On Credit Risk Management Models: CreditMetrics vs. KMV

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Abstract

This paper compares and improves the two primary default models—CreditMetrics and KMV models. CreditMetrics characterizes the past changes in credit quality through a credit transition matrix, and hence generates forecasts of the credit asset portfolio distribution. This approach focuses on a direct analysis of the relationship between credit statuses of inter-enterprise. On the contrary, KMV model focuses on an indirect interpretation of "Expected Default Frequency" (EDF) that promptly reflects the market expectations and changes in credit status. It estimates the probability of default using the information of a firm's assets as well as the volatility of the market value of these assets. Furthermore, I specify the model selections under different settings.

Keywords: Credit Risk Management; CreditMetrics model; KMV model; Credit Transition Matrix (Markov); Expected Default Frequency (EDF)

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1 Introduction

1.1 Background

Under the trend of financial globalization, financial risk has become the key challenge of risk management, especially with the continuous development of financial industry and innovation of financial product. As credit exposures have multiplied, the need for more sophisticated risk management techniques for credit risk has also increased. However, there are three main difficulties of modern credit risk management (CRM).

(1) Difficulty to quantify and measure

The difficulties of credit risk quantitative analysis and modeling mainly are due to the lack of data and vulnerable validity test of the model, which is caused by information asymmetry, long holding period, infrequent defaults, etc. Propelled by market risk quantification models and development of the credit derivatives, the emergence of techniques, such as CreditMetrics KMV, CreditRisk+, etc. makes CRM more precise and more scientific.

(2) "Credit paradox"

Risk management theory requires that banks should follow the principle of investment decentralization and diversification. However, the main reason for this credit paradox lies in the following aspects: (i) for most of the credit rating of SMEs, the banks obtain the information of credit status mainly from SMEs' long-term business relationship with them, which makes the bank prefer to concentrate their loans on a limited number of existing clients; (ii) in their marketing strategy, some banks only focus loan objects in a certain field or industry where they are good at; (iii) the miniaturization of business caused by diversification is disadvantageous for banks to gain economies of scale; (iv) investment opportunities in the market will sometimes force banks to invest in a limited number of sectors or regions.

(3) Pricing difficulty

Credit risk belongs to non-systemic risk. Theoretically, it could be avoided by a diversified investment completely. CAPM and APT models are only fit for systemic risks, such as interest rate risk, exchange rate risk, inflation risk, etc. In fact, accurate measurement of risk is a prerequisite to price for any risk.

The development of credit derivatives is still in its elementary stage, and the pure credit risk transactions are uncommon. The market cannot provide a comprehensive and reliable basis for credit risk pricing. Although it is instructive to compare the yield to maturity of other financial instruments, such as government bonds, corporate bonds, etc., the approach is limited to some major categories of credit risk, which can hardly be nailed down to a specific credit instrument.

CRM methods are gradually developed from qualitative to quantitative. Effective basis and means of modern CR model provide credit risk prevention, using statistical analysis of historical data and quantitative evaluation of the group or individual credit level.

1.2 Literature Review

Traditional credit risk management methods include internal rating classification model, Altman-Z score model, etc. Methods, such as 5C "expert judgment," are flawed by its subjective estimation; Logistic Regression model, Altman-Z score model, etc. rely too much on the financial indicators, which are historical data and hence unreliable to predict the future situation.

Since the 1990s, the most striking model for measuring credit risk is JP Morgan's CreditMetrics CRM system. The approach is based on credit rating, calculating the probability of default (PD), and then deriving the probability of credit migration (moving from one credit quality to another). [1] It models the full forward distribution of the values of any bond or loan portfolio. It assumes that the changes in values are only related to credit migration, while interest rates are determined (i.e., no market risk). The model covers almost all the credit products, including traditional commercial loans, commitment, fixed income securities, commercial contracts, swaps contracts futures contracts and other derivative products.

KMV differs from CreditMetrics as it relies upon the "Expected Default Frequency" (EDF) for each issuer, rather than upon the average historical transition probability rated for each credit category. The focus in the KMV model is on the relationship between the characteristics of the company's equity and its asset. [7] In actual practice, KMV uses an empirically based "distance-to-default"(DD) measure to produce a PD for each firm at any given point in time. To calculate the probability, the model subtracts the face value of the firm's debt from an estimate of the market value of the firm and then divides this difference by an estimate of the volatility of the firm (scaled to reflect the horizon of the forecast). The resulting DD is then substituted into a CDF (cumulative density function) to calculate the probability that the value of the firm will be less than the face value of debt at the forecasting horizon. [8]

This paper is organized as follows. In Section 2 and Section 3, the default models—CreditMetrics and KMV are analyzed. This part presents their frameworks including basic assumptions, and shows how to evaluate credit risk by the models. Through a credit migration (or transition) matrix, CreditMetrics characterizes past changes in credit quality, and hence generates forecasts of the credit asset portfolio distribution. While for KMV model, crucial inputs into the estimation of the probability of default are firm's assets as well as the volatility of the market value of the assets. Some general results are concluded in Section 4, where we make comparisons between the two models, and discuss how the models may be extended to get new dependence structures between defaults. Applications and some future works are pointed out in section 5.

2 CreditMetrics

2.1 Model Overview

Most of the previous work focused on estimating the likelihoods of default for individual firms (Moody's and S&P have long done this, and many others have started to do so). While CreditMetrics accepts any assessment of PD as an input so that firms could be classified into discrete groups (such as rating categories). [4] To fully assess credit risk (volatility) within a portfolio, the volatility is estimated according to changes in credit quality, not just the expected loss (EL). CreditMetrics constructs a distribution of historically estimated credit outcomes (rating migrations including potential default).

The key assumptions in CreditMetrics are:

1. Firms within the same rating class are assumed to have the same default rate and the same transition probabilities.

2. The actual default rates (migration probabilities) are set equal to the historical default rate (migration frequencies).

3. The default is defined in a statistical sense (non-firm specific) without explicit reference to the process which leads to default.

2.2 Rating process and Parameter setting

In this section, we focus on the process to specify a rating system: rating categories, combined with the probabilities of migrating from one credit quality to another over the credit risk horizon.

Referring to Fei Fei et al. [5], a credit rating is a financial indicator of an obligor's level of creditworthiness. Let the credit rating of a firm at time t be denoted R(t), $R(t) \in S = \{0,1,2,...,N\}$, where S is the rating space with state 1 and N representing, respectively, the best and worst credit quality; state 0 represents default, which occurs if the value of a company's assets at T is below the value of its liabilities at time T. For instance, the S&P's rating system (AAA, AA, A, BBB, BB, B, CCC) together with the default state implying N = 7. The rating definitions provided by the agencies are qualitative, which makes their mapping onto specific quantitative risk measures crucial. [8] The rating process counts on several credit factors.

Referring to Frey et al. [3], suppose $X_i(t), d_1^i(t)$ respectively stand for the values of assets and liabilities for an obligor i at time t. Let $X(t) = (X_1(t), \ldots, X_m(t))$ be an m-dimensional random vector, $i \in \{1, \ldots, \mathrm{m}\}$.Let $-\infty = d_0^i(t) < d_1^i(t) < \ldots < d_N^i(t) = +\infty$ be a sequence of cut-off levels.

$$S_i(t) = j \Leftrightarrow d_{j-1}^i(t) < X_i(t) \le d_j^i(t),\ j \in \{0,1,\ldots,N\} \qquad (2.1)$$

Denote the marginal distribution functions (MDF) of X by $F_i(x) = P(X_1 \le x)$. Then the default probability of company i is given by

$$PD_i = F_i(d_1^i)$$

X is assumed to have a multivariate normal distribution and X_i is interpreted as a change in asset value for obligor i over the time horizon of interest. d_1^i is chosen so that PD_i is the same as the historically observed default rate for companies of a similar credit quality. The components of X can be written as

$$X_i = \sum_{j=1}^{a} \beta_{i,j} \lambda_j + \sigma_i \varepsilon_i + \mu_i \qquad (2.2)$$

for $a < m$, vector $= (\lambda_1, \ldots \lambda_a) \quad N(0, \Lambda)$, and $\varepsilon_1, \ldots, \varepsilon_m$ are independent standard normally distributed random variables , which are also independent of Λ.

After we define the rating rule, the first step in the CreditMetrics methodology establishes the likelihood of migrations between any possible credit quality states during the risk horizon for each obligor. [10] Here, we estimate the credit migration (or transition) probabilities over [t, $t + \Delta t$]. Denote

$$M(\Delta t) \equiv M(t, t+\Delta t) = \begin{pmatrix} m_{11}(\Delta t) & m_{12}(\Delta t) & \ldots & m_{1N}(\Delta t) \\ m_{21}(\Delta t) & \ldots & \ldots & m_{2N}(\Delta t) \\ \ldots & & & \\ m_{N1}(\Delta t) & m_{N1}(\Delta t) & \ldots & m_{NN}(\Delta t) \end{pmatrix} \qquad (2.3)$$

where $m_{ij}(\Delta t) \equiv P(R(t+\Delta t) = j \mid R(t) = i) \geq 0, \forall i, j \in S$. The probability of credit rating migrations in one year for default measures the credit quality migration likelihoods, State N is treated as an "absorbing" state, $m_{NN}(\Delta t)$=1, $m_{Ni}(\Delta t) = 0$. Usually N stands for "default (D)," (or in some cases, "not rated (NR)") implying that R(t) will settle to the default steady-state as $t \to \infty$. Here, two assumptions are noted:

(1) Markovian behavior:

$$P(R(t+\Delta t) = j \mid R(t), R(t-1), R(t-2)...) = P(R(t+\Delta t) = j \mid R(t)), \forall j \in S \qquad (2.4)$$

(2) Time-homogeneity:

$$M(t, t+\Delta t) = M(t-k, t-k+\Delta t), \forall k \qquad (2.5)$$

The migration matrix, which characterizes past changes in credit quality of these firms, is then all that is needed to generate forecasts of the credit asset portfolio distribution in the future. Moreover, in the continuous time-homogeneous Markov framework, the objective is to estimate a generator matrix which is used to compute the credit transition matrix, allowing for forecasts over any time horizon. [6]

To present a case study, we use S&P's rating categories, the transition matrix for one-year average transition rate is:

Table 2.1 Example of migration matrix

Global Corporate Average Transition Rates, One-Year(1981-2010) (%),

Initial rating	Year-end rating (%)								
	AAA	AA	A	BBB	BB	B	CCC	D	NR
AAA	87.91	8.08	0.54	0.05	0.08	0.03	0.05	0.00	3.25
AA	0.57	86.48	8.17	0.53	0.06	0.08	0.02	0.02	4.06
A	0.04	1.90	87.29	5.37	0.38	0.17	0.02	0.08	4.75
BBB	0.01	0.13	3.70	84.55	3.98	0.66	0.15	0.25	6.56
BB	0.02	0.04	0.17	5.22	75.75	7.30	0.76	0.95	9.79
B	0.00	0.04	0.14	0.23	5.48	73.23	4.47	4.70	11.71
CCC	0.00	0.00	0.19	0.28	0.83	13.00	43.82	27.39	14.48

Source: Table 33

http://www.standardandpoors.com/ratings/articles/en/us/?articleType=HTML&assetID=1245302234237

The stylized fact that these matrices tend to be diagonally dominant means that most of the time there is no migration at all. [6] Generation of transition matrix can be found in Antonov, Anatoliy, and Yanka Yanakieva [18], in which the matrix could be adjusted according to the credit year quality and the systematic component or using an aggregation schema.

The second step is to specify the risk horizon. It is usually one year, although multiple horizons could be chosen, like 1±10 years, when one is concerned by the risk profile over a longer period as it is needed for long-dated illiquid instruments. [15]

Next, we specify the forward pricing model, which includes the forward discount curve at the risk horizon(s) for each credit category, and, in the case of default, the value of the instrument which is usually set at a percentage, named the "recovery rate", of face value or "par".

In the final step of rating, we derive the forward distribution of the changes in portfolio value. All the information above can be translated into the forward distribution of the changes in portfolio value consecutive to credit migration.

In the next phase, CreditMetrics estimates the correlations between the equity returns of obligors. By a Monte Carlo simulation, it generates the full distribution of the portfolio values at the credit horizon of one year. We derive the thresholds asset return for each rating category and estimation of the correlation between each pair of obligors. The joint default and migration correlations are driven by the correlations of the asset values of the obligors. Since the asset values are not observable, equity correlations of traded firms are used as a proxy for the asset correlations. [10]

After that, we infer the correlations between changes in credit quality directly from the joint distribution of equity returns.[15] Given the spread curves which apply for each rating, the portfolio is revalued, and further, the percentiles of the distribution of the future values of the portfolio are derived.

Michel Crouhy et al. [15] also introduced how to derive the capital charge related to credit risk:

FV (forward value) = V(1 + PR), where PR is the promised return

EV (expected value) = V(1 + ER), where ER is the expected return

EL (expected loss) = FV − EV

Capital = EV–V(p), where V(p) is the value of the portfolio in the worst case scenario at the confidence level of p%.

The empirical examples can be found in Gupton [6], Crouhy et al. [15]. Results show that for high-grade investment bonds, the spreads tend to increase with time to maturity, while for low grade, it tends to be wider at the short end of the curve than at the long end.

2.3 Strengths and Weaknesses

Strengths:

(1) In aggregating volatilities across the portfolio, CreditMetrics applies estimates of correlation. Thus, although the relevant time horizon is usually longer for credit risk, the method computes credit risk on a comparable basis with market risk. [4]

(2) It adapts to a wider range, including not only traditional commercial loans but also modern financial derivatives. The distribution of the portfolio value is calculated using the normal distribution and Monte Carlo simulation method, which avoids rigid assumptions of the normality of return on assets. [13]

(3) Provide measure a single asset for the assets in the portfolio credit VAR quantitative analysis. The scientific method of CreditMetrics can be applied to compare different industries, which is integrated with the credit rating.

Weaknesses:

(1) The major weakness of CreditMetrics is the reliance on transition probabilities based on average historical frequencies of defaults and credit migration.[15] The methodology assumes that PD and the risk-free rate remain unchanged. Because the bond's future value (and thus its risk) will have little variation if credit quality is believed to have not

changed, the use of the matrix based on agency rating transitions results in a significant understatement of risk. [8]

(2) The correlations in credit quality changes are not directly observable for all pairs of obligors. The evaluation is based on the joint probability of asset returns. One might also argue that there is little correlation between different firms' rating changes and defaults, claiming that each firm is in many ways unique and its changes in credit quality often are driven by events and circumstances specific to that firm. [4]

(3) The index of credit rating is influenced by external factors, such as industry, economic cycles, and economic conditions, etc. But the rating may be more static, instead of adapting to the dynamic environment.

3 KMV model

3.1 Model Overview

Black and Scholes (1973) proposed that one could view the equity of a company as a call option. This insight provided a coherent framework for the objective measurement of credit risk. [7] Sreedhar T Bharath and Tyler Shumway [8] pointed out that KMV used "distance-to-default"(DD) as the state variable for credit quality, which essentially is computed using the Merton (1974) model for pricing defaultable securities. Given the asset characteristics (i.e., value and volatility) and the default point, KMV model can be used to calculate a simple, robust measure of the company's default risk-the number of standard deviation moves required to bring the company to the default point within a specified time horizon. [7]

3.2 Parameter setting

With the use of the KMV default database, Bharath, Sreedhar, and Tyler Shumway [8] found that the empirical probabilities could be substituted for the theoretical probabilities by measuring the empirical distribution with sufficient accuracy. As it is explained, the differences between individual companies are expected to be reflected in their asset values, volatilities, and capital structures, all of which are accounted for in their DDs.

The model treats the company's equity as the standard call option based on the value of its assets, its liabilities as the exercise price of the call option. It is assumed that the default happens if the value of a company's capital assets is less than its liabilities. The main inputs to estimate credit quality are the value and volatility of a firm’s equity. According to the Black-Scholes option pricing formula,

$$E = VN(d_1) - De^{-rT}N(d_2) = E(V,\sigma,r,D,T) \tag{3.1}$$

where E is the equity value, V is the current market value of a company's assets, D is the company's default point (which depends on the nature and extent of the company's fixed obligations. [7]), and N(.) is the normal CDF(cumulative distribution function).

$$d_1 = \frac{\ln(V/D) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, d_2 = \frac{\ln(V/D) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

where σ is the volatility of asset returns, T is debt maturities, r is risk-free return (alternatively, the expected market return to the assets per unit of time μ_A is used, as it is in Stephen Kealhofer [7]). We aim to determine the unknown variables V and σ. KMV introduces the volatility of equity σ_E, and its relationship with σ:

$$\sigma_E = \frac{VN(d_1)}{E}\sigma = f(V, \sigma, r, D, T) \tag{3.2}$$

We apply Newton iteration method here:

$$\begin{pmatrix} V^{(k+1)} \\ \sigma^{(k+1)} \end{pmatrix} = \begin{pmatrix} V^{(k)} \\ \sigma^{(k)} \end{pmatrix} - \begin{pmatrix} \frac{\partial E}{\partial V} & \frac{\partial E}{\partial \sigma} \\ \frac{\partial f}{\partial V} & \frac{\partial f}{\partial \sigma} \end{pmatrix}^{-1} \begin{pmatrix} E(V^{(k)}, \sigma^{(k)}) - E \\ f(V^{(k)}, \sigma^{(k)}) - \sigma_E \end{pmatrix} \tag{3.3}$$

DD is defined as $DD = \frac{V - D}{V\sigma}$.

If the value of company's assets is normally distributed, the DD reflects the standard deviation of the distance-to-default.

$$V_t = V_0 \exp\{(r - \sigma^2/2)t + \sigma\sqrt{t}Z_t\} \tag{3.4}$$

$$DD = \frac{\ln(V_0/D) + (r - \sigma^2/2)}{\sigma\sqrt{T}} \tag{3.5}$$

where Z_t applies to a normal distribution N(0,1).

Further, we can deduce the Expected Default Frequency as EDF = N (-DD). KMV Company owns a huge historical database about company's default account, and the model could be based on EDF. However, for those lacking of a similar database, DD could be an alternative as it is applied in Zhang et al.[9]. Empirical experiment of the model can be found in references [8,9,13,14,15].

3.3 Strengths and Weaknesses

Strengths:

(1) KMV model is a dynamic model that could reflect the changes in the level of credit risk promptly. Based on the structural model of the modern firm theory and option theory, KMV model possesses foresight of the dynamic, especially when it's difficult to obtain financial and other credit information or to guarantee its authenticity.

(2) KMV utilizes the credible information inherent in the volatility of stock price, the macroeconomic conditions as well as credit risk profile of the enterprise. [9] Stephen Kealhofer [7] applied the power curve and the intra-cohort analyses to demonstrate that KMV model is a more accurate predictor of default than are agency debt ratings (Moody's or S&P ratings), and EDFs contain all the information in ratings.

(3) The base measurement method can not only reflect the characteristics of different enterprise risk levels, but also the degree of difference. This property makes the model more accurate, and more appropriate for the loan pricing.

Weaknesses:

(1) Due to lack of timeliness of financial data of listed company and analysis of the financial situation of the creditor (without considering moral hazard under asymmetric information situation), KMV may not accurately measure the risk.

(2) KMV does not provide enough analysis on the correlation of the changes in corporate credit quality. Chuang Wang, Gang Yan [13] pointed out that it's likely that the company could undergo structural change during the contract period. It only focuses on the analysis of the single company's credit status reflected in the price change.

(3) The model cannot distinguish between the different types of debt, such as repayment priorities, guarantees, contracts, etc. Again, it's shaky to assume the normal distribution of company's assets value.

4 Comparisons and Conclusions

Both the CreditMetrics and KMV model are widely used in the current financial industry as scientific methods to measure credit risk, and thus provide quantitative decentralized investment and credit decisions. Frey, Rüdiger, Alexander J. McNeil et al. [2,3] concluded KMV or CreditMetrics as latent variable models, which essentially descend from the firm-value model of Merton (Merton 1974). Both models work with a Gaussian dependence structure for the latent variable vector X, and default occurs if a random variable X falls below some threshold. And hence dependence between defaults is caused by dependence between the corresponding latent variables.

CreditMetrics focuses on the direct analysis of the relationship between inter-enterprise credit statuses. It is a tool for assessing portfolio risks due to defaults and changes in the obligors' credit quality such as upgrades and downgrades in credit ratings. However, its accuracy relies upon two critical assumptions: firms within the same rating class have the same default rate, and the actual default rate is equal to the historical average default rate. [15] The credit rating as index remains static for quite a long period.

On the contrary, KMV model focuses on the analysis of the company is reflected in the price change information in their credit status, while changes in corporate credit have not given enough analysis. The measurement index of KMV model, EDF, stems from the changes of market stock price, which makes the model in accordance with changes to update the input data. And therefore the model reflects market expectations and changes of credit status promptly. Studies [7, 9] also showed that KMV does a superior job of predicting and measuring default risk when compared with conventional credit measures. In the meantime, KMV model is considered as forward-looking, since the indicator EDF contains the judgment by investors about the future development of the corporate. CreditMetrics is mainly dependent on the credit status change history data backward (backward-looking). By contrast, KMV provides forward-looking analysis, which overcomes the reliance on historical data backward-looking model of mathematical statistics.

The following table concludes some of the differences between the two discussed models.

Table 4.1 Comparison between Creditmetrics and KMV

Models	**CreditMetrics**	**KMV**
Definition of risk	MTM	MTM/DM
Risk-driven factors	volatility of assets value	volatility of PD (discrete)
Correlation between credit events	multivariate gaussian distribution	i.i.d. or relevance to the expected PD
Classification system	external rating system (Moodys or Standard & Poors, etc.)	distance-to-default
Dynamism	static	dynamic
Modeling methodology	backward-looking	forward-looking
Measurement approach	ordinal measurement	base measurement

Note: MTM (market to market)

The CreditMetrics or KMV models can accommodate a wide range of different correlation structures for the variables, which is an advantage in modeling a portfolio where obligors are exposed to several risk factors and where the exposure to different risk factors differs across obligors (such as a portfolio of loans to companies from different industries or countries).[3] However, Frey et al.[2] claimed that it's not enough to describe dependence between defaults only according to asset correlations, which might not fully specify the dependence structure of the latent variables.

Besides, a core assumption of the two models is the multivariate normality of the latent variables, which may lead to inaccuracy. Therefore, it is necessary to improve both models in the sense of dependence structure and distribution limit.

5 Applications and Future Work

Zhang, Chen, and Wang (2007) [9] claim that KMV model fits better to identify the credit risk for SMEs (listed small and medium-sized enterprises) in China. Models like CreditMetrics rely too much on the credit rating system, and the time lag of rating will affect the model performance dramatically. M Crouhy, D Galai, R Mark [15] also pointed out that CreditMetrics have chosen the equity price as a proxy for the asset value of the firm that is not directly observed.

As concluded by Kealhofer, Stephen[7], another class of model is Bernoulli mixture model, such as CreditRisk+, where default events have a conditional independence structure conditional on common economic factors. It is claimed there are some advantages of Bernoulli mixture models: (1) easier to simulate in Monte Carlo risk analyses; (2) more convenient for statistical fitting purposes; (3) understandable regarding the behavior of the distribution of the common economic factors.

Douglas W. Dwyer et al. [14] proposed RiskCalc v3.1 as a powerful default prediction technology available for assessing middle-market credit risk. It combines the RiskCalc v1.0 framework (the leading middle-market modeling approach in industry) with the KMV's DD value.

Overall, there is still space to improve credit risk measurement are still under study. credit rating also plays an increasingly important role. Caused by asymmetric information, moral hazard is one of the important elements of the credit risk concerning market risk. To record and report credit conditions promptly becomes the premise of the investors to guard against credit risk. The analyzed CMR models are directly dependent on the credit rating and its changes.

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