Tropical Cyclone Structure

Introduction

To this point in the semester, we have only briefly touched upon the salient structural features of a tropical cyclone. We now describe these in greater detail. Included in this are structural aspects of both the primary (horizontal) and secondary (vertical) circulations and the axisymmetric and asymmetric structure of a tropical cyclone. We begin by introducing the basic structure of a tropical cyclone, using that as a launching point for exploring the tropical cyclone secondary circulation and asymmetric structure in greater detail.

Key Concepts

- What are the salient characteristics of a tropical cyclone's primary circulation?
- What are the salient characteristics and dynamics of a tropical cyclone's secondary circulation?
- What is the typical structure of a tropical cyclone rain band?
- What are secondary eyewalls and what impact do they have on tropical cyclone intensity?

Tropical Cyclone Structure: Overview

Tropical cyclones, as areas of low pressure, are characterized by cyclonic tangential and inflowing radial winds. The cyclonic winds can extend out to over 1000 km from the center in the lower troposphere; this radial extent decays with increasing height. Tropical cyclones are warm-core features, meaning that their intensity (as measured by the cyclonic tangential wind speed) decreases with increasing height. A tropical cyclone is most intense just above the boundary layer, where frictional dissipation terminates, and weakest in the upper troposphere, where the winds become anticyclonic and evacuate mass outward. Radial inflow is typically maximized within the boundary layer, with weaker inflow into the middle troposphere. Radial inflow rapidly decelerates upon reaching the tropical cyclone eyewall, where convergence leads to tropospheric-deep ascent within the eyewall. Compensatory descent for such strong ascent occurs in a concentrated manner in the eye and in a diffuse manner at radii larger than the radius of maximum winds.

The warm-core structure of a tropical cyclone can be viewed to first approximation as a hydrostatic response to a radially constrained, middle-to-upper tropospheric warm potential temperature anomaly near the tropical cyclone's center. This warm anomaly primarily results from latent heat energy extracted from the underlying surface that is released aloft within updrafts as vapor condenses and subsequently freezes. A small but non-negligible contribution to this warm anomaly is also observed from subsidence warming within the eye. The hydrostatic approximation implies that higher warm anomaly altitudes result in greater surface pressure falls for a given anomaly magnitude; however, Stern and Nolan (2012) document that the warm anomaly is not always maximized at high altitudes and that the relationship between warm anomaly structure and cyclone intensity is not entirely linear.

In a planar view, a mature tropical cyclone is characterized by a nearly cloud-free region near its center, termed the eye. The minimum sea-level pressure is found at the center of the eye. For weaker tropical cyclones without eyes, the minimum sea-level pressure is typically found at the where the warm anomaly associated with the tropical cyclone is strongest. The primary eyewall is found at the outermost radius of the eye. Here, intense convection and modestly strong updrafts (~5-10 m s^{-1}) are often found. The eyewall is often the location of the radius of maximum winds. On average, the eyewall and radius of maximum winds are typically found approximately 35 km from the center of the tropical cyclone; however, much smaller and much larger radii are often observed.

The eyewall region is characterized by an equivalent potential temperature maximum. Isosurfaces of equivalent potential temperature are nearly vertical within the eyewall, implying that equivalent potential temperature is nearly constant with increasing height. Above the level of non-divergence, these surfaces flare radially outward. A local minimum in equivalent potential temperature is found in the middle to upper troposphere within the eye itself, reflecting drying associated with concentrated descent into the eye.

The eyewall and radius of maximum winds within a mature tropical cyclone slope outward with increasing height at an angle approaching 45°. This implies that the outward displacement of the eyewall in the upper troposphere (relative to its location at the surface) is approximately equivalent to its height above the sea surface. The physical reasoning behind this sloping structure lies with the conservation of angular momentum and the warm-core structure of the tropical cyclone. As air parcels ascend within the eyewall, angular momentum is approximately conserved. For a rotating body such as a tropical cyclone, recall that angular momentum is a function of the radius from the center of rotation (*r*) and the tangential wind speed (*v*), i.e.,

(1)
$$m = rv + \frac{fr^2}{2}$$

Since *v* decreases with increasing height, *r* must increase for *m* to remain constant.

A moat region, or region of predominantly stratiform precipitation, is found radially outward of the eyewall. Continuing radially outward, mature tropical cyclones may possess a secondary eyewall. These secondary eyewalls form in response to the accumulation of heat energy, angular momentum, and vertical vorticity at some critical radius. The precise dynamics behind secondary eyewall formation remain unclear, however. Kossin and Sitkowski (2009) suggest that secondary eyewall formation is associated with high values of maximum potential intensity, small values of vertical wind shear, weak upper tropospheric zonal winds, a deep layer of underlying warm water, and high middle-to-upper tropospheric relative humidity. In other words, factors that promote high tropical cyclone intensity also promote secondary eyewall formation.

Secondary eyewall formation temporarily halts tropical cyclone intensification, as secondary eyewall formation effectively cuts off radial inflow into the inner eyewall. As the secondary eyewall matures and begins to contract to smaller radii, compensating descent acts to erode the inner eyewall and clear out the moat region. After several hours to multiple days, the inner eyewall has completely dissipated, leaving a modestly weaker tropical cyclone. The larger eye and broadening of the tropical cyclone's wind field that result from this process, however, result in a stronger storm (as assessed by area-integrated kinetic energy). Reintensification is possible after the culmination of an eyewall replacement cycle assuming an otherwise favorable environment and the delayed formation of another secondary eyewall.

Beyond the secondary eyewall are the rain bands of the tropical cyclone. Such rain bands can be characterized as primary, secondary, or distant. Distant rain bands are composed of deep, moist convection along confluence lines at large radii (*r* > 200 km) and occur primarily where the environmental convective available potential energy (CAPE) is largest. Significant vertical motions (on the order of tens of meters per second both upward and downward) and lightning activity are often found with distant rain bands, implying a predominantly convective nature to these features. Tornadic activity is possible along distant rain bands, particularly with those found in the right-front quadrant of landfalling tropical cyclones where the lower-tropospheric helicity is often maximized with environmental westerly vertical wind shear.

Secondary rain bands are typically found somewhat radially inward of a principal rain band. Often, the two intersect. Secondary rain bands in more intense tropical cyclones are believed to be manifestations of vortex Rossby waves (e.g., Montgomery and Kallenbach 1997), which are waves that propagate on the radial gradient of the azimuthal-mean cyclone vertical vorticity, similar to mid-latitude Rossby waves that propagate along the meridional potential vorticity gradient. They propagate cyclonically and outward at a rate of speed much less than the mean tangential flow of the tropical cyclone. Wave energy associated with a vortex Rossby wave accumulates at what is called a stagnation radius, whereby the strong tangential flow of the tropical cyclone shears and mixes the energy about the tropical cyclone. This shearing and mixing process is often referred to as axisymmetrization.

The principal rain band lies predominantly in the tropical cyclone inner core (*r* < 200 km). Principal rain bands tend to remain stationary in a storm-relative reference frame (i.e., they don't rotate around the cyclone to a significant extent). The precise dynamics behind their formation are unclear, however. A principal rain band is characterized by new convection on its upwind flank, mature convection in its core, and more stratiform-like precipitation on its downwind flank. Principal rain bands are typically found radially inward of a localized middle tropospheric jet, termed a secondary horizontal wind maximum. Convection within principal rain bands slopes radially outward with height, as does eyewall convection, but is generally constrained to within the lowest 8-10 km of the atmosphere.

Individual convective elements within principal rain bands are associated with both updrafts and downdrafts. Downdrafts are typically found radially inward of updrafts and result in a sharp horizontal edge to the region of precipitation associated with the rain band. Low equivalent potential temperature air with such downdrafts can subsequently interact with the eyewall convection and reduce its vigor, as described in our earlier lecture on tropical cyclone intensity change. Updrafts transport high equivalent potential temperature) air upward within the rain band. Dynamically, updrafts tilt horizontal vorticity, found beneath the secondary horizontal wind maximum, into the vertical and subsequently amplify it via vortex stretching. Vertical advection accumulates vertical vorticity within the middle troposphere, subsequently intensifying the secondary horizontal wind maximum.

Inner-core convection is typically most intense during the local nighttime hours whereas outer rain bands are typically most intense during the local daytime hours. The oscillation between these two states is known as the “tropical cyclone diurnal cycle,” most readily identified in intense tropical cyclones, and can be observed as an outward-moving pulse in infrared satellite imagery (Dunion et al. 2014). The most-likely causes of these pulses are cloud-radiation feedbacks that vary between day and night (e.g., daytime heating of the boundary-layer by insolation; nighttime cooling by outgoing longwave radiation) and inertia-gravity waves that are convectively generated by latent heating in and near the cyclone’s inner core (Dunion et al. 2014, 2019).

Secondary Circulation: Sawyer-Eliassen Framework

As previously described, a tropical cyclone's secondary circulation is characterized by radial inflow at low levels, ascent near its center, and radial outflow near the tropopause. Colloquially, this is sometimes referred to as the cyclone's "in-up-out," or axisymmetric, circulation. This circulation is thermally direct in nature because ascent is found where it is warmest. Compensating descent occurs at larger radii where it is relatively cooler. An exception is in the eye, where locally warm air descends.

The Sawyer-Eliassen non-linear balance framework enables us to analytically describe the structure of this circulation as a function of the structure of the tropical cyclone and its environment. Furthermore, it also enables us to describe how this circulation evolves to the imposition of external heat (e.g., latent heat release) or momentum (e.g., trough interaction) forcing. In the following, we derive this equation from first principles of the atmosphere in a radial coordinate system.

Mathematical Derivation

Below are the governing equations represented in a two-dimensional (*z*, *r*) cylindrical coordinate system. In this regard, we view the secondary circulation as axisymmetric. The equations are as follows:

$$m^2 = r^3 \frac{\partial \phi}{\partial r} \tag{2}$$

$$\frac{dm^2}{dt} = F \tag{3}$$

$$\frac{\partial \Phi}{dz} = \frac{g}{\theta_0}\theta \tag{4}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0 \tag{5}$$

$$\frac{d\theta}{dt} = Q \tag{6}$$

$$\Phi = \phi + \frac{f^2 r^2}{8} \tag{7}$$

$$z = \left[1 - \left(\frac{\rho}{\rho_0}\right)^{K}\right]\frac{c_p \theta_0}{g} \tag{8}$$

$$\frac{1}{r^3}\frac{\partial m^2}{\partial z} = \frac{g}{\theta}\frac{\partial \theta}{\partial r} \tag{9}$$

Except as noted below, all variables have their standard meaning. Subscripts of *0* denote base-state values.

Equation (2) defines gradient wind balance.

Since angular momentum is a function of tangential velocity v, (3) is the tangential momentum equation. Changes in squared angular momentum along the motion are exclusively related to prescribed momentum forcing F.

Equation (4) is the hydrostatic equation.

Equation (5) is the flux form of the continuity equation. Recall that u denotes radial and not zonal wind.

Equation (6) is the thermodynamic equation, indicating that changes in potential temperature following the motion are exclusively related to diabatic heating at a rate given by Q.

Equation (7) defines the geopotential.

Equation (8) defines a pseudoheight vertical coordinate, used to simplify the mathematics and interpretation thereof. The exponent K is equal to R_d/c_p.

Finally, (9) is the thermal wind relationship, relating vertical wind shear (i.e., the angular momentum field) to horizontal temperature gradients. We assume that a tropical cyclone is initially in thermal wind balance in the absence of any prescribed heat (Q) or momentum (F) forcing. Prescribed heat or momentum forcing destroys thermal wind balance, whereas the Sawyer-Eliassen equation allows us to describe how the tropical cyclone's secondary circulation *must* respond to prescribed heat (Q) and/or momentum (F) forcing in order to restore (or attempt to restore) thermal wind balance.

First, expand the total derivatives in (3) and (6) to obtain:

(10)
$$\frac{\partial m^2}{\partial t} + u\frac{\partial m^2}{\partial r} + w\frac{\partial m^2}{\partial z} = F$$

(11)
$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial r} + w\frac{\partial \theta}{\partial z} = Q$$

Next, take the partial derivative of (10) with respect to z and multiply the result by $1/r^3$:

(12)
$$\frac{\partial}{\partial t}\left(\frac{1}{r^3}\frac{\partial m^2}{\partial z}\right) + \frac{\partial}{\partial z}\left(u\frac{1}{r^3}\frac{\partial m^2}{\partial r} + w\frac{1}{r^3}\frac{\partial m^2}{\partial z}\right) = \frac{1}{r^3}\frac{\partial F}{\partial z}$$

Similarly, take the partial derivative of (11) with respect to r and multiply the result by g/θ_0:

(13)
$$\frac{\partial}{\partial t}\left(\frac{g}{\theta_0}\frac{\partial \theta}{\partial r}\right) + \frac{\partial}{\partial r}\left(u\frac{g}{\theta_0}\frac{\partial \theta}{\partial r} + w\frac{g}{\theta_0}\frac{\partial \theta}{\partial z}\right) = \frac{g}{\theta_0}\frac{\partial Q}{\partial r}$$

Note that in obtaining (12) and (13), the partial derivatives with respect to time have been commuted with the partial derivatives with respect to z and r, respectively.

Before proceeding, it is useful to define several additional terms to simplify (12) and (13):

(14) $$N^2 = \frac{g}{\theta_0}\frac{\partial\theta}{\partial z}$$

(15) $$B = -\frac{g}{\theta_0}\frac{\partial\theta}{\partial r} = -\frac{1}{r^3}\frac{\partial m^2}{\partial z}$$

(16) $$I = \frac{1}{r^3}\frac{\partial m^2}{\partial r} = \left(f + \frac{1}{r}\frac{\partial(rv)}{\partial r}\right)\left(f + \frac{2v}{r}\right)$$

(14-16) define static stability, baroclinicity, and inertial stability, respectively. These are akin to A, B, and C from the Kuo-Eliassen equation used to describe the Hadley cell. Applying these definitions to (12) and (13) results in the following:

(17) $$\frac{\partial}{\partial t}(-B) + \frac{\partial}{\partial z}(Iu - Bw) = \frac{1}{r^3}\frac{\partial F}{\partial z}$$

(18) $$\frac{\partial}{\partial t}(-B) + \frac{\partial}{\partial r}(-Bu + N^2 w) = \frac{g}{\theta_0}\frac{\partial Q}{\partial r}$$

Next, subtract (17) from (18) to eliminate the time partial derivatives and obtain:

(19) $$\frac{\partial}{\partial r}(N^2 w - Bu) + \frac{\partial}{\partial z}(Bw - Iu) = \frac{g}{\theta_0}\frac{\partial Q}{\partial r} - \frac{1}{r^3}\frac{\partial F}{\partial z}$$

Equation (19) describes the response in the zonal and vertical motion fields to imposed heat and/or momentum forcing. However, as there are two unlinked unknowns given by *u* and *w*, this equation is difficult to solve. To link these two variables and thus make solving the diagnostic equation simpler, the streamfunction is used. The streamfunction in this coordinate system is defined by:

(20) $$u = -\frac{\partial\psi}{\partial z}, w = \frac{1}{r}\left(\frac{\partial(r\psi)}{\partial r}\right)$$

Substituting (20) into (19) results in the following:

(21) $$\frac{\partial}{\partial r}\left(N^2 \frac{1}{r}\frac{\partial(r\psi)}{\partial r} + B\frac{\partial\psi}{\partial z}\right) + \frac{\partial}{\partial z}\left(B\frac{1}{r}\frac{\partial(r\psi)}{\partial r} + I\frac{\partial\psi}{\partial z}\right) = \frac{g}{\theta_0}\frac{\partial Q}{\partial r} - \frac{1}{r^3}\frac{\partial F}{\partial z}$$

Equation (21) is the Sawyer-Eliassen non-linear secondary circulation diagnostic equation. It highlights the relationship between the specified heating *Q*, momentum forcing *F*, and the streamfunction ψ as modulated by coefficients representing static stability, inertial stability, and baroclinicity. *The streamfunction responds to attempt to restore the thermal wind balance that the specified heating and/or momentum forcing disrupts!* While thermal wind balance restoration is never truly achieved, the concepts of balance destruction and restoration nevertheless enable us to consider how radial and vertical motions are impacted by prescribed heating and/or momentum forcing. Solutions to the Sawyer-Eliassen diagnostic model are contained within the lecture materials.

Physical Understanding

First, we focus upon describing basic solution characteristics. A localized heat source forces ascent through the region of heating. This promotes convergence beneath and divergence above the heat source. Compensating descent occurs at larger radii. A localized cyclonic momentum source forces enhanced radial outflow through the region of cyclonic momentum. The totality of the resultant streamfunction response takes on a dipole structure in the vertical. Beneath the momentum source, ascent (descent) is promoted radially inward (outward) of the momentum source. The opposite is true above the momentum source: ascent (descent) is promoted radially outward (inward) of the momentum source. Enhanced radial inflow is promoted both above and below the cyclonic momentum source.

Inertial stability, baroclinicity, and static stability modulate these basic solutions. For both localized heat and cyclonic momentum sources, such motions are constrained in the vertical for weak inertial stability (small I) and constrained in the horizontal for strong inertial stability (large I). For a barotropic vortex (zero B), the forced vertical and radial motions are predominantly upright; for a baroclinic vortex (non-zero B), they are somewhat vertically tilted, with the degree of tilting directly proportional to the baroclinicity. The vertical extent of a given streamfunction response is restricted for large static stability (large N^2). For small static stability (small N^2), the vertical extent of a given streamfunction response is much less restricted.

Physically, localized heat and momentum sources destroy thermal wind balance. To restore thermal wind balance, the effects of the heat or momentum source must be mitigated through a balanced response in the cyclone's secondary circulation. For the heat source, this occurs via adiabatic cooling with the forced ascent. For a cyclonic momentum source, the radial outflow (implying upper-tropospheric divergence and corresponding anticyclonic flow) through the momentum source erodes and expels it from the cyclone.

To first order, localized heat and cyclonic momentum sources within the upper troposphere act to enhance the secondary circulation of a tropical cyclone, particularly when each source is located within the middle-to-upper troposphere. Strengthening the secondary circulation enhances the rate at which latent heat is transported upward within the inner core, in turn intensifying the primary circulation. More specifically, a localized heat source leads to enhanced cyclonic tangential flow near and just inside of the radius of maximum heating. Weakened cyclonic tangential flow is found closer to the center of the tropical cyclone. A localized cyclonic momentum source results in enhanced cyclonic tangential flow radially inward of the radius at which the cyclonic momentum source is found.

The impacts of heat and momentum forcing on tropical cyclone intensity depend on initial cyclone size and intensity as well as initial heating or momentum source location. For initial size and intensity, the greatest impact is observed for relatively small, intense tropical cyclones; i.e., those for which the inertial stability is largest. Weaker and/or larger tropical cyclones exhibit a somewhat weaker response. For source location, localized heating located at or inside of the radius of maximum winds (RMW) is most efficient at intensifying the tropical cyclone; expanding the horizontal extent of the heat source or moving it radially outward from the RMW renders the heating less efficient. Similarly, localized cyclonic momentum forcing located at or inside the RMW is most efficient at intensifying the tropical cyclone; expanding its horizontal extent or moving it radially outward from the RMW renders the cyclonic momentum forcing less efficient at intensifying the tropical cyclone.

The radial structure of the primary circulation response to heat forcing provides a nice context from which the contraction of the RMW often observed with mature tropical cyclones can be explained. As the maximum acceleration in the tangential flow of the tropical cyclone is found radially inward of the RMW, the RMW will tend to move inward until equilibrium between the RMW and the radial location of the maximum response to the heat source is achieved. As the above implies, this context also makes clearer the process by which the primary eyewall is eroded by heat forcing found in association with a maturing secondary eyewall at larger radii.

With respect to the impact of cyclonic momentum forcing upon the intensity of a tropical cyclone, it is important to note that such forcing is often associated with an upper-tropospheric trough, which itself is often accompanied by vertical wind shear and dry middle-/upper-tropospheric air. A tropical cyclone will only intensify if the positive impact of the cyclonic-momentum source exceeds the negative impact of dry air import and vertical wind shear.

Observational and Full-Physics Modeling Support

Observations from several platforms, including airborne Doppler radar, geostationary satellite, and ground- and satellite-based lightning detection networks, and output from high-resolution numerical model simulations support the analytical insights derived from the Sawyer-Eliassen framework, particularly those related to localized heat forcing. While convective bursts (i.e., vertical hot towers) frequently occur near a tropical cyclone's center whether it is weakening or intensifying, they are preferentially found inward from the RMW for intensifying tropical cyclones, particularly those that are rapidly intensifying (e.g., Rogers et al. 2013, Hazelton et al. 2017). The region radially inward from the RMW is characterized by strong inertial stability, which confines the radial extent of the diabatic heating that accompanies convective bursts in the intensifying cases to the region inside the RMW where the cyclone's warm thermal anomaly is strongest. Further, convective bursts in intensifying tropical cyclones are often found upshear, where they may act to reduce the vortex's shear-induced tilt (e.g., Stevenson et al. 2014). This is also connected to the radial extent of the associated diabatic heating: a more-upright heating source horizontally concentrates the heating, thus resulting in a more localized but stronger response in the vortex's mass and wind fields.

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