<u>Assignment #3: Effects of the Numerical Approximations</u>

Part II: Numerical Dispersion and Numerical Diffusion

Due: 5 November 2019 (By the start of class, preferably via e-mail. *Include your code!*)

Objective

Through practical experiments using a highly simplified numerical model, quantify the effects of numerical dispersion on model solutions and demonstrate benefits and drawbacks of implicit and explicit numerical diffusion on forecast quality and accuracy.

Introduction

This assignment uses the same numerical model formulation as in Assignment 2. Please refer to the description of that assignment for relevant instructions. All integrations are for 100,000 s.

Continued Exercises in 1-D Advection

1) (25 pts) Conduct three model simulations using a *second-order centered-in-time, second-order centered-in-space* finite difference scheme: one each for C = 0.8, 0.5, and 0.1.

    a. (7 pts) Create a plot of the three model solutions at the final time for each. Use different line colors and/or line styles for each and provide a legend. Describe how the final model solutions differ from each other.

    b. (9 pts) Compute and plot the power spectra as a function of wavelength from each simulation at the final time for each. Describe differences between the spectra from each simulation. What information is given by these spectra about the contributions of implicit numerical damping and numerical dispersion to the final solution?

    c. (9 pts) The wavelength and Courant number dependence of the phase and group velocities for this finite difference scheme is depicted in Fig. 3.25 of the textbook. Given this information, describe why the final model solutions in 1(a) differ.

2) (25 pts) Conduct three model simulations using a *second-order centered-in-time, second-order centered-in-space* finite difference scheme with a Courant number of 0.1, once using a Gaussian wave with $c = 8$ (as compared to $c = 4$ in the previous question and assignment), and once each for the following two waves:

$$h(x) = \frac{100}{b}\left(x - b\left\lfloor \frac{x}{b} + \frac{1}{2} \right\rfloor\right)\left((-1)^{\left\lfloor \frac{x}{b} + \frac{1}{2} \right\rfloor}\right) \qquad \text{(triangle wave)}$$

If $h(x)$ from the triangle wave < 0, $h(x)$ = -100.0

(square wave)

If $h(x)$ from the triangle wave $\geq$ 0, $h(x)$ = 100.0

For the triangle wave, $\lfloor \ \rfloor$ indicates the "floor" function. For this wave, let b = 10, such that the triangle and square waves are periodic over the model domain.

a. (7 pts) Create a plot of the three model solutions at the initial and final times for each. Use different line colors at the initial time and different line styles (with colors that match the initial time) at the final time and provide a legend. Describe how the wave structures evolve with time and how the final model solutions differ.

b. (9 pts) Compute and plot the power spectra as a function of wavelength from each simulation at the final time for each. Describe differences between the spectra from each simulation. What information is given by these spectra about the contributions of implicit numerical damping and numerical dispersion to the final solution? Why might this match your answer to 1(b)?

c. (9 pts) Given the initial wave structures and Fig. 3.25 of the textbook, describe why the final model solutions in 2(a) differ.

In the first two questions, we demonstrated the deleterious effects of numerical dispersion on the modeled representation of various wave-like features. Now, we wish to demonstrate how implicit and explicit numerical diffusion can be used to mitigate these effects. For the following questions, we return to our initial Gaussian wave with b = 50 and c = 4.

3) (25 pts) The 1-D advection equation underlying our model can be rewritten with a second-order explicit diffusion term as follows:

$$\frac{\partial h}{\partial t} = -U\frac{\partial h}{\partial x} + K\frac{\partial^2 h}{\partial x^2}$$

Here, K is a positive-definite diffusion coefficient. Using a *second-order centered-in-time, second-order centered-in-space* (for first **and** second partial derivatives) finite difference scheme, with $K = 3.0\text{x}10^3$ m^2 s^{-1} and C = 0.1, integrate the model forward.

a. (7 pts) Create a plot with the model solutions at the initial, second, 350th, 700th, and last times. Use different line colors and/or line styles for each and provide a legend. Describe how the model solution evolves with time. How does the solution at the final time compare to that in 1(a) for C = 0.1?

b. (9 pts) The scale selectivity of this second-order explicit diffusion term is depicted in Fig. 3.34 of the course text (note that *K* is different between that figure and the application here). Describe how this scale selectivity impacts the final solution as compared to that in 1(a).

c. (9 pts) Compute and plot the power spectrum as a function of wavelength at four times: the initial time, the 350th time, the 750th time, and the final time. Does this analysis support your answer to 3(b)? Discuss.

4) (25 pts) Using a *Runge-Kutta 3 in time, third-order upwind-biased* finite difference scheme and a Courant number of 0.1, integrate the model forward.

d. (7 pts) Create a plot with the model solution at the initial, second, 350th, 700th, and last times. Use different line colors and/or line styles for each and provide a legend. Describe how the model solution evolves with time.

e. (9 pts) Compute and plot the power spectrum as a function of wavelength at four times: the initial time, the 350th time, the 750th time, and the final time. Describe how the power spectrum changes in time, particularly as a function of wavelength. (You may wish to zoom in on particular wavelengths in answering this question.)

f. (9 pts) Compare your plots to those from 1(a) and 1(b) in Assignment 2. Describe how and why the model solutions differ between the two differencing schemes.