Public versus Private Education in Rural-Urban Model: Income Inequality, Growth and Urbanization

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Abstract
Based on the work of Glomm and Ravikumar (1992), we present a model to analyze the implication of two education regimes on income inequality, growth and dynamics of urbanization with special concentration on duality feature in China. In static rural-urban agent model, we find that the public education system is more effective in narrowing the income inequality and impelling rural development, while the private schooling leads to higher growth in urban area. Besides, private education will expedite the overall economic growth unless inequality is sufficiently high. In the dynamic model, we devise a mechanism of rural-urban migration and further investigate the urbanization process and its impact on inequality and growth in public education regime. We find that the urbanization process appears to be accelerating and rural community will disappear in the end. Along with the urbanization process, income inequality is declining.

Keywords: Education, Income Inequality, Urbanization, Growth

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1. Introduction
Along with booming growth of Chinese economy during last 30 years after economic reform, the income gap between citizens is widening at an alarming speed. The Gini Coefficient has been soaring ceaselessly from 0.302 in 1978 to 0.462 in 2006 (Kanbur and Zhang, 2005). Many authors, like Chen et al (2005) and Dennis Tao Yang (1999) consider that the gap between urban and rural area is a significant fraction in interpreting the Chinese Gini coefficient.

Intrigued by the worrying regional disparity, we aim to present a model to analyze the implication of different education regimes on income inequality, growth and dynamics of urbanization. Built on the Glomm and Ravikumar (1992), this paper emphasizes the duality feature of Chinese economy and the heterogeneity of agents in two areas—rural and urban. In our model, the human capital investment is the engine of economic growth. The agents in two areas are different in two perspectives—the initial human capital level and preference over leisure. Two alternative education systems—public education system and private education system—are discussed. In public schooling system, the government uses tax revenue collected from rural and urban agents to fund public education. In private education system, each agent determines the optimal amount of education for his child endogenously.

Recently, income inequality in China has received considerable attention from researchers worldwide. Two key factors remain the heart of the studies on rural-urban inequality in China. Primarily, many works point out the importance of human capital and education in examining the rural-urban inequality. G. Wan (2004) uses a regression based approach to demonstrate that equalization of human capital across regions and households in rural China is important to close the gap of income between urban and rural. Terry Sicular et al. (2007) find that the only household characteristic that contributes substantially to the gap is education. Fan et al. (2004) and Zhang and Fan (2004) find that public investment in rural education would have the most impact to deal with the problem. Besides, the rural-urban migration has profound influence on the rural-urban difference and it cannot be neglected in the discussion of inequality in China.

In the paper, we take into consideration the heterogeneous preference of rural and urban agents. There is growing evidence that shows the rural and urban residents differ in their leisure involvement, in terms of the amount of time or the types of leisure activities. Rural residents enjoy less leisure time than their counterparts in urban areas. Many Chinese social scholars attribute the phenomenon to the prevailing belief in the rural area that earning money
should come before enjoying life. Besides, the types of leisure activities they involved are disparate. Zheng, He, and Chen (2004) shows how rural resident's leisure activity is closely related to regional economic development by rendering evidence that the pattern of leisure involvement is varied from place to place even within the same province.

In section 2 and 3, we develop the basic framework of the representative agent and then derive the equilibrium path of human capital accumulation. Preferences are logarithmic and depend on three components: leisure, consumption and equality of schooling for descendant. We introduce the heterogeneity of agents’ value for leisure through an exogenous parameter. With Cobb-Douglas learning technology, it is obvious that, for both urban and rural agent, the time allocated to learning is constant over time and independent of parental human capital and the quality of schooling for offspring. And rural agent is always willing to devote more to learning than the urban agent.

In section 4, we compare merits of the alternative education regimes in terms of inequality and growth both theoretically and numerically in the static setting, and further consider the educational choice of residents in a majority voting system. A consistent measure with Gini coefficient—the ratio of human capital level between urban and rural agent—is utilized to measure inequality. We find that income converges faster in public education regime. Besides, the private education is always more favorable to the growth of urban area, while the public education may expedite the development of rural area. It is instructive to compare our results with Glomm and Ravikumar (1992). They find, as we do, that income inequality is declining more quickly in public education system, which is also in conformity with existing literature like G Saint-Paul and T Verdier (1993), D de la Croix and M Doepke (2003). However, the rural-urban model renders more insights on regional development. Moreover, we replicate a political process of educational choice in majority voting system, and we conclude that the choice depends on the share of rural population and the degree of inequality.

In section 5, we establish a mechanism of urbanization and discuss its impact on income inequality and economic growth by simulation under the lognormal initial wealth distribution assumption. In public education regime, when the hardworking rural agent’s human capital level surpasses the average human capital level of his counterpart in urban area, his descendant will share the same preference with the urban residents over leisure with urban residents. In other words, the process of rural to urban migration is accomplished. This mechanism perfectly captures what is going on in China. Myriads of old generations born in rural areas fought long and hard in their youth. And their endeavor finally pays off and their
offspring can enjoy a decent life in big city. Also, the mechanism defined above satisfies the features of a successful theory of the urbanization process mentioned in Lucas (2004).

We find that the urbanization process appears to be accelerating and the rural community will disappear in the end. This finding is in conformity with some studies (Y Zhao, 1997; Zhang and Fan 2004) which point out that promoting rural education would hasten rural–urban migration. Besides, along with the urbanization process, not only the overall income inequality is declining but also the inter-group and within-group inequality. However, the growth in the dynamic model is slower than that in static model. The results are robust to the specification of parameters in the model.

Section 6 summarizes the discussion and offers a few concluding comments.

2. The Static Model
Consider an overlapping generation economy, in which individuals are classified as rural agent and urban agent based on their initial human capital. Individuals born at time \( t \) allocate their time between leisure and human capital accumulation. At \( t+1 \), individuals allocate their income (measured by human capital level) between consumption and education expenditure for children. Also, each individual born at time \( t \) gives birth to another at the beginning \( t+1 \) so that population remains constant over time. We normalize the population to 1, and denote \( \alpha \) as the share of rural population.

At this time, it’s convenient to assume the agents living in each area are homogenous. Thus a simple representative agent model in each generation is helpful in this setting. At time \( t=0 \), there is an initial generation of old agents in which the rural and urban agent are endowed with \( h_j'(0) = h_{j0} \) (\( j=1,2 \)) respectively, and \( h_{10} < h_{20} \).

Formally, the agents’ preferences over leisure are different in the two areas. And it is reasonable to suppose that agent in rural area is more willing to give up leisure to accumulate human capital. Therefore, the utility of the representative agent \( j \) (\( j = 1,2 \) represent the agent in rural area and urban area respectively) born at time \( t \) can be specified by:

\[
\lambda_j \ln l_j^t + \ln c_{j+1}^t + \ln e_{j+1}^t
\]

where \( l_j^t \) is leisure at time \( t \), \( c_{j+1}^t \) is consumption at \( t+1 \), and \( e_{j+1}^t \) is the expenditure for schooling at \( t+1 \) for agents born at time \( t \); \( \lambda_j \) measures the agents’ preferences over leisure, and we have \( \lambda_1 < \lambda_2 \), implying that leisure is more preferable for the urban agent. The accumulation function of human capital is
where $h_t^j$ is the stock of human capital of the corresponding parent. Also, we assume $\beta, \gamma, \delta \in (0, 1)$, implying all factors exhibit diminishing returns. Under private education regime, the quality of schooling is determined endogenously by each agent through maximization problem. Each individual allocates his income between his own consumption and quality of education for his descendent.

To avoid ambiguity, in public education regime, we use $H_t$ as human capital, $E_t$ as the quality of schooling. And at time $t+1$, the earnings of individual in two areas are taxed at the rate $\tau_{t+1}^j$ respectively, which highlights the policy difference in different regions. The tax rate is determined by the representative agent endogenously. Average quality of public schooling at time $t+1$ is

$$E_t = \alpha \tau_{t+1}^1 H_{t+1}^1 + (1-\alpha) \tau_{t+1}^2 H_{t+1}^2$$

3. Equilibrium

3.1 Equilibrium under public education

Under the public education regime, the individual’s optimization problem can be solved in two steps. First, we solve optimal leisure $l_t^j$, consumption $c_{t+1}^j$ and the expenditure for schooling $e_{t+1}^j$ (j = 1, 2) by maximizing

$$\lambda_j \ln l_t^j + \ln c_{t+1}^j + \ln E_{t+1}$$ (1)

Subject to

$$c_{t+1}^j = (1-\tau_{t+1}^j) H_{t+1}^j$$ (2)

$$H_{t+1}^j = \theta (1-l_t^j)^{\beta} E_t^j (H_t^j)^{\delta}$$ (3)

Given the $E_t, H_t, E_{t+1}, \tau_{t+1}^j$

Next, agents preferred tax rate $\tau_{t+1}^j$ should be derived from the maximization problem

$$\ln (1-\tau_{t+1}^j) H_{t+1}^j + \ln E_{t+1}$$ (4)

where

$$E_{t+1} = \alpha \tau_{t+1}^1 H_{t+1}^1 + (1-\alpha) \tau_{t+1}^2 H_{t+1}^2$$ (5)

**Definition 1:** The equilibrium for the public education regime is a set of sequence $\{l_t^j\}_{t=0}^\infty$, $\{H_t^j\}_{t=0}^\infty$, $\{c_t^j\}_{t=0}^\infty$, $\{E_t\}_{t=0}^\infty$ and $\{\tau_{t+1}^j\}_{t=0}^\infty$ such that (i) $l_t^j, c_{t+1}^j$ are the optimal choices of an agent.
born at time t whose parent's human capital is $h_j^t$; (ii) the human capital of each agent is determined by $H_{i,t+1}^j = \theta(l^{j^l} - l^{j^l})^\delta E_i^{\tau_j^l}(H_j^l)^\delta$; (iii) the tax rate $\tau^l_j$ is preferred by the majority of old agents in two areas at time $t$; and (iv) the quality of schools at time $t$ is defined by $E_i = \alpha \tau_j^l H_j^l + (1-\alpha)\tau_j^2 H_j^2$

We derive the optimal human capital investment and consumption by first order conditions. It’s obvious that the time spent to human capital accumulation by representative agents born at time $t$ is

$$1-l^j_t = \frac{\beta}{\beta + \lambda_j}$$

(6)

Since $\lambda_1 < \lambda_2$, we have $l^1_t < l^2_t$, implying that rural agent devotes more time in accumulating human capital. Clearly the leisure time is independent of tax rate, but it depends on the individual type, which generates different results compared to Glomm and Ravikumar model.

The rural agent invest more time in human capital accumulation, since the leisure is less preferable compared with the urban agent. The human capital accumulation function turns out to be

$$H_{i,t+1}^{j^l} = \theta \left( \frac{\beta}{\beta + \lambda_j} \right)^\delta E_i^{\tau_j^l}(H_j^l)^\delta$$

(7)

The equation (7) describes the evolution of representative agent’s human capital in two areas. The preferred tax rates in two areas are given by

$$\tau_j^1 = \frac{2\alpha H_1^l - (1-\alpha)H_2^2}{3\alpha H_1^l}$$

(8)

$$\tau_j^2 = \frac{2(1-\alpha)H_2^2 - \alpha H_1^l}{3(1-\alpha)H_2^2}$$

(9)

We notice that when $(1-\alpha)H_2^2 > 2\alpha H_1^l$, $\tau_j^1 < 0$, the tax rate of rural agent in time $t$ is negative, i.e. the urban residents subsidize the rural area by funding public education. And we can calculate the quality of schooling in public education regime,

$$E_i = \frac{\alpha H_1^l + (1-\alpha)H_2^2}{3}$$

(10)

It is a function of average income uncorrelated with tax rate. And substitute (10) into (7), the human capital levels of agents in two areas are interdependent, and the equilibrium path of human capital accumulation is uniquely determined by
Note that in public education the learning technology exhibits spillovers: the agent’s future human capital depends not only his own stock today but also the average human capital stock.

3.2 Equilibrium under Private Education

In the private education system, optimal leisure $l^j_t$, consumption $c^j_{t+1}$ and the quality of education $e^j_{t+1}$ ($j = 1, 2$) are determined by maximizing

$$\lambda_j \ln l^j_t + \ln c^j_{t+1} + \ln e^j_{t+1}$$

Subject to

$$c^j_{t+1} = h^j_{t+1} - e^j_{t+1}$$

$$h^j_{t+1} = \theta(1-l^j_t)^\beta (e^j_t)^\gamma (h^j_t)^\delta$$

Given $e^j_t$ and $h^j_t$.

**Definition 2:** The equilibrium for the public education regime is a set of sequence \{l^j_t\}_{t=0}^{\infty}, \{h^j_t\}_{t=0}^{\infty}, \{c^j_t\}_{t=0}^{\infty}, \{e^j_t\}_{t=0}^{\infty}$ such that (i) $l^j_t, c^j_{t+1}, e^j_{t+1}$ are the optimal choices of an agent born at time $t$ whose parent’s human capital is $h^j_t$; (ii) the human capital of each agent is determined by $h^j_{t+1} = \theta(1-l^j_t)^\beta (e^j_t)^\gamma (h^j_t)^\delta$

Each representative agent born at time $t$ will allocate the future consumption and quality of education to be

$$c^j_t = e^j_t = \frac{1}{2} h^j_t$$

The time devoted to human capital accumulation is then determined as

$$1-l^j_t = \frac{2\beta}{2\beta + \lambda_j}$$

We conclude that $l^1_t < l^2_t$ which is identical with public education system. Besides, comparing (6) and (16), it is clear that in private education regime each agent is more willing to give up leisure to accumulate human capital. Under private education, learning not only impacts the consumption level but also the quality of schooling for offspring. The human capital accumulation function in private education regime is
The equilibrium path of human capital accumulation is also uniquely determined. Note that this result is same with the Glomm and Ravikumar model (1992), because the duality and preference parameter don’t affect some crucial characteristics of private education regime.

4. Comparisons between different regimes

In this section, we compare the two education systems to get more insights in perspectives of income inequality and economic growth. In section 4.1, we propose an inequality measurement to investigate the income convergence in both education regimes, and simulate the evolution of inequality to support proposition 1&2. In section 4.2, we focus on the growth for rural and urban areas in both regimes, and simulate the systems.

4.1 Income Inequality

In Glomm and Ravikumar model (1992), human capital follows lognormal distribution, and the inequality is measured by variance, which was proved to be consistent with Gini Coefficient (which is generally conceived as a standard measure of inequality) by McDonald and Ransom (1979). However, in our setting of representative agents, the distribution is simple. The ratio of relative human capital level is promising to serve as an inequality measurement.

Denote

\[ h_{t+1}^l = \theta \frac{1}{2^n} \left( \frac{2\beta}{2\beta + \lambda_j} \right) (h_t^l)^{r+\delta} \]

Intuitively, \( \xi_t \) reflect the degree of inequality. In the appendix, we prove that \( \xi_t \) is a measurement in conformity with Gini index. With the measurement, it’s also convenient to make the comparison between the education systems.

4.1.1 Public Education regime

Denote \( A_j = \theta \left( \frac{\beta}{\beta + \lambda_j} \right) \), the accumulation function in public education regime can be simplified as

\[ H_{t+1} = A_j E_t (H_t^l)^{\delta} \]

Thus the \( \xi_t \) can be expressed as follows
\[ \xi_i = \frac{H_i^2}{H_i^1} = A \left( \frac{H_i^{2-1}}{H_i^{1-1}} \right)^\delta = \left( \frac{H_i^{2-1}}{H_i^{1-1}} \right)^\delta \]

Since \( \delta \in (0,1) \) and \( \frac{A}{A} = \left( \frac{\beta + \lambda_1}{\beta + \lambda_2} \right)^\delta < 1 \), we have

\[ \xi_i = \frac{H_i^2}{H_i^1} < \frac{H_i^{2-1}}{H_i^{1-1}} = \xi_{i-1} \quad (19) \]

From the above relationship, we can conclude that the income inequality is declining under public education. The result is not surprising since the public education equalizes the quality of schooling over agents. Also, the assumption in preference indicates that rural agent will eventually make up the initial inequality in human capital. It is predictable that the human capital of rural agent will eventually exceed the human capital of urban agent, as

\[ \xi_i \xrightarrow{t \to \infty} \left( \frac{A}{A} \right)^{1-\delta} < 1 \]

The reversal in human capital level is fascinating and triggers us to introduce a mechanism of rural-urban migration which will be further discussed in dynamic model.

4.1.2 Private education regime

Denote \( B_j = \theta \frac{1}{2^y} \left( \frac{2\beta}{2\beta + \lambda_j} \right)^\delta \) then human capital accumulation is \( h_{i+1}^j = B_j(h_i^j)^{1+\delta} \)

Thus the \( \xi_i \) can be expressed as follows

\[ \xi_i = \frac{h_i^2}{h_i^1} = \frac{B_2}{B_1} \left( \frac{h_i^{2-1}}{h_i^{1-1}} \right)^{1+\delta} = \xi_0 \left( \frac{B_2}{B_1} \right)^{1+\delta} \]

Obviously, \( \frac{B_2}{B_1} = \left( \frac{\lambda_1 + 2\beta}{\lambda_2 + 2\beta} \right)^\delta \) < 1 and the parameter \( \gamma + \delta \) is pivotal in the analysis of inequality here. We discuss the following cases:

(1) \( \gamma + \delta = 1 \), \( \xi_i = \frac{h_i^2}{h_i^1} = \left( \frac{B_2}{B_1} \right)^{\xi_0} \), since \( \frac{B_2}{B_1} < 1 \), the income inequality is declining.

(2) \( \gamma + \delta > 1 \),

\[ \xi_i = \left( \frac{B_2}{B_1} \right)^{1+\delta} \left( \frac{B_2}{B_1} \right)^{-\gamma-\delta} \]
When $\xi_0$ is sufficiently large ($\xi_0 > \frac{1}{(y+\delta)^{-1}}$), $\xi_t \xrightarrow{t \to \infty} \infty$. It means that the income gap is ceaselessly widening.

(3) $\gamma + \delta < 1$, $\xi_t \xrightarrow{t \to \infty} 1$ which means that the income inequality will eventually be closed.

We summarize the above analysis in the following proposition

**Proposition 1** (a) In public education regime, income inequality will decline over time; (b) In private education regime, inequality will decline if $\gamma + \delta \leq 1$; income gap will be enlarged when $\gamma + \delta > 1$, if initial inequality is sufficiently large.

Since quality of schooling and human capital are the two channels through which accumulation takes place, the relationship between $\gamma + \delta$ and 1 is crucial in determining the convergence of income. Actually, the income convergence in public education has been analyzed by many authors such as Tamura (1991). In his model, the learning technology exhibits spillovers, which is consistent with our model when public education is stalled. However, in private education regime, things turn out to be ambiguous. Besides, even if $\gamma + \delta \leq 1$, public education system is more effective in dealing with the income disparity. We summarize this result in proposition 2.

**Proposition 2** If $\gamma + \delta \leq 1$, both public and private education will narrow the income gap between agents in two areas. But in the public education regime, the gap declines faster, and reversal of human capital is even possible in this static setting when rural agent always holds constant preference structure.

4.1.3 Numerical experiments

The statistics from National Bureau of Statistics show that the relative share of rural population is nearly 1 (i.e. $\alpha \approx 0.5$). According to Terry Siculor, Yue Ximing, Björn Gustafsson and Li Shi(2007), the estimated the income gap between urban and rural area is 3 (i.e. $\xi_0 \approx 3$). We take the above data as given and simulate the evolution of income inequality.
The idea of the previous analysis is captured by the above graph. (Note $\gamma + \delta > 1$). In private education regime, the income inequality is widening at an incredible speed. As we can see from the graph, the income ratio surpasses 12 when $t=5$. In public education regime, the income gap will be eliminated eventually and income reversal will arise in the end. We will further analyze it in the section 5.

4.2 Economic Growth and Choice of Education Regime

In this section, we analyze the regional development and overall growth in the two education systems. Before analyzing, we first clarify the notation of overall growth and regional development.

**Definition 3**: $G_1(t), G_2(t)$ are respectively growth rates of rural area and public area in public education regime; $g_1(t), g_2(t)$ are respectively growth rates in private education regime; $G(t)$, $g(t)$ are gross growth rate in public and private education regime.

The growth rate can be calculated as follows ($j=1,2$)

$$G_j(t) = \frac{H_{t+1}^j - H_t^j}{H_t^j} = A_j(H_t^j)^{\delta-1} E_T^{\gamma} - 1 \quad (20)$$

$$g_j(t) = \frac{h_{t+1}^j - h_t^j}{h_t^j} = B_j(h_t^j)^{\delta+\gamma-1} - 1 \quad (21)$$
where $W_t = \alpha H_t^1 + (1-\alpha) H_t^2$, $w_t = \alpha h_t^1 + (1-\alpha) h_t^2$ are the total wealth of society for public education and private education regimes.

Consider two education economies with the same human capital level at time $t$, $H_t^j = h_t^j$ ($j = 1, 2$). The relationships of growth concerning regional development are summarized in the following table. (Proof in Appendix)

<table>
<thead>
<tr>
<th>$\xi_t$</th>
<th>Rural growth</th>
<th>Urban growth</th>
<th>Overall growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3(2\beta + 2\lambda_1)^\beta - 2\alpha \leq \frac{2\beta + \lambda_1}{2(1-\alpha)}$</td>
<td>$G_1(t) \leq g_1(t)$</td>
<td>$G_2(t) &lt; g_2(t)$</td>
<td>$G(t) &lt; g(t)$</td>
</tr>
<tr>
<td>$3(2\beta + 2\lambda_1)^\beta - 2\alpha &gt; \frac{2\beta + \lambda_1}{2(1-\alpha)}$</td>
<td>$G_1(t) &gt; g_1(t)$</td>
<td>$G_2(t) &lt; g_2(t)$</td>
<td>—</td>
</tr>
</tbody>
</table>

Based on the above table, we conclude that private education regime is more effective for urban development since it ensures higher quality of schooling and more effort in human capital accumulation of urban community. For rural area, things turn out to be ambiguous. In public education regime, rural citizens are able to enjoy higher quality of schooling which is inaccessible in private education regime, but less effort will be invested into accumulation. In terms with the growth of the economy, if $\xi_t$ is relatively small, the gross growth rate in private regime is higher; if $\xi_t$ is relatively large, the relationship may reverse.

If the choice of educational system is endogenized, we can also analyze it in a majority voting system. In our model, the share of rural population $\alpha$ and initial human capital gap matter a lot. We summarize the conclusion in the following proposition.

**Proposition 3** Consider two education economies with the same human capital level at time $t$, $H_t^j = h_t^j$ ($j = 1, 2$). If $\alpha < 0.5$, the private education system will dominate; if $\alpha > 0.5$, the educational choice for time $t+1$ depends on the inequality in time $t$: (a) if $\xi_t$
is relatively small, private education will be implemented; (b) if $\xi_t$ is relatively large, majority of agents will choose public education.

The following two graphs depict the result of simulation for economic growth.

(Figure 4.2) Regional economic growth

(Figure 4.3) Gross economic growth

In Figure 4.2, it is clear that the private education consistently leads to higher growth in urban community. However, for rural development, which education system is better depends on the degree of inequality. The choice of educational system can be analyzed (Note $\alpha > 0.5$ here). In a majority voting system, private education will be chosen except for the first two periods. Figure 4.3 shows that the economy consistently grows faster under private education.
compared with public education. The private education tends to be predominant in the long term.

5. Dynamic Model

In the above section, we have demonstrated that in public education regime, the income of rural agent may ultimately surpass the urban agent. As discrepancy of initial human capital is declining, we postulate that their preference will also tend to be the same. Thus it provides a way to introduce the urbanization mechanism as follows: if the income of rural resident reaches the average income level of urban residents, his child may share the same utility function with the urban residents. In other words, his child behaves like an urban resident.

The above urbanization mechanism is quite appealing. When a typical rural resident accumulate enough human capital through the endeavor of not only himself but also his ancestors, it is reasonable to assume that leisure is more preferable for him at this stage of life. However, he is too old to remake the choice. It is natural to assume his preference will be inherited by his child, and his child becomes an urban resident. This mechanism perfectly captures what is going on in China. Myriads of old generations born in the rural areas fought long and hard in their youth. Their hard work finally pays off and their children can enjoy decent life in big city.

To better illustrate the evolution of urbanization process, income inequality and economic growth, we simulate the dynamics of the above system. In private education system, the economy undergoes a polarization of the two areas, implying that no migration will take place. Consequently, we concentrate on urbanization in public education, mainly concerning three aspects: speed of urbanization, evolution of inequality and growth. We assume the initial distribution of human capital is lognormal and the urban residents are endowed with higher mean value. \( \alpha(t) \) denotes the share of rural population at time \( t \).

The results show that overall inequality is diminishing along with the urbanization process and the rural community will disappear in the end. Besides, the urbanization process is accelerating over time. Moreover, in comparison with static model, growth rate is lower when rural-urban migration is allowed. The results are robust to the specification of parameters in the model.
5.1 Urbanization Rate
To investigate the influence of preference parameter on urbanization, we simulate the process with various sets of $\lambda$. Generally, the simulation results are robust to the setting of key parameters. We list several results of numerical experiments as follows:

<table>
<thead>
<tr>
<th>$\alpha(t)$</th>
<th>$t=1$</th>
<th>$t=2$</th>
<th>$t=3$</th>
<th>$t=4$</th>
<th>$t=5$</th>
<th>$t=6$</th>
<th>$t=7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1=0.8, \lambda_2=1$</td>
<td>0.5000</td>
<td>0.4520</td>
<td>0.3750</td>
<td>0.2630</td>
<td>0.1120</td>
<td>0.0110</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\lambda_1=0.6, \lambda_2=1$</td>
<td>0.5000</td>
<td>0.4210</td>
<td>0.2240</td>
<td>0.0270</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\lambda_1=0.4, \lambda_2=1$</td>
<td>0.5000</td>
<td>0.4120</td>
<td>0.1500</td>
<td>0.0020</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\alpha(t)$</th>
<th>$t=1$</th>
<th>$t=2$</th>
<th>$t=3$</th>
<th>$t=4$</th>
<th>$t=5$</th>
<th>$t=6$</th>
<th>$t=7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1=0.6, \lambda_2=1$</td>
<td>0.5000</td>
<td>0.4210</td>
<td>0.2240</td>
<td>0.0270</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\lambda_1=0.6, \lambda_2=0.8$</td>
<td>0.5000</td>
<td>0.4450</td>
<td>0.3370</td>
<td>0.1560</td>
<td>0.0100</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\lambda_1=0.6, \lambda_2=0.7$</td>
<td>0.5000</td>
<td>0.4580</td>
<td>0.3680</td>
<td>0.2390</td>
<td>0.0820</td>
<td>0.0070</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Parameter $\lambda$ reflects not only the residents’ preference over leisure but also their effort in learning. Higher value of $\lambda$ will lead to lower effort level that individual is willing to implement, thereby influencing the speed of urbanization. As we can see from the above two tables, the small $\lambda_1$ or large $\lambda_2$ will expedite the urbanization process, and vice versa.

Parameters $\theta, \gamma$ are key parameters that control the economic growth, and if they are raised, the urbanization will complete in a relative longer term. Larger $\theta, \gamma$ will magnify the initial human capital gap, thereby slowing the process of urbanization. From simulation results, we also find that the urbanization process appears to be accelerating.

5.2 Income inequality
To analyze the impact on income inequality in the dynamic model, we depict the evolution of inequality in the following two graphs.
The above graphs confirm the equilibrating power of public schooling system on human capital over residents in the process of urbanization. In the Figure 5.1, the distribution is “shrinking” towards its mean value. The Figure 5.2 illustrates the diminishing trend of variance of human capital in both rural and urban areas. Together with Figure 5.1, it implies the decline of overall income inequality. The public education system ensures that the relative poor individuals will be subsidized from those wealthier, and provides residents in both of the areas the same quality of education. Besides, the migration from rural to urban also contributes to this trend by increasing the number of middle income residents in the urban community and decreasing number of high income residents in rural community.
5.3 Economic growth

The growth of economy also warrants our examination. The graph below compares the evolution of growth rates between the static and the dynamic model.

![Graph comparing growth rates between static and dynamic models.](image)

(Figure 5.3) Growth in static and dynamic model

The gross growth rate of economy is higher in the static model. Urbanization actually inhibits migrated residents' desire to work harder. In the absence of enough hardworking individuals, there is no doubt that the growth of economy will slow down. The implication of result is that hardworking labor force is crucial for the prosperity of the economy. The booming growth in China during the last 30 years can be partly attributed to the enduring effort of myriads hardworking individuals.

6. Conclusions

Based on Glomm and Ravikumar (1992), we have presented a model to analyze the implication of two education regimes on income inequality, growth and dynamics of urbanization with special concentration on duality feature in China. We find the following results: (1) In the static model, income inequality will decline over time in public education regime. The income gap can also be eliminated in private education regime when education level and parent's bequests together exhibit decreasing rate of returns. (2) The private education regime is conducive to the development of urban area, while the public education system benefits rural area more. Meanwhile, private education yields higher overall growth unless inequality is sufficiently high. (3) In the dynamic model, the speed of urbanization seems to be increasing along the process and the rural community will eventually disappear.
Along with urbanization process, the income inequality is decreasing quickly and growth of economy is slowing down. From our analyses, an important policy prescription for Chinese government is that expanding public investment in education for rural areas can be a viable strategy for alleviating the income inequality, boosting the development of rural areas and impelling rural–urban migration.
Appendix

1) $\xi_i$ is a reasonable measurement of income inequality which is consistent with Gini coefficient.

Proof

Initially, $\xi_i(0) = \frac{h_{i0}}{h_{i0}} > 1$

In a static setting, the Gini Coefficient can be calculated as below

$$G_t = 1 - \frac{\alpha^2 h_1^1 + (1-\alpha)(2\alpha h_1^1 + (1-\alpha)h_1^2)}{\alpha h_1^1 + (1-\alpha)h_1^2}$$

$$= 1 - \frac{(2\alpha - \alpha^2) + (1-\alpha)^2 \xi_i}{\alpha + (1-\alpha)\xi_i} = \alpha - \frac{\alpha}{\alpha + (1-\alpha)\xi_i}$$

$G_t$ is the function of $\xi_i$ and $G_t'(\xi_i) > 0$

The result is satisfying, so $\xi_i$ can be utilized to measure income inequality between rural and urban area.

2) Proof for Proposition 3

For notational ease, here we denote the relative inequality under private education is $\zeta_i = \frac{h_2^i}{h_1^i}$.

Consider two education economies with the same human capital level at time $t$, that is $H^j_t = h^j_t, (j = 1, 2)$.

$$G(t) = \frac{G_1(t)}{(1-\alpha)\xi_i} + 1 + \frac{\alpha}{\alpha + (1-\alpha)\xi_i}$$

(A1)
\[ g(t) = \frac{g_1(t)}{(1-\alpha)\xi_t/y + 1} + \frac{g_2(t)}{\alpha/(1-\alpha)\xi_t + 1} \quad (A2) \]

\[ G_j(t) = A_j(h_j^r)^{\delta_j-1} - 1 = A_j(h_j^r)^{\delta_j-1}\left(\frac{\alpha h_j^r + (1-\alpha)h^2}{3}\right)^\gamma - 1 \quad (A3) \]

\[ g_j(t) = B_j (h_j^r)^{\delta_j+\gamma-1} - 1 \quad (A4) \]

(1) For urban agent, we prove that \( G_2(t) < g_2(t) \) as follows:

\[ G_2(t) < g_2(t) \text{ is equivalent to } B_2 (h_2^r)^\gamma > A_2 E_i^\gamma \]

Since \( h_2^2 > h_1^2 > \frac{2\alpha}{1+2\alpha} h_1^2 \), then we have \( 2E_i = \frac{2[\alpha h_1^l + (1-\alpha)h_2^2]}{3} < h_2^2 \), and hence

\[ \left(\frac{h_2^2}{E_i}\right)^\gamma \frac{B_2}{A_2} = \left(\frac{h_1^l}{E_i}\right)^\gamma \left(\frac{2\beta + 2\lambda_1}{2\beta + \lambda_2}\right)^\delta > 1 \quad (A5) \]

Therefore we can conclude that \( G_2(t) < g_2(t) \)

(2) For rural agent,

if \( 2E_i < h_2^2 \), \((\xi_i = \frac{h_2^2}{h_1^l} < \frac{3-2\alpha}{2-2\alpha})\). Similar to the derivation as \((A5)\), we have \( G_i(t) < g_i(t) \)

If \( \xi_i \geq \frac{3-2\alpha}{2-2\alpha} \), the comparison between \( G_i(t) \) and \( g_i(t) \) is also related to parameter \( \beta \) and \( \lambda_i \)

\[ G_i(t) > g_i(t) \iff \left(\frac{h_i^l}{E_i}\right)^\gamma \frac{B_i}{A_i} = \left(\frac{h_1^l}{E_i}\right)^\gamma \left(\frac{2\beta + 2\lambda_i}{2\beta + \lambda_i}\right)^\delta < 1 \iff \xi_i > \frac{3(2\beta + 2\lambda_i)^\delta}{2\beta + \lambda_i} \]

Obviously, if \( \xi_i \) is sufficiently large, \( G_i(t) > g_i(t) \) can be satisfied.

(3) The final step is to determine the relationship between gross growth rate \( g(t) \), \( G(t) \).

\[ \frac{3(2\beta + 2\lambda_i)^\delta}{2\beta + \lambda_i} - 2\alpha \]

When \( \xi_i < \frac{3(2\beta + 2\lambda_i)^\delta}{2\beta + \lambda_i} \), we have proved \( G_i(t) \leq g_i(t) \) and \( G_2(t) < g_2(t) \), and hence it is clear that \( G(t) < g(t) \). However, if \( \xi_i \) is sufficiently large, the relationship may be reverse since \( G_i(t) > g_i(t) \). Q.E.D.
Reference


Belton Fleisher, Haizheng Li, Min Qiang Zhao, “Human capital, economic growth, and regional inequality in China,” Journal of Development Economics


