Fundamentals of Atmospheric Radiation and its Parameterization

Source Materials

The following notes draw extensively from *Fundamentals of Atmospheric Physics* by Murry Salby and Chapter 8 of *Parameterization Schemes* by David Stensrud to discuss both fundamental tenets of atmospheric radiation and its parameterization. The latter directs readers to several additional papers, particularly Stephens (1984, *Mon. Wea. Rev.*), for further details.

Introduction

The sun is the primary source of energy for the Earth. Solar energy enters the atmosphere in the form of short wavelength (or *shortwave*) radiation. We typically refer to this as *insolation*, which is a portmanteau of "incoming solar radiation." Spatial and temporal variability in insolation drives small- and large-scale atmospheric circulations, ranging from turbulent planetary boundary layer vertical mixing on the small scales to the Hadley cell and monsoon circulation on the large scales.

In an overarching sense, insolation varies as a function of time of day (diurnal cycle), time of year (seasonal cycle), and latitude, as depicted in Fig. 1. Each influence the angle at which insolation intersects the Earth and thus the atmospheric path length over which radiation must travel in order to reach the surface at a given location. Insolation also varies as a function of attenuation within Earth's atmosphere. A numerical model must accurately parameterize the interactions of insolation with land, including surface land use characteristics; water bodies; clouds and other atmospheric sources of water molecules across its various forms; other air molecules; and aerosol. These interactions can result in absorption and subsequent terrestrial reemission as *longwave radiation*, the evolution and impacts of which must also be accurately parameterized. As one might expect, this has important implications for accurately simulating radiative heating within the atmosphere.

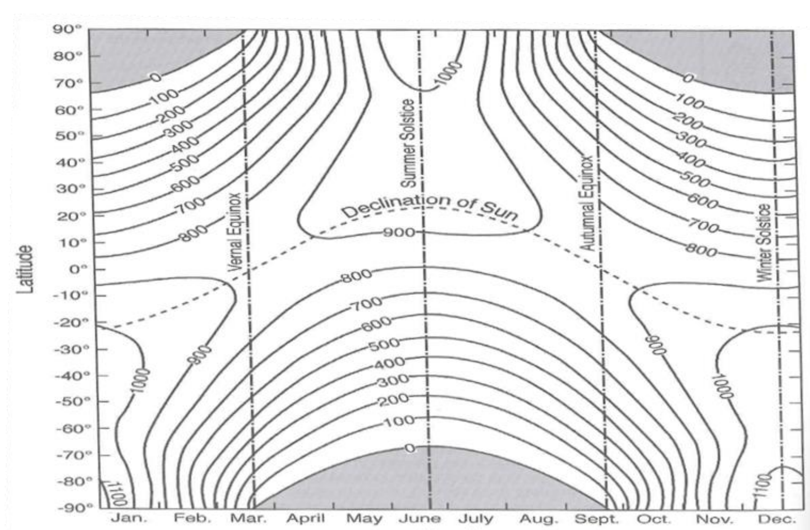

Figure 1. Shortwave radiation flux incident at the top of the atmosphere as a function of latitude (*y*-axis) and time of year (*x*-axis). Figure reproduced from Warner (2011), their Fig. 4.20.

The relevant radiative transfer processes – absorption, emission, transmission, and scattering – are molecular-level processes, far below the scales that a numerical model can explicitly resolve. In addition, radiative transfer is complex: it depends on numerous atmospheric constituents, not all of which are known at a given time and location, and over a broad spectrum encompassing both short and long wavelengths. It is typically evaluated over many distinct, finite-depth vertical layers and involves many calculations per layer. As a result, these processes and their impact upon the simulated atmosphere are parameterized in numerical models. Before we introduce how radiation is parameterized, however, we wish to first discuss the relevant physics of radiative transfer.

Radiation Fundamentals

There are four important processes to consider when attempting to describe radiative transfer:

- **Absorption**: the ability for an atmospheric constituent or the surface to take or soak up (i.e., absorb) radiation incident upon it.
- **Scattering**: the reflection of radiation by an atmospheric constituent or the surface.
- **Emission**: the ejection or release of radiation by an atmospheric constituent or the surface.
- **Transmission**: the passing through (i.e., without absorption or scattering) of radiation by an atmospheric constituent.

Further, there are two radiative transfer quantities that we wish to define:

- **Intensity (Radiance)**: the flow rate of energy *in a given direction* per a unit area. This is usually defined on a per-wavelength basis; i.e., not integrated over all wavelengths.
- **Flux (Irradiance)**: the area-integrated intensity that is fluxed, or transported, across the planar surface comprising the area over which the integration is performed. The **total flux** is the irradiance integrated over all wavelengths.

Energy transfer in the atmosphere involves radiation in two *distinct* wavelength bands: shortwave, which describes radiation emitted by the sun, and longwave, which describes terrestrial radiation. *Planck's law* gives the emitted monochromatic intensity (or radiance) for a given wavelength and temperature of an emitting blackbody surface:

$$B_{\lambda}(T) = \frac{2hc^2}{\lambda^5\left(e^{\frac{hc}{K\lambda T}} - 1\right)}$$

Note that a *blackbody* is a perfect absorber of incident and perfect emitter of absorbed radiation. Here, B is the radiance for a given wavelength λ and is written as a function of temperature T, h is Planck's constant (6.6261×10^{-34} J s^{-1}), c is the speed of light, and K is the Boltzmann constant (1.381×10^{-23} J K^{-1}). As T increases, so too does B. However, as T increases, the wavelength λ_{max} at which B is maximized decreases. This is illustrated by *Wien's Displacement Law*, which can be derived from Planck's law:

$$\lambda_{\max} = \frac{2897}{T}$$

where 2897 is in K µm and T is in K. As a result of these relationships, the radiance emitted by the sun is larger than that with terrestrial radiation, but the wavelength of maximum radiance emitted by the sun is smaller than that with terrestrial radiation. The latter of these insights is depicted in Fig. 2 below. The sun, with an average temperature of approximately 6000 K, has λ_{max} of 0.48 µm, which lies within the visible portion of the electromagnetic spectrum. Its radiance ranges from 0.15-3 µm, encompassing the ultraviolet to near-infrared portions of the electromagnetic spectrum. Conversely, the Earth, with an average temperature of approximately 288 K, has λ_{max} of 10 µm, which lies within the infrared portion of the electromagnetic spectrum. Its radiance ranges from 5-100 µm, encompassing the far-infrared to microwave portions of the electromagnetic spectrum.

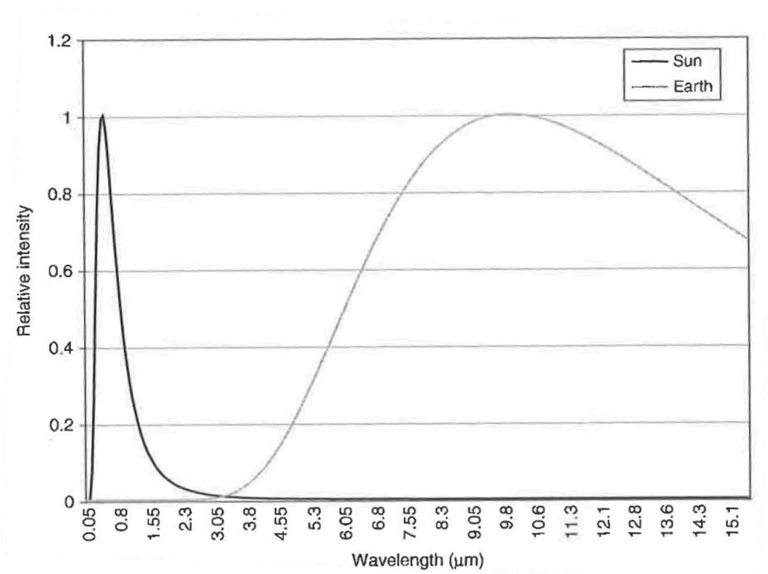

Figure 2. Relative intensity (intensity *B* normalized by *B* at the wavelength where it is maximized) as a function of wavelength for solar radiation (dark curve) and terrestrial radiation (light curve). Figure reproduced from Stensrud (2007), their Fig. 8.1.

Insolation is attenuated by a large number of atmospheric constituents. For example, ultraviolet radiation (λ < 0.3 µm) is nearly entirely absorbed at high altitudes by oxygen (O_2) and ozone (O_3); it is the loss of ozone, manifest in the ozone hole, that results in large ultraviolet radiation incident on the Earth's surface. In general, however, the remaining insolation is unaffected until reaching the troposphere, where water vapor (H_2Ov) and carbon dioxide (CO_2) are the primary absorbers. Here, most insolation except for that at visible wavelengths (0.4-0.7 µm) is absorbed. Insolation attenuation as a function of wavelength is depicted in Figs. 3 and 4.

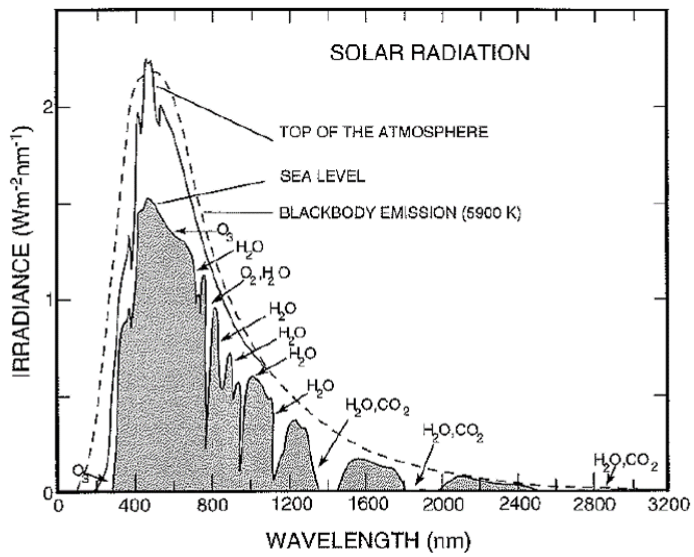

Figure 3. Irradiance (or area-integrated radiance) as a function of wavelength (nm, where 1000 nm = 1 μm) for a blackbody emitter with T = 6000 K (e.g., the sun; dashed line), observed at the top of the atmosphere (solid line) and at the Earth's surface (shaded area). Atmospheric absorbers are indicated by arrows and labels at the wavelengths where they meaningfully contribute to insolation attenuation. Figure reproduced from Salby (1996), their Fig. 1.27.

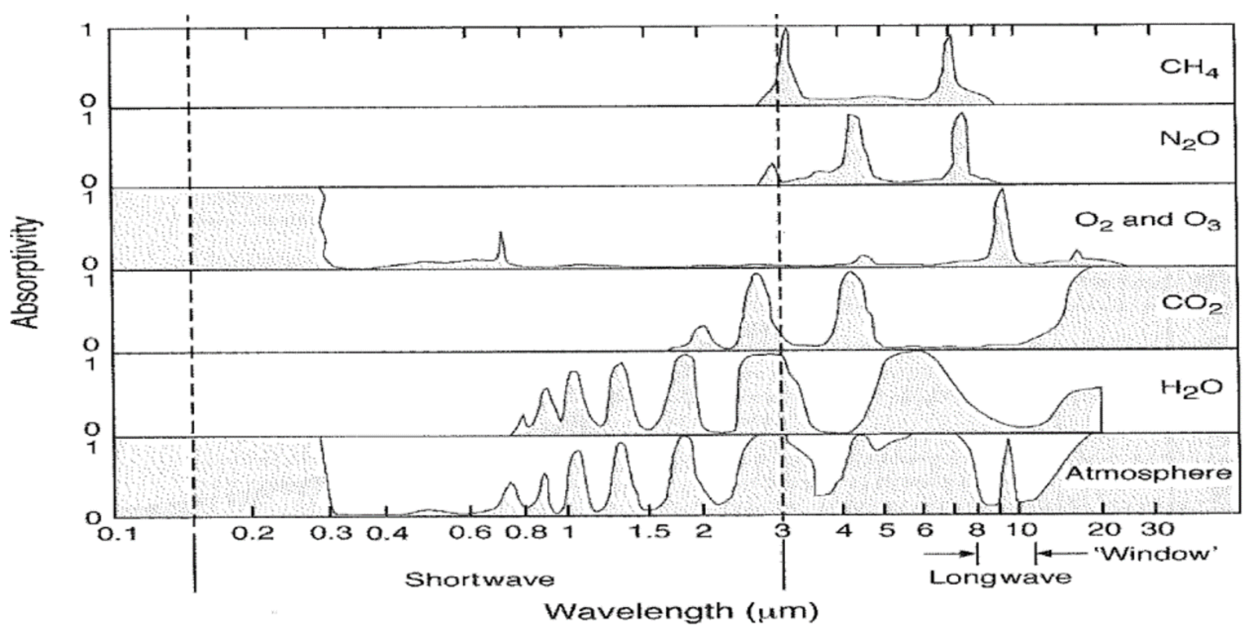

Figure 4. Absorptivity (i.e., fraction of incident radiation absorbed) as a function of wavelength (x-axis) and atmospheric constituent (first five rows; the sixth row is the sum of all atmospheric constituents). An absorptivity of 1 indicates that all total absorption by that constituent. Figure reproduced from Warner (2011), their Fig. 4.21.

The Earth reemits a large fraction of the insolation that it absorbs. As noted earlier, this reemission occurs at longer wavelengths than the incident insolation. Reemitted radiation is assumed to be isotropic, or uniform in all directions, with half directed upward and half directed downward. Much of the longwave radiation emitted by the Earth's surface is completely absorbed by the atmosphere – again primarily by water vapor and carbon dioxide, in bands centered at approximately 6.3 μm and 15 μm respectively, but also by trace gases including ozone, methane (CH_4), and nitrous oxide (N_2O) at specific wavelengths. Most of this absorption occurs within the troposphere. A notable exception is in the atmospheric window, where λ ~ 8-12 μm; here, absorption is small except for ozone around 9.6 μm, and most upward-directed longwave radiation at these wavelengths escapes to space. Naturally, some of the emitted longwave radiation is absorbed and then reemitted further. Longwave radiation attenuation by absorption can be inferred from Figs. 4 and 5.

Before proceeding, we wish to define two further terms relevant to atmospheric radiation:

- **Albedo**: the fractional amount of insolation incident upon the top of the atmosphere that is reflected and/or scattered back to space by the Earth's surface, atmosphere, and clouds.
- **Emissivity**: the fractional amount of radiation (per unit area – an irradiance – per unit time) emitted by an emitter such as the sun or the Earth's surface. Nominally, this is the fraction of absorbed radiation that is reemitted. A blackbody emitter has emissivity equal to 1.

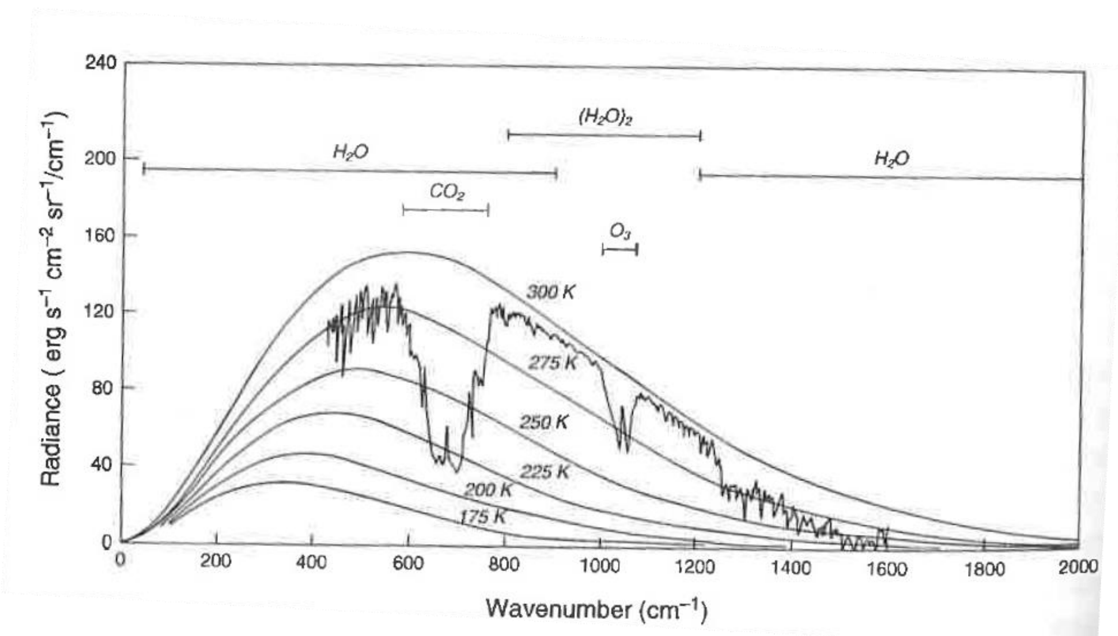

Figure 5. Theoretical radiance curves as a function of temperature and wavelength (solid labeled curves, with the *x*-axis being wavelength and not wavenumber; each computed using Planck's law) and the actually observed longwave radiance (jagged solid line). For an atmospheric temperature of 288 K, in the absence of atmospheric absorbers the observed longwave radiance should follow a smooth curve. The extent to which it does not, particularly around 6-8 μm and 10 μm, provides a measure of atmospheric attenuation by absorption. Primary attenuating constituents are labeled at the top of the chart. Figure reproduced from Stensrud (2007), their Fig. 8.2.

Global-Mean Energy Budget

A very small part of the complexity of radiative transfer is highlighted through the global-mean energy budget. First, let us consider insolation. The global-mean shortwave radiation flux incident at the top of the atmosphere is approximately 343 W m^{-2}. Of this, roughly 20% is absorbed by the atmosphere, 49% (169 W m^{-2}) is absorbed by the surface, 26% is reflected by the atmosphere, and 5% is reflected by the Earth's surface. This, including breakdowns of atmospheric absorption and reflection, is depicted in the left-hand side of Fig. 6.

To maintain thermal equilibrium, the insolation absorbed by the surface must be reemitted. Given a global-mean surface temperature of 288 K, Wien's displacement law indicates that this emission is at longer wavelengths. From the Stefan-Boltzmann law, i.e.,

$$F = \varepsilon \sigma T^4$$

where ε is the emissivity (= 1 for a blackbody emitter) and σ = 5.67 x 10^{-8} W m^{-2} K^{-4}. For T = 288 K and assuming ε = 1, the global-mean longwave radiation flux from the surface is 390 W m^{-2}. Of this, 368 W m^{-2} is absorbed within the atmosphere and 22 W m^{-2} escapes through the atmospheric window to space. The global-mean atmospheric longwave radiation flux is 542 W m^{-2}, of which 327 W m^{-2} is directed toward and absorbed by the surface. This is depicted in the right-hand side of Fig. 6.

At the surface, there is a net absorption of 169 W m^{-2} shortwave and 327 W m^{-2} longwave radiation, totaling 496 W m^{-2}, and a net emission of 390 W m^{-2} longwave radiation. This results in a net flux of +106 W m-2, which is offset by sensible (16 W m^{-2}) and latent (90 W m^{-2}) heat fluxes from the surface to the atmosphere. The 542 W m^{-2} global-mean atmospheric longwave radiation flux is approximately derived from shortwave absorption (66 W m^{-2}), longwave absorption (368 W m^{-2}), and the absorption of sensible and latent heat fluxes (106 W m^{-2}).

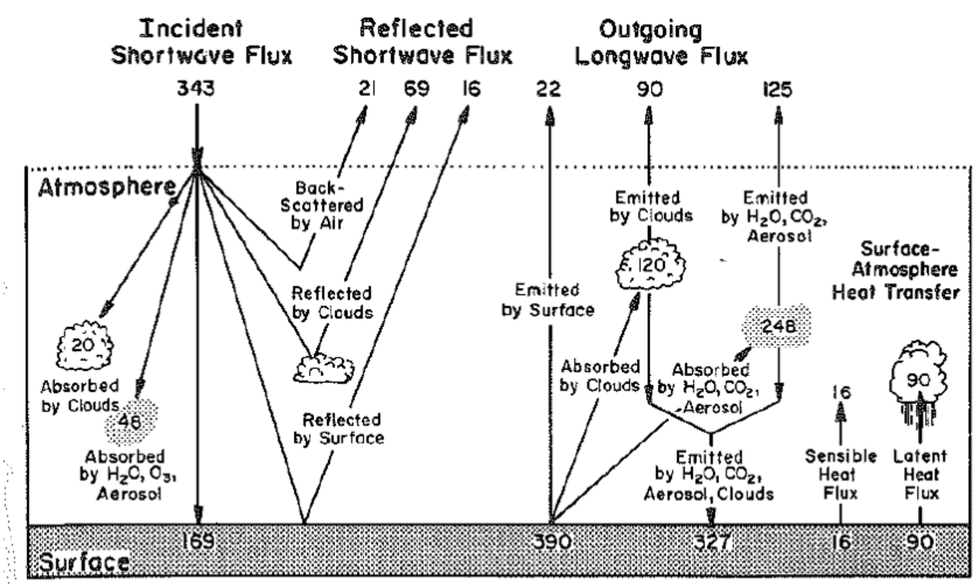

Figure 6. Global-mean energy budget. Figure reproduced from Salby (1996), their Fig. 1.27.

Radiation parameterizations must accurately and efficiently represent the physical processes that influence not only the global-mean energy budget but also local departures from this mean budget. For instance, there is substantial temporal variability in the magnitude of the shortwave radiation flux incident at the top of the atmosphere, with higher magnitudes found during local summer, as depicted in Fig. 1. There is also substantial spatiotemporal variability in the concentrations of atmospheric constituents that influence radiative transfer, yet observations of these constituents (except for water vapor) are not always available nor carried as either tracers or predicted quantities in numerical weather prediction models. Whether or not clouds are present, yet alone what types of clouds and their characteristics, also influence radiative transfer. Further, the Earth's surface is non-uniform; as previously noted, factors such as topography (influencing the atmospheric depth over which radiation must travel) and land use characteristics influence radiative transfer. Thus, while the global-mean energy budget gives us a general framework for radiative transfer, accurate depiction of atmospheric radiation is far more complex than it would otherwise imply.

Fundamentals of Absorption, Emission, and Scattering

In the absence of scattering, the absorption of radiation is described by *Lambert's law*:

$$\frac{dI_\lambda}{I_\lambda} = -\rho\sigma_{\alpha\lambda}ds$$

where I_λ is the intensity (or radiance) at a given wavelength, ρ is the mass per unit volume of the absorbing atmospheric constituents, $\sigma_{\alpha\lambda}$ is a mass absorption coefficient, and ds is an incremental distance over which the radiation travels along a slant path. For isotropic radiation, or that directed uniformly in all directions from an emitter, intensity is related to flux (or irradiance) by the relationship:

$$F_\lambda = \pi I_\lambda$$

where F_λ is the component of radiation normal to a plane parallel to Earth's surface.

Integrating Lambert's law over ds, for $s = 0$ to $s = s$, we obtain:

$$I_\lambda(s) = I_\lambda(0)\exp\left(-\int_0^s \rho\sigma_{\alpha\lambda}ds\right)$$

The *optical path length*, or the weighted dimensionless distance traveled by radiation, is a measure of the cumulative depletion of a beam of radiation as it passes through a given layer. It is given as:

$$u = \int_0^s \rho\sigma_{\alpha\lambda}ds$$

Such that:

$$I_\lambda(s) = I_\lambda(0)\exp(-u)$$

In the absence of scattering, the **absorptivity** and **transmissivity** of a medium are related to the optical path length by:

$$\Im_\lambda(s) = e^{-u} \qquad\qquad a_\lambda(s) = 1 - e^{-u}$$

such that the sum of the absorptivity and transmissivity equals 1. Formally, however, note that the physical relationship that holds is transmissivity plus absorptivity plus reflectivity equals 1, where the latter is a measure of scattering; i.e., scattering cannot be neglected.

To maintain thermal equilibrium, a substance that absorbs radiation must also emit it. The basis for describing thermal emission is the theory of *blackbody radiation*, where the emitted radiation is uniquely a function of the emitter's temperature (Planck's law), is the maximum possible at all wavelengths (a perfect emitter with emissivity equal 1), and is isotropic (intensity is independent of the direction of emission). Non-blackbodies have an emissivity equal to the absorptivity, from *Kirchhoff's law*, such that absorbers reemit all of the radiation that they absorb.

Scattering refers to the extraction and subsequent reemission of radiation by matter. It is generally non-isotropic; radiation is scattered with intensity that varies by direction and in wavelengths and directions different from those of the incident radiation. The mass scattering coefficient $\sigma_{s\lambda}$ gives a measure of the radiation lost due to scattering. From this, a total *mass extinction coefficient* can be defined that combines the effects of absorption and scattering:

$$k_\lambda = \sigma_{\alpha\lambda} + \sigma_{s\lambda}$$

This can be used to rewrite Lambert's law as follows:

$$\frac{dI_\lambda}{I_\lambda} = -\rho k_\lambda ds$$

Or, in integral form,

$$I_\lambda(s) = I_\lambda(0)\exp(-u) \qquad \text{where} \qquad u = \int_0^s \rho k_\lambda ds$$

Radiative Transfer and Radiation Parameterization

Expanding on Lambert's law, the general form of the *radiative transfer equation* is given by:

$$\frac{dI_\lambda}{\rho k_\lambda ds} = -I_\lambda + J_\lambda$$

This is identical to an algebraic reorganization of Lambert's law (wherein $-I_\lambda$ denotes the loss of radiance at a given wavelength) except for the addition of J_λ, a source term that represents the sum of emission and scattering. Of these terms, emission is equal to the output of Planck's law and is sometimes written separately; (e.g., $B_\lambda(T) + J_\lambda$), where J_λ is exclusively associated here with scattering. For longwave radiation, scattering can generally be neglected; however, the same is not true for shortwave radiation. In total, the radiative transfer equation describes the loss of radiation by absorption and scattering offset by the gain of radiation from emission and scattering.

To this point, we have considered radiative transfer over a slant path *ds*. For a layer with thickness *dz*, *ds* only equals *dz* when incident radiation is normal to the layer; e.g, when the *zenith angle* is zero. Otherwise, *ds* is larger than *dz*, such that radiation is incident upon a progressively larger number of atmospheric molecules for larger zenith angles.

We can define an *optical depth* as:

$$\tau = \int_{z_1}^{z_2} \rho k_\lambda dz$$

From which it can be shown that:

$$\tau = -u \cos\zeta$$

where ζ is the zenith angle. Where $|\zeta| > 0$, $\cos\zeta < 1$, such that the optical depth is smaller than the optical path length. The two only equal for a zenith angle of zero.

We can rewrite the radiative transfer equation in terms of optical depth:

$$-\cos\zeta \frac{dI_\lambda}{d\tau} = -I_\lambda + J_\lambda$$

It is an integral form of this equation that radiation parameterizations (or radiative transfer models) attempt to solve. In such models, the Earth's curvature is generally neglected, isotropic emission is assumed, and atmospheric constituents are said to vary more in the vertical than the horizontal. Given that their wavelengths are distinct, shortwave and longwave radiances are computed by separate parameterizations. Shortwave parameterizations compute downward-directed radiances as only insolation must be considered, whereas longwave parameterizations compute both upward- and downward-directed radiances. For each, the parameterizations attempt to simplify the integral equations into less computationally expensive forms while also making assumptions as to the value of the mass extinction coefficient (and thus optical depth) at varying altitudes.

The primary influence of a radiation parameterization is on temperature. Consequently, a radiation parameterization contributes to the time tendency of the model's thermodynamic variable. Usually, this takes the form of a vertical radiation flux convergence term; e.g.,

$$\frac{\partial T}{\partial t} \propto \frac{1}{pc_p}\frac{\partial}{\partial z}\left(F_D - F_U\right)$$

Here, the model thermodynamic variable is temperature T, F_D represents the net downward flux (shortwave plus longwave), while F_U represents the net upward flux (longwave only). Implicitly, this formulation assumes that horizontal flux convergence is negligible.

Consider a case where the net downward flux exceeds the net upward flux. Parameterized warming results if this quantity increases with height; e.g., there is more downward flux and/or less upward flux at higher altitudes. For a net downward flux that is less than the net upward flux, parameterized warming results if this quantity becomes more negative with height; e.g., less downward flux or more upward flux at higher altitudes.

A radiation parameterization is typically not called every model time step. While radiation fluxes can change over short time scales, they generally do not meaningfully change over intervals on the order of the model time step. Instead, they are typically called once very few minutes (of real time) during a model simulation. This helps mitigate the computational expense of the model simulation. Ideally, a radiation parameterization will be coupled to a microphysics parameterization so that it has appropriate inputs for atmospheric water in all of its forms. Radiation parameterization outputs are important inputs for land-surface models that simulate soil temperature and moisture and thus exert a substantial control on surface sensible and latent heat fluxes. Our inability to observe and reliably predict the evolution of atmospheric constituents such as carbon dioxide, ozone, methane, and aerosol such as dust, sand, and ash, limits the extent to which radiation parameterizations can provide accurate forecasts for weather and especially climate applications.