and trailing stratiform region are shown. In (b), primarily stratiform precipitation with isolated convective cells in Iowa is shown. Images courtesy College of DuPage, as archived by the NCAR/MMM Image Archive.

Why do we care about cumulus? In addition to precipitation production and thus their integral role in the hydrological cycle, latent heat release in deep cumulus is an important contributor to tropical cyclone intensity. Latent heating also acts to vertically redistribute potential vorticity, thus altering the synoptic-scale flow (e.g., ridge building aloft fostered by upper tropospheric diabatic heating). In addition, horizontal gradients of latent heating in deep cumulus drive the major circulations of the tropics, including the Hadley cell, Walker cell, and monsoons. Shortcomings in deep cumulus parameterization are a leading source of error in model forecasts of the Madden-Julian Oscillation. More generally, cumulus clouds influence the local radiation budget, blocking incoming shortwave and trapping outgoing longwave radiation.

Deep cumulus, or convection, typically has a horizontal scale of $O(10 \text{ km})$. Consequently, models with horizontal grid spacing of approximately 4 km and finer can resolve convection, if somewhat crudely; such models are said to be convection-allowing. Convection-resolving models are those that can correctly resolve the statistical properties of deep cumulus clouds; in general, this requires a horizontal grid spacing of $O(100 \text{ m})$. Models with horizontal grid spacing of 5 km or greater are said to be convection-parameterizing, as they must parameterize deep cumulus. Shallow cumulus are more difficult to partition; while they typically have a horizontal scale of $O(1 \text{ km})$ or less, they are not always parameterized in convection-allowing model simulations, and this so-called “grey zone” is an active area of research. From here on out, we focus primarily on tenets of deep cumulus parameterization unless otherwise specified.
Figure 2. (a) Total cumulus-related diabatic warming (°C day$^{-1}$; red) and its convective (blue) and stratiform (or mesoscale; yellow) components. (b) As in (a), except total cumulus-related drying (cm day$^{-1}$; positive values indicate drying). Figures reproduced from Johnson (1984, *Mon. Wea. Rev.*), via D. Stensrud’s “An Overview of Convection Parameterization” resource.
Basic Principles – Trigger Functions

Cumulus parameterizations must determine the presence, intensity, and impact of cumulus clouds. The constructs by which the presence of cumulus are determined are known as trigger functions. The intensity of vertical motions, and thus latent heating, is related to the closure assumptions used in the parameterization; e.g., what latent heating and precipitation must result in order to eliminate the instability giving rise to the parameterized cloud? The impact of cumulus clouds on resolved-scale atmospheric variables can be determined either prognostically or diagnostically; in the latter, model variables may be relaxed to those given by a set of reference profiles.

Trigger functions are unique to each cumulus parameterization. They are constructed in terms of what the parameterization developers felt were most important to cumulus development. What this is physically, however, is not immediately clear. Is cumulus development primarily related to local buoyancy, moisture content, large-scale forcing magnitude, or smaller-scale forcing? Truly, all of these are important, albeit to varying extents depending upon the horizontal scale of the simulated cumulus, and a theoretical analytical trigger function that appropriately weights all inputs has yet to be developed. A listing of factors comprising the trigger functions of multiple parameterizations is provided in Fig. 3; we will discuss the Betts-Miller-Janjić (BMJ), Kain-Fritsch (KF), and Tiedtke trigger functions in more detail later in this lecture.

Figure 3. A listing of the physical parameters comprising, to extents that vary between parameters and parameterizations, trigger functions for several cumulus parameterizations. Figure reproduced from D. Stensrud’s “An Overview of Convection Parameterization” resource.
Although the trigger functions and closure assumptions vary between cumulus parameterizations, there are some common traits that exist between parameterizations. As one might expect for deep, moist convection, the principles of *convective available potential energy* (CAPE) and *convective inhibition* (CIN) underlie many cumulus parameterizations. We define CAPE and CIN here as they are traditionally defined within the construct of parcel theory (e.g., no entrainment or detrainment of an air parcel from or to its environment, although cumulus parameterizations do parameterize these processes and their effects on buoyancy), where CAPE is the maximum energy available to an ascending air parcel and CIN is the energy necessary to lift a parcel pseudoadiabatically to its level of free convection (LFC; level beyond which an ascending air parcel is positively buoyant):

$$\text{CAPE} = \int_{LFC}^{EL} Bdz \approx g \int_{LFC}^{EL} \frac{T_{\text{parcel}}^v - T_{\text{env}}^v}{T_{\text{env}}^v} dz$$

$$\text{CIN} = -\int_{SL}^{LFC} Bdz \approx -g \int_{SL}^{LFC} \frac{T_{\text{parcel}}^v - T_{\text{env}}^v}{T_{\text{env}}^v} dz$$

Here, $B$ is buoyancy, $SL$ is a parcel’s starting vertical level, where $SL = 0$ defines the surface; $EL$ is the equilibrium level for an ascending air parcel, $T_v$ is virtual temperature, superscripts of $\text{env}$ denote that of the environment, and superscripts of $\text{parcel}$ denote that of the ascending air parcel. An example of CAPE and CIN for an ascending surface-based parcel is given in Fig. 4. It should be noted that cumulus parameterizations do not evaluate buoyancy in the simplified form as seen here; rather, they include a full treatment of hydrometeor mass, entrainment, and detrainment.

![Figure 4.](image-url) Hypothetical skew $T – \ln p$ diagram, depicting observed (or environmental) temperature by the thin solid line and the parcel trace for an ascending surface-based air parcel by the thick solid line. The hatched (shaded) regions depict CIN (CAPE) for the ascending air parcel. The EL,
LFC, and lifting condensation level (LCL) are denoted. Figure reproduced from Warner (2011), their Fig. 4.5.

Nominally, deep cumulus cannot form in the absence of CAPE and sufficient lift to overcome any CIN that may be present. To first order, this can serve as a theoretical construct by which a trigger function may be developed: is there non-zero CAPE at one or more vertical levels collocated with sufficient lifting on one or more horizontal scales to lift an air parcel to its LFC?

Keeping with this theme, there are two primary classes of deep cumulus parameterizations: deep-layer and low-level control schemes. Deep-layer control schemes relate convective development and intensity to the creation of CAPE by large-scale processes (e.g., radiative heating, advection). This is physically sound; positive buoyancy leads to upward parcel accelerations, the magnitude of which are larger when positive buoyancy is larger. By contrast, low-level control schemes relate convective development to the removal of CIN by sustained boundary layer ascent, particularly that on the meso- and smaller-scales. This is physically sound; ascent on multiple spatiotemporal scales is often required for deep, moist convection to initiate, particularly isolated surface-based convection in continental environments. Note the scale distinctions in these definitions; deep-layer control schemes nominally assume a quasi-equilibrium state between the processes (e.g., radiation) that produce CAPE and the parameterized convection that eliminates such CAPE, while low-level control schemes instead do not allow for CAPE to be consumed until or unless CIN is removed by low-level ascent. In practice, as both sets of processes are important for deep, moist convection to initiate, most parameterizations include elements of both deep-layer and low-level control in their trigger functions (e.g., Fig. 3).

In the next few sections, in lieu of a general discussion of closure assumptions and parameterized impacts on model fields, we describe basic characteristics of several cumulus parameterizations. We focus on one scheme that is primarily deep-layer control, the Betts-Miller-Janjic scheme used in the North American Mesoscale (NAM) model, and two schemes that are primarily low-level control, the Kain-Fritsch scheme and the Tiedtke scheme used by the ECMWF model. All three parameterize both shallow and deep cumulus. All three are available in the WRF-ARW model.

**Cumulus Parameterization Example: Betts-Miller-Janjic (BMJ) Scheme**

The BMJ scheme assumes a quasi-equilibrium state between the processes that create CAPE and the deep, moist convection that consumes it; i.e., deep, moist convection consumes CAPE as fast as large-scale processes create it. It starts by first identifying the most unstable parcel within the lowest 200 hPa above ground level and determining if this parcel has non-zero CAPE. If so, it then determines the cloud base and cloud top that could result from the parcel’s ascent. Assuming that the resulting cloud depth is at least 200 hPa, it then identifies the appropriate reference profile for the point being considered. The database of reference profiles was generated from field campaign observations in tropical convection. After the reference profile has been identified, its temperature and moisture traces are adjusted so that its vertically integrated enthalpy \( (c_pT + L_vq) \) is unchanged from that of the point considered. If the adjusted reference profile results in drying when vertically integrated over the cloud depth (if not also warming), precipitation is said to result and the scheme
activates. The model temperature and moisture profiles are relaxed toward the adjusted reference profiles over a typical convective time scale (approximately 1 h).

If this condition is not met, or if the cloud depth is less than 200 hPa, the shallow cumulus portion of the scheme is activated. Like the deep cumulus scheme, the shallow cumulus scheme identifies appropriate reference profiles, adjusts them so that they conserve enthalpy (separate for \( T \) and \( q \)), and then relaxes the model profiles toward the adjusted reference profiles over a characteristic time scale. As expected from theory, turbulent vertical mixing parameterized by the shallow cumulus scheme warms and dries below the cloud as it cools and moistens above the cloud.

The precipitation resulting from the BMJ deep cumulus parameterization is a consequence of the vertically integrated moisture excess, or the parameterized drying produced by the scheme:

\[
P = \int_{p_b}^{p_t} \frac{q_{v,r} - q_v}{\tau g} \, dp
\]

Here, \( p_b \) is the pressure at the cloud base, \( p_t \) is the pressure at the cloud top, \( q_{v,r} \) is the water vapor mixing ratio of the adjusted reference profile, \( q_v \) is the water vapor mixing ratio at the point being considered, \( \tau \) is the adjustment timescale, and \( g \) is the gravitational constant. This formulation is, as you might expect, sensitive to the atmospheric moisture content; greater environmental moisture results in greater precipitation production. However, it is not particularly sensitive to the horizontal grid spacing (not shown); i.e., it produces approximately the same result at \( \Delta x = 100 \) or 10 km.

![Figure 5](image.png)

**Figure 5.** Decision tree for the BMJ shallow (left) and deep (right) cumulus schemes. Figure reproduced from Betts and Miller (1993, *Meteor. Monographs*).
Figure 6. (a) First-guess BMJ reference profiles (red and green) for a modeled sounding (grey lines). (b) Adjusted reference profiles (thick black lines, annotated with blue arrows) for the case in (a). Figures reproduced from Baldwin et al. (2002, Wea. Forecasting), their Figs. 1 and 3, as presented in modified form in the D. Stensrud reference.

Figure 7. Adjusted BMJ reference profiles (thick black lines) for a modeled shallow cumulus case (grey lines). Figure reproduced from Baldwin et al. (2002, Wea. Forecasting), their Fig. 9.

The decision tree followed by the BMJ shallow and deep schemes is depicted in Fig. 5. Reference profiles for a representative deep cumulus case are depicted in Fig. 6a, and their adjusted versions are depicted in Fig. 6b. Adjusted reference profiles for a representative shallow cumulus case are depicted in Fig. 7.
**Cumulus Parameterization Example: Kain-Fritsch (KF) Scheme**

The KF scheme starts by mixing vertically adjacent layers over the 60 hPa nearest the ground; this is the first potential *updraft source layer*. The LCL for this updraft source layer is then determined. Next, the grid-scale vertical motion at the LCL is used to parameterize the effects of lifting on the environmental LCL temperature; ascent cools and descent warms. Taking this as a perturbation of opposite sign to the parcel’s LCL temperature, the perturbed parcel LCL temperature is compared to the environmental LCL temperature. If the former exceeds the latter, the parcel is retained as a potential updraft source layer. In this sense, the KF scheme practically requires grid-scale ascent for deep cumulus to be initiated. The parcel is then allowed to ascend freely with a vertical velocity that is proportional to the grid-scale vertical velocity plus an estimate from parcel theory, including entrainment, detrainment, and hydrometeor loading (buoyancy reduction from hydrometeor mass). If the vertical velocity remains positive over a minimum cloud depth of 2-4 km that depends on the environmental LCL temperature – shallower depth when colder, deeper depth when warmer – deep cumulus is activated. For moist environments, the entrainment of environmental air is less deleterious to upward motion than it is in drier environments, and thus this criterion is more likely to be met in moister rather than drier environments in marginal cases. If at this or an earlier stage the updraft source layer is eliminated from consideration, the process is repeated for successive model levels above the surface until the search moves above the lowest 300 hPa above ground.

The KF scheme modifies the modeled temperature, water vapor mixing ratio, and cloud water mixing ratio fields in response to both deep and shallow cumulus. For deep cumulus, mass is vertically rearranged by the updraft (as above), downdraft (driven by the evaporation of condensate generated within the updraft, factoring in both entrainment and detrainment as well), and grid-scale mass fluxes. This continues, in conjunction with latent heating, until 90% of the CAPE – here computed for an entraining parcel, not in the traditional parcel theory construct in which the parcel is assumed to remain isolated from its environment – is removed. This is done through the lowering of equivalent potential temperature in the updraft source layer and warming of the environment aloft. The time scale over which this occurs ranges from 30-60 min and is a function of the ratio of the horizontal grid spacing to the horizontal velocity in the cloud layer; larger horizontal grid spacing and/or smaller horizontal velocities result in longer adjustment time scales.

Idealized pre- and post-convection profiles for the KF deep cumulus scheme are given in Fig. 8.
Figure 8. (a) Hypothetical sounding (grey lines), updraft source layer (grey shading), and updraft and downdraft profiles (red and blue lines, respectively), the latter three assessed by the KF deep cumulus scheme. (b) The resulting sounding (thick black lines) after the KF deep cumulus scheme has been active for the full convective adjustment time. Although CAPE has been reduced, it has not been eliminated. Figures reproduced from the D. Stensrud reference, his slides 67 and 70.

The precipitation produced by KF-parameterized deep cumulus is a function of the precipitation efficiency $E$ and the total vertical water vapor and cloud water flux 150 hPa above the LCL $S$:

$$ P = ES $$

Precipitation produced by the KF scheme is, like the temperature and mixing ratio adjustments, at least weakly scale-sensitive, with greater precipitation production at finer horizontal grid spacings (not shown). Note that the KF scheme is designed to account for convective precipitation only; it is assumed that stratiform precipitation occurs on large-enough scales that it can be resolved by a microphysical parameterization.

The KF shallow cumulus scheme is similar to the BMJ shallow cumulus scheme in that it activates if the deep cumulus scheme’s triggering conditions are met except for the diagnosed cloud depth; i.e., the shallow scheme activates if the diagnosed cloud depth is less than the required 2-4 km. In this formulation, grid-scale subsidence is not allowed to explicitly hinder the ability for shallow cumulus to be activated; there is no negative parcel temperature perturbation applied in the absence of grid-scale ascent. However, the ascending parcel must still have an LCL temperature warmer than its environment for shallow cumulus to be triggered. Absent a positive perturbation from grid-scale ascent, this only occurs in the presence of a superadiabatic near-surface layer, such as may be found during the day over strongly heated lands but is exceedingly rare over the oceans. In the KF scheme, shallow convection is only activated if deep convection is not activated. Further, the updraft source layer that produces the deepest shallow cloud layer is that which is activated. The impact of shallow cumulus upon the model temperature and mixing ratio fields is determined by relating the shallow cumulus updraft mass transport to the turbulence in the sub-cloud layer. As in the deep cumulus scheme, this acts over an adjustment time scale of 30-60 min. Precipitation
produced by the shallow cumulus scheme, though not physically realistic, is fed back to the grid-scale-resolved fields as an additional moisture source.

**Cumulus Parameterization Example: Tiedtke Scheme**

The original Tiedtke scheme is based on observations that deep cumulus frequently occurs in areas of strong low-level convergence and that shallow cumulus frequently occurs beneath subsidence inversions in response to boundary layer turbulence.

For deep cumulus, a deep layer of conditional instability and large-scale moisture convergence are required. Large-scale low-level convergence is well-known to produce lift that may enable an air parcel to become positively buoyant, whereas moisture convergence may deepen moist air depth in the boundary layer and thus mitigate against losing buoyancy due to entrainment as air parcels ascend toward their LCL. The moisture convergence requirement is associated with an assumption that deep, moist convection consumes water in equilibrium with the environmental supply. This is not necessarily supported by observations, however; deep, moist convection often consumes water at a rate faster than that at which the environment can supply it. Regardless, in the Tiedtke scheme, if conditional instability and large-scale moisture convergence are present, deep cumulus are said to exist, with the resulting impact on the three-dimensional enthalpy profile diagnosed as a function of condensation and evaporation (the Tiedtke scheme does not consider frozen condensate and the associated latent heat release) as well as updraft and downdraft mass fluxes.

Both organized (large-scale) and turbulent entrainment and detrainment influence the updraft and downdraft mass fluxes. Entrainment is assumed to occur in the lower half of the cloud, beneath the level of maximum vertical velocity and thus over the layer of large-scale convergence, whereas detrainment is assumed to primarily occur in the upper half of the cloud, such as seen in organized outflow. Here, larger clouds entrain less than smaller clouds, entrainment is larger but less negative in moister than in drier environments, and deeper clouds detrain more than shallow clouds.

Because the moisture convergence assumption is not supported by observations, recent changes to the Tiedtke scheme have reposed the trigger condition in terms of CAPE and have added further requirements that the cloud depth be at least 200 hPa and the vertically averaged (over the cloud depth) relative humidity be at least 80% for deep cumulus to be present. This new version of the Tiedtke scheme largely maintains the original treatment of organized and turbulent entrainment and detrainment and their corresponding impacts on updraft and downdraft mass fluxes and, thus, the vertical enthalpy profile. Like the KF scheme, it does so under the assumption that the CAPE is removed by modifying the vertical enthalpy profile over an adjustment period.

For shallow cumulus, the Tiedtke scheme is based upon observations that the net upward moisture flux at cloud base is approximately equal to the turbulent moisture flux at the surface; e.g., surface evaporation provides the requisite moisture and boundary layer turbulence transports it upward, where it is consumed by shallow cumulus that form. A small percentage of the resulting shallow cumulus clouds are assumed to penetrate the overlying inversion layer, wherein they detrain and
thus cool and moisten the environment. The treatment of shallow cumulus is otherwise similar to that for deep cumulus.

**Forecast Sensitivity to Cumulus Parameterization**

In lieu of focusing on output from individual case study analyses, as is done in the course text and the Stensrud reference, we focus on idealized and multiple-case analyses to attempt to document differences between cumulus parameterizations as manifest in their impacts on model variables.

1. **Single-column model examples (from the D. Stensrud presentation)**

A single-column model is simply a one-dimensional version of a numerical model. In this example, WRF-ARW is used as a single-column model to illustrate forecast differences arising from the choice of cumulus parameterization. A fixed set of other parameterizations is utilized such that all forecast differences are directly or indirectly due to the choice of cumulus parameterization, although it should be noted that the results formally only apply to the specific model configuration and sounding (deeply moist and unstable) used. Three cumulus parameterizations are considered, BMJ, Grell, and KF, and all simulations are run for 4 h. The results are depicted in Fig. 9.

![Figure 9](image-url)

**Figure 9.** Simulated temperature (red; K) and water vapor mixing ratio (blue; g kg\(^{-1}\)) change after 4 h of a single-column model simulation using the (a) BMJ, (b) Grell, and (c) KF cumulus schemes. Figure reproduced the D. Stensrud reference, his slide 80.

There are broad similarities in the impacts to modeled temperature and mixing ratio fields between simulations; for example, all produce warming maximized in the mid-troposphere, although the magnitude and altitude of maximum warming vary slightly between schemes. All three result in boundary layer cooling due to evaporation of falling hydrometeors into a sub-saturated boundary layer, although the magnitude and depth of cooling vary slightly between schemes. All three also result in notable drying in the bottom of the cloud layer due to precipitation formation.
However, there are also notable differences between the three schemes. The KF and Grell schemes remove moisture from the cloud layer via precipitation, whereas the BMJ scheme adds moisture in much of the cloud layer (although the vertical integral of moisture change is still negative, required in order for the deep cumulus scheme to activate). Boundary layer cooling is deeper in the BMJ scheme due to the interplay between its shallow scheme and the boundary layer parameterization that results in a deeper simulated boundary layer. The KF scheme parameterizes updrafts that overshoot the equilibrium level or tropopause, and the resulting radiative cooling aloft results in cooling not seen in the other simulations.


In this study, a cycled ensemble data assimilation system is used to document a warm bias associated with the KF scheme’s treatment of shallow cumulus in maritime environments. Cycled ensemble data assimilation systems work by first running an $N$ member ensemble of model forecasts forward in time by a desired amount – here, and typically, 6 h – to obtain an ensemble of first-guess analyses at that time. Available observations are then assimilated to update the first-guess analyses, which are then used to advance the $N$ member ensemble forward to the next analysis time. Departures of the first-guess analyses from observations, if persistent over time, can be used to identify numerical model biases.

A 96-member mesoscale ($\Delta x = 36$ km) cycled ensemble data assimilation system was run over a north Atlantic basin domain from mid-August to mid-September 2008. Two versions of the cycled assimilation system were identically configured except for the cumulus parameterization; one used the KF scheme while the other used the Tiedtke scheme. The KF scheme underforecast shallow cumulus over water relative to the Tiedtke scheme. This resulted in too large potential temperature at and above the top of the boundary layer (~900 hPa) and too small potential temperature in the boundary layer that was ascribed to a lack of turbulent vertical mixing by shallow cumulus (Fig. 10). This was argued to result from the different trigger formulation functions for shallow cumulus between parameterizations; the KF scheme does not trigger if there is grid-scale descent at the LCL, appropriate over land but not subtropical ocean, whereas the Tiedtke scheme does not have this requirement.

In turn, the subtropical ridge predicted by the KF scheme was too intense compared to that in the Tiedtke scheme, which degraded tropical cyclone track and intensity forecasts that used ensemble analyses generated by the KF-based cycled ensemble data assimilation system as initial conditions.
Figure 10. Area- (over the southwestern North Atlantic Ocean) and time- (over 48 h) averaged model temperature tendencies due to cumulus (red), boundary layer (purple), microphysics (cyan), longwave radiation (green), and shortwave radiation (orange) parameterizations, as well as vertical advection (gray) and the sum of all tendencies (black) for numerical simulations using the (a) KF and (b) Tiedtke cumulus parameterizations. Figure reproduced from Torn and Davis (2012, Mon. Wea. Rev.), their Fig. 5.