

Trade-Wind Inversion

Introduction

The trade-wind inversion is one of the most prominent manifestations of temperature inversions within the troposphere. Trade-wind inversions are commonly found in association with the subtropical anticyclones connected to the descending branch of the time-averaged Hadley cell circulation. The trade-wind inversion has impacts upon the sensible weather of the boundary layer across the subtropics and northern reaches of the tropics, particularly manifest in stability, cloud cover, and precipitation. In the following, we first describe the basic characteristics of the trade-wind inversion and its impacts upon sensible weather in the tropics and subtropics. In so doing, we focus specifically upon its structure and impacts in the eastern North Pacific Ocean during Northern Hemisphere summer, although this can easily be generalized to any of the other ocean basins (e.g., eastern South Pacific Ocean, eastern North and South Atlantic Ocean, etc.) in which a trade-wind inversion is present. We then develop lapse rate tendency equations to help us understand the factors influencing trade-wind inversion formation and intensity.

Key Concepts

- What are the climatological structure and intensity of subtropical anticyclones?
- How and why do vertical profiles of temperature and moisture vary with increasing distance from the center of a subtropical ridge?
- What physical processes contribute to inversion formation and intensity?

Trade-Wind Inversion Structure and Impacts

Trade-wind inversions are commonly found in the eastern regions of the subtropical oceans and the western coastal regions of adjacent continents. The trade-wind inversion is characterized by a layer in which temperature increases with increasing height above the surface layer, in which temperature decreases with increasing height. Associated with the trade-wind inversion are the northeasterly (Northern Hemisphere) or southeasterly (Southern Hemisphere) trades diverging from the subtropical anticyclones toward tropical latitudes. Trade-wind inversion strength, measured as the increase in temperature from the bottom to the top of the inversion, varies primarily as a function of the distance from the western coastal regions and centers of the subtropical anticyclones aloft. The trade-wind inversion is strongest and is found at lower altitudes to the east, near the continental coastlines. It weakens and its base is found at progressively higher altitudes with increasing distance toward the west and toward the Equator. The diurnal and annual/seasonal cycles also exert influences upon trade-wind inversion strength. There is only modest variability in the thickness of the inversion layer, however.

Most generally, the trade-wind inversion serves to limit clouds and turbulent mixing to below a height slightly above inversion base. Given the zonal variability in trade-wind inversion strength and structure, however, it stands to follow that the specific impacts of the trade-wind inversion on sensible weather also vary in the zonal direction. Near the coast, where the inversion base is low, convective precipitation is largely absent. The only precipitation that occurs in these regions during the summer months is infrequent light drizzle from stratus clouds. Owing to relatively light winds beneath the inversion in the near-coastal environment, fog and haze are relatively common occurrences along and near the coastlines.

Stratus and stratocumulus clouds are most common (~30%) a short distance away from the coast and less common farther away from the coast. Similarly, average cloud cover (of all cloud types) is greatest (~60%) a short distance off of the coast and decreases farther away from the coast. To the west, where the inversion is less persistent, weaker, and higher in altitude, trade wind cumulus clouds are frequent and convective precipitation in the form of rain showers from these clouds is possible. Thunderstorm activity is infrequent across the entire region(s) covered by the trade-wind inversion.

Trade-Wind Inversion Formation and Modulation

To discuss the factors influencing trade-wind inversion formation and modulation, we develop a mathematical framework by which the temporal change of the vertical gradient in potential temperature can be assessed. We start with the first law of thermodynamics, which can be expressed as:

$$(1) \quad c_p \frac{T}{\theta} \frac{D\theta}{Dt} = \sum_i H_i$$

where H_i are all heating sources and sinks, including latent heating, sensible heating, and radiation. We also find it helpful to recall Poisson's equation, given by:

$$(2) \quad \theta = T \left(\frac{p_o}{p} \right)^{\frac{R}{c_p}}$$

If we solve (2) for T and substitute into (1), we obtain:

$$(3) \quad \frac{D\theta}{Dt} = \frac{1}{c_p} \left(\frac{p_o}{p} \right)^{\frac{R}{c_p}} \sum_i H_i$$

We can expand the total derivative in (3) as follows:

$$(4) \quad \frac{\partial \theta}{\partial t} = -\vec{v}_h \cdot \nabla_h \theta - \omega \frac{\partial \theta}{\partial p} + \frac{1}{c_p} \left(\frac{p_o}{p} \right)^{\frac{R}{c_p}} \sum_i H_i$$

where subscripts of h denote horizontal quantities and operators.

It can be shown that:

$$\nabla \cdot (\mathbf{v}\theta) = \mathbf{v} \cdot \nabla \theta + \theta(\nabla \cdot \mathbf{v})$$

In an isobaric coordinate system as we utilize here, the continuity equation states that $\nabla \cdot \mathbf{v} = 0$, such that:

$$\nabla \cdot (\mathbf{v}\theta) = \mathbf{v} \cdot \nabla \theta$$

Using this to rewrite (4), we obtain:

$$(5) \quad \frac{\partial \theta}{\partial t} = -\nabla_h \cdot (\mathbf{v}_h \theta) - \frac{\partial}{\partial p} (\omega \theta) + \frac{1}{c_p} \left(\frac{p_o}{p} \right)^{c_p} \sum_i^R H_i$$

If we write potential temperature and velocities as the sum of a mean (overbar) and perturbation (prime) quantity, e.g., $u = \bar{u} + u'$, substitute into (5), and take the Reynolds average of the result, we obtain:

$$(6) \quad \frac{\partial \overline{(\theta + \theta')}}{\partial t} = -\frac{\partial}{\partial x} \left(\overline{(u + u')(\theta + \theta')} \right) - \frac{\partial}{\partial y} \left(\overline{(v + v')(\theta + \theta')} \right) - \frac{\partial}{\partial p} \left(\overline{(\omega + \omega')(\theta + \theta')} \right) + \frac{1}{c_p} \left(\frac{p_o}{p} \right)^{c_p} \sum_i^R \overline{H_i}$$

We can simplify this using Reynolds' postulates, where for any two variables a and b , $\overline{\overline{a}} = \overline{a}$, $\overline{a'} = 0$, $\overline{a'b} = 0$, and $\overline{a'b'} \neq 0$. Doing so, we obtain:

$$(7) \quad \frac{\partial \bar{\theta}}{\partial t} = -\frac{\partial}{\partial x} (\overline{u\theta}) - \frac{\partial}{\partial x} (\overline{u'\theta'}) - \frac{\partial}{\partial y} (\overline{v\theta}) - \frac{\partial}{\partial y} (\overline{v'\theta'}) - \frac{\partial}{\partial p} (\overline{\omega\theta}) - \frac{\partial}{\partial p} (\overline{\omega'\theta'}) + \frac{1}{c_p} \left(\frac{p_o}{p} \right)^{c_p} \sum_i^R \overline{H_i}$$

In (7), the second and fourth right-hand side terms represent turbulent (i.e., small-scale) horizontal mixing and are said to be negligibly small. Invoking the relationship that $\nabla \cdot (\mathbf{v}\theta) = \mathbf{v} \cdot \nabla \theta$, we can rewrite (7) as:

$$(8) \quad \frac{\partial \bar{\theta}}{\partial t} = -\bar{u} \frac{\partial \bar{\theta}}{\partial x} - \bar{v} \frac{\partial \bar{\theta}}{\partial y} - \bar{\omega} \frac{\partial \bar{\theta}}{\partial p} - \frac{\partial}{\partial p} (\overline{\omega'\theta'}) + \frac{1}{c_p} \left(\frac{p_o}{p} \right)^{c_p} \sum_i^R \overline{H_i}$$

The lapse rate of potential temperature is defined as:

$$(9) \quad \Gamma = -\frac{\partial \theta}{\partial p}$$

If we take the derivative of (8) with respect to p , multiply by -1, and substitute with (9), we obtain:

$$(10) \quad \frac{\partial \bar{\Gamma}}{\partial t} = \frac{\partial \bar{v}}{\partial p} \cdot \nabla \bar{\theta} - \bar{v} \cdot \nabla \bar{\Gamma} - \bar{\omega} \frac{\partial \bar{\Gamma}}{\partial p} - \bar{\Gamma} \frac{\partial \bar{\omega}}{\partial p} + \frac{\partial^2}{\partial p^2} (\overline{\omega'\theta'}) - \frac{\partial}{\partial p} \left[\frac{1}{c_p} \left(\frac{p_o}{p} \right)^{c_p} \sum_i^R \overline{H_i} \right]$$

In a *dry* sense, for large values of (10), such as in an inversion, the atmosphere is said to be very stable. For small values of (10), the atmosphere is said to be stable. For (10) equal to zero, the atmosphere is said to be neutral. For negative values of (10), the atmosphere is said to be unstable.

From (10), we can gauge how various physical processes increase or decrease the lapse rate and thus influence inversion strength. We now consider how each of the six terms on the right-hand side of (10) impacts the local rate of change of the lapse rate.

(a) Differential horizontal potential temperature advection

This term represents changes in stability associated with vertical variation in horizontal potential temperature advection. For small horizontal $\bar{\theta}$ gradients and weak vertical wind shear (of the large-scale mean flow) across the trade-wind inversion, we consider this term to be negligibly small.

(b) Horizontal lapse rate advection

This term alone cannot create an inversion; rather, it can only horizontally transport a pre-existing inversion. As horizontal $\bar{\Gamma}$ gradients are small, as can be inferred from observations contained within the lecture materials, this term is a small contributor to the strength of the trade-wind inversion at any given location. It is largest in the central oceans, where persistent easterly flow acts to transport a stronger inversion (and thus more stable conditions) from the east to the west.

(c) Vertical lapse rate advection

As with horizontal lapse rate advection, this term also cannot create an inversion; however, it can lead to impacts upon the altitude and/or depth of a pre-existing inversion. Large-scale subsidence ($\bar{\omega} > 0$) associated with the subtropical ridge can act to push an inversion closer to the surface. Overall, however, as large-scale vertical motions and vertical $\bar{\Gamma}$ gradients are fairly small in magnitude away from the core of the subtropical ridge, this term is also a small contributor to the strength of the trade-wind inversion at any given location.

(d) Divergence term

From continuity, the vertical gradient of the large-scale vertical motion field can be expressed as equivalent to the large-scale horizontal divergence field. For divergence ($\nabla \cdot \bar{\mathbf{v}} > 0$), $-\frac{\partial \bar{\omega}}{\partial p}$ must also be positive. Requiring that the lapse rate not be dry adiabatic or superadiabatic, where potential temperature is constant or decreases with height, $\bar{\Gamma}$ must also be positive. With this entire term positive, $\frac{\partial \bar{\Gamma}}{\partial t}$ is also positive, implying a stronger inversion and greater stability with time. With strong divergence away from the subtropical ridge, as can be inferred from the streamline analysis contained within the lecture materials, we find that this term is the biggest contributor to the presence of the trade-wind inversion! As divergence weakens with westward extent, the influence of this term becomes weaker yet remains a significant contributor to the inversion over the central oceans.

(e) Turbulent vertical heat flux

This term reflects the large-scale averaged effects of turbulent vertical motions within clouds. Consider the vertical distribution of potential temperature, whether within clouds or over the large-scale environment. Potential temperature increases with height, particularly near the base of the inversion. As a result, cloud-scale turbulent descent mixes higher potential temperature downward and cloud-scale turbulent ascent mixes lower potential temperature upward. This describes a

situation in which ω' and θ' are positively correlated, wherein a downward-directed heat flux acts to weaken an inversion.

Demonstrating this mathematically in the context of (10), as clouds and their associated turbulent vertical motions are found at and below inversion base over a somewhat shallow layer, $\overline{\omega'\theta'}$ is maximized in the vertical near inversion base and is relatively small above and below inversion base.

The second derivative can be approximated as:

$$(11) \quad \frac{\partial^2}{\partial p^2}(\overline{\omega'\theta'}) = \frac{(\overline{\omega'\theta'})_{top} + (\overline{\omega'\theta'})_{bottom} - 2(\overline{\omega'\theta'})_{mid}}{(\Delta p)^2}$$

As we are computing (11) at inversion base, where $(\overline{\omega'\theta'})_{top}$ is largest, the $(\overline{\omega'\theta'})_{mid}$ term dominates the computation. Because ω' and θ' are positively correlated, $(\overline{\omega'\theta'})_{mid}$ is positive. This means that $-2(\overline{\omega'\theta'})_{mid}$ is negative. Since the denominator of (11) is positive-definite, the second derivative as a whole is thus negative. This implies $\frac{\partial \overline{\Gamma}}{\partial t} < 0$, reflecting a decrease in inversion strength and stability with time. Thus, clouds destabilize and counterbalance the strong stabilizing effect of large-scale divergence, particularly to the east where cloud coverage (particularly stratus clouds) is greatest.

(f) Heating terms

For the purposes of drawing insight into the impacts of diabatic processes upon inversion strength and presence, this term can be interpreted as:

$$(12) \quad -\frac{\partial}{\partial p}(\sum \overline{H_i})$$

where H_i is approximated by:

$$(13) \quad H_i = H_{con} + H_{ncon} + H_{evp} + H_{sen} + H_{rad}$$

In (13), the five terms are latent heat release associated with convection, latent heat release associated with non-convective rainfall, latent heat release due to evaporation, sensible heating, and radiative heating/cooling, respectively. If we neglect rain processes, H_{con} and H_{ncon} can be neglected, leaving us with only evaporation, sensible heating, and radiative processes.

Evaporation: By definition, evaporation is a cooling process ($H < 0$). It occurs beneath the inversion via the entrainment of environmental dry air into moist, cloudy environments. If we compute its vertical derivative near and beneath inversion base, greater evaporative cooling is noted near inversion base, where drier air from atop the inversion more readily mixes into the cloudy, saturated boundary layer, with less evaporative cooling at lower altitudes. Thus, its vertical derivative with respect to p is positive, given the conventions on H and ∂p , and the entire term is negative given the leading negative sign on (12). As a result, evaporation acts as a destabilizing term. Its magnitude

is comparable to that of other destabilizing effects and there is minimal variation in its magnitude across an oceanic basin.

Sensible Heating: To gauge the impact of sensible heating on inversion strength, we must consider the temperature of the underlying surface, given here by the sea-surface temperature. In the eastern portion of an ocean basin, sea-surface temperature is less than that of the atmosphere. Sensible heat transfer is downward (negative) in this case as the air loses heat to the ocean. Intuitively, this promotes a near-surface inversion and acts as a stabilizing, inversion building term. In the western portion of an ocean basin, with greater distance from the cold oceanic currents that characterize the eastern portions of the world's major oceans, sea-surface temperature is warmer than that of the atmosphere. Sensible heat transfer is upward (positive) in this case as the air is warmed by the ocean. This promotes a less stable environment, thereby acting as an inversion weakening term.

Radiation: Consider again a cloud trapped just beneath the base of the inversion. At the top of the cloud, heat is lost to space via both emission and reflection, constituting radiative cooling ($H < 0$). At the bottom of the cloud, radiation emitted from the cloud is largely trapped between it and the surface, constituting boundary layer radiative heating. This reduces the magnitude of Γ and weakens the inversion. Mathematically, as with evaporation, its vertical derivative with respect to p is positive, given the convention on ∂p , and the entire term is negative given the leading negative sign on (12). As a result, radiative processes act as a destabilizing term. Magnitudes of this destabilization are greatest in the east where cloud coverage is greater and weakest in the west where cloud coverage is more sporadic.

Considering all of these factors together, divergence and sensible heating (in the eastern oceans only) act to build or strengthen an inversion. Cloud-scale turbulence, evaporation, radiative processes, and sensible heating (in the western oceans only) act to erode or weaken an inversion. Advection can locally strengthen or weaken an inversion but cannot create an inversion. The observed weakening of the trade-wind inversion with westward extent is largely a factor of weakened large-scale divergence with westward extent.

In the previous discussion, we have considered only dry processes. Given that we are in a moist environment, if we want to truly assess stability, we need to consider the lapse rate of equivalent potential temperature θ_e , where

$$(13) \quad \Gamma_m = -\frac{\partial \theta_e}{\partial p}$$

As depicted within the lecture materials, $\overline{\Gamma}_m$ beneath the inversion is negative (i.e., equivalent potential temperature decreasing with height, an unstable situation), whereas above the inversion it is positive (i.e., equivalent potential temperature increasing with height, a stable situation).

An equation analogous to (10) can be developed to describe $\frac{\partial \overline{\Gamma}_m}{\partial t}$, with similar physical forcings acting upon it. Such an equation takes the form:

$$(14) \quad \frac{\partial \overline{\Gamma}_m}{\partial t} = -\frac{\partial \overline{v}_h}{\partial p} \cdot \nabla \overline{\theta} - \overline{v} \cdot \nabla \overline{\Gamma}_m - \overline{\omega} \frac{\partial \overline{\Gamma}_m}{\partial p} - \overline{\Gamma}_m \frac{\partial \overline{\omega}}{\partial p} + \frac{\partial^2}{\partial p^2} (\overline{\omega' \theta_e'}) - \frac{\partial}{\partial p} \left[\frac{1}{c_p} \left(\frac{p_o}{p} \right)^{\frac{R}{c_p}} \sum_i \overline{H}_i \right]$$

In (14), H_i is composed of surface evaporation, radiative processes, and sensible heating. Herein, our stability criteria are $\frac{\partial \overline{\Gamma}_m}{\partial t} < 0$ for destabilization and $\frac{\partial \overline{\Gamma}_m}{\partial t} > 0$ for stabilization. We now want to consider how each of the terms on the right-hand side of (14) act to modify the lapse rate of equivalent potential temperature.

(a) Differential horizontal equivalent potential temperature advection

As before, this term represents changes in stability associated with vertical variation in horizontal equivalent potential temperature advection. For small horizontal $\overline{\theta}_e$ gradients and weak vertical wind shear (of the large-scale mean flow) across the trade-wind inversion, this term is negligibly small.

(b) Horizontal lapse rate advection

As before, horizontal lapse rate advection can only act to move an inversion around and thus, on their own, cannot create or destroy an inversion. Because the inversion is stronger to the east, $\overline{\Gamma}_m$ beneath the inversion is more negative to the east. Prevailing easterly flow thus acts to destabilize the central oceans. This term is small in magnitude in the eastern oceans.

(c) Vertical lapse rate advection

Also as before, vertical lapse rate advection can only act to raise or lower an inversion. Across the entire basin, with weak large-scale vertical motion, this term is relatively small. It is on the cloud scales, as manifest via the turbulence term, where vertical transport and mixing processes become important.

(d) Divergence

Because $\overline{\Gamma}_m$ beneath the inversion is negative, large-scale divergence acts as a destabilizing term in this framework. The magnitude of this influence is relatively small, however, compared to the dry case.

(e) Turbulent heat flux

We must consider the vertical equivalent potential temperature distribution across the inversion. Equivalent potential temperature is at a minimum at inversion base and increases above and below the inversion. Thus, ascent and descent alike will act to mix higher equivalent potential temperatures into the inversion and lower equivalent potential temperature above and below the

inversion. This acts to stabilize the environment by weakening $\overline{\Gamma}_m$ within the boundary layer. This is the largest contributor to stability in this framework.

(f) Heating terms

Surface Evaporation: Evaporation of moisture from the underlying ocean by trade wind-induced surface heat exchange (e.g., latent heat flux) moistens the near-surface layer. This raises equivalent potential temperature near the surface, acting to destabilize the boundary layer. Therefore, surface evaporation is a strong destabilizing term.

Sensible Heating: As before, in the eastern subtropical oceans, sea-surface temperature is less than the atmospheric temperature. The sensible cooling of the near-surface layer that results reduces equivalent potential temperature in this layer, increasing stability beneath the trade-wind inversion. In the central subtropical oceans, sea-surface temperature is greater than the atmospheric temperature. The sensible warming of the near-surface layer that results increases equivalent potential temperature in this layer, decreasing stability beneath the trade-wind inversion.

Radiative Processes: As before, radiative processes act to cool near the top of the inversion and warm beneath the inversion, affecting both θ and θ_e in a like sense. The mathematical and physical interpretations are thus the same as for the dry case, such that radiation destabilizes the boundary layer.

Thus, from the context of moist static stability, turbulent fluxes and sensible heating (in the eastern oceans) act to stabilize the boundary layer beneath the trade-wind inversion by making $\overline{\Gamma}_m$ less negative. Divergence, surface evaporation, radiative processes, and sensible heating (in the central oceans) act to destabilize the boundary layer beneath the trade-wind inversion by making $\overline{\Gamma}_m$ more negative. Advection can lead to the transport of enhanced or reduced instability within the boundary layer but is a relatively small contributor overall.

References

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