Low-frequency oscillation

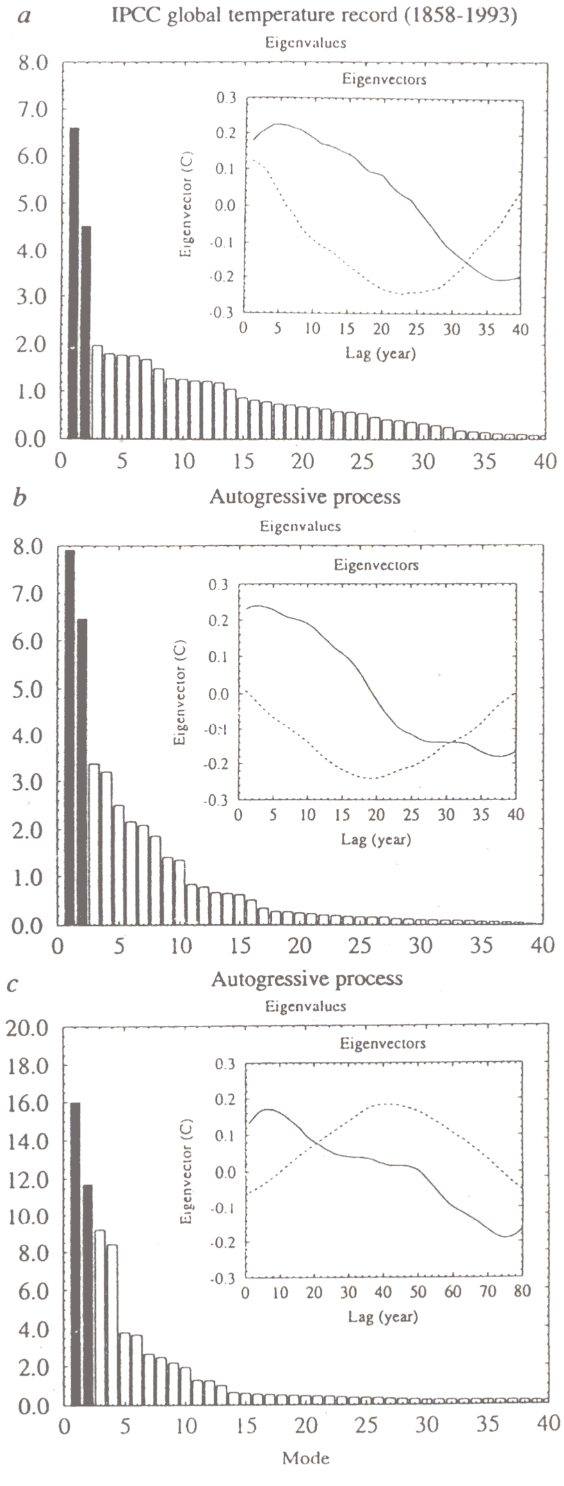
SIR — The singular spectrum approach (SSA) has recently been used to identify a low-frequency oscillation in the global surface air temperature record[1]. SSA[2,3] considers M lagged copies of a series $X(t)$ sampled at equal intervals τ, $X_i = X(t_0 + i\tau)$, $i = 1, N$, and estimates the eigenvalues λ_k and the corresponding eigenvectors ρ_k of their time-lag covariance matrix C, where $1 < k < M$. The eigenvectors are of interest if, among the k eigenvalues, there exist several whose magnitudes are significant. As such, SSA can be used to separate signal from noise.

The figure (a) shows the estimated eigenvalues, λ_k, of the time-lag covariance matrix C (with $M = 40$) for the detrended IPCC global temperature record[4]. Detrending the data is recommended in such cases (when M represents a fairly large fraction of N); otherwise the first eigenvalues representing the trend can mask or alter long-period cycles that might be present. Here we detrend the data throughout by removing the least-squares line. In agreement with the earlier work[1], we observe the first two eigenvalues appreciably higher than the so-called 'noise floor'. These two eigenvalues correspond to eigenvectors (insert in the figure) that exhibit an oscillation of approximately 60–70 years. Eigenvalues far from the 'noise floor' are often considered to represent deterministic components, whereas all others are assumed to correspond to random processes. As shown below, however, the situation is not always so simple.

The figure (b) shows the eigenvalues and eigenvectors from a detrended time series generated from a stochastic process $X(t+1) = \alpha X(t) + \epsilon(t)$ (first-order linear Markov process) and having the same length of 136 values, where $\epsilon(t)$ is white noise and α the lag-1 autocorrelation of the IPCC temperature record. Signals generated in this manner are simple autocorrelated ($AR(1)$) noise containing no deterministic oscillations.

We observe that b is almost identical to a, indicating that the very low-frequency periodicity of 65–70 years in the IPCC record can arise naturally from the simplest of stochastic processes. The eigenvalues and eigenvector for the same stochastic process but of length 272 (twice the length of the original record) are shown in c. Again we observe two eigenvalues that stand above the rest, and that correspond to eigenvectors with periods commensurate with M. Such spurious periodicities indicate that caution must be used in interpreting eigenvalues associated with low-frequency modes as they may be an artefact of SSA; a proper statistical evaluation must always accompany any evaluation of suspected oscillations in data. Our result suggests that the common practice used in spatial eigenvector analysis of attributing the most significance to the first several eigenvalues is erroneous when applying the SSA.

By decomposing 1,000 detrended $AR(1)$ time series (surrogates) of length 136 and using the same alpha as the original record, we find that the 99%/1% confidence band overlaps the first two eigenvalues obtained from the data. These results do not change if alpha is taken from the detrended IPCC record. Although explaining the largest percentage of



a, Eigenvalues of the time-lag covariance matrix from the detrended IPCC global temperature anomaly record over the period 1885–1993. The first two eigenvalues extend appreciably above the so-called noise floor and correspond to eigenvectors (insert) exhibiting an oscillation of 60–70 years. b, Eigenvalues of the time-lag covariance matrix from a detrended time series of equal length to the IPCC record and generated from a red-noise stochastic process (see text). As in a, the first two eigenvalues are appreciably above the others and correspond to eigenvectors (insert) exhibiting an oscillation of 60–70 yr. c, Eigenvalues of the time-lag covariance matrix from a detrended red-noise time series of length twice that of the record used in b (272 values). As before, there are two outstanding eigenvalues corresponding to eigenvectors with periodicities of half the record length (insert).

variance in the temperature record, these first two eigenmodes (eigenvalue/eigenvector pairs) are clearly indistinguishable from low-frequency eigenmodes of the simplest autocorrelated noise. We note that our confidence limits, obtained by using the eigenvectors of each surrogate series as basis functions (that is, each surrogate has a different set of basis functions), are very similar to the confidence bands obtained by using a fixed, single set of basis functions from the original record[5]. We speculate that the reason for this similarity is that the eigenvectors from the data are not necessarily a better estimate of the population eigenvectors than are the eigenvectors obtained from the surrogates. Indeed, when a bootstrap procedure is applied to the time series residuals (assuming an $AR(1)$ process), we find large error variances for the eigenvector components suggesting that either method of computing confidence limits is appropriate at least for this record.

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1. Schlesinger, M. E. & Ramankutty, N. *Nature* **367**, 723–726 (1994).
2. Broomhead, D. S. & King,G. P. *Physica* **20**D, 217–236 (1986).
3. Fraedrich, K. *J. atmos. Sci.* **43**, 419–432 (1986).
4. Folland, C. K. *et al.* in *Climate Change 1992 – The Supplementary Report to the IPCC Scientific Assessment* (eds Houghton J. T. *et al.*) 135–170 (Cambridge Univ. Press, 1992).
5. Allen, M. R. & Smith, L. A. *Geophys. Res. Lett.* **21**, 883–886 (1994).