Review of Economic Dynamics 18 (2015) 101–117

Contents lists available at ScienceDirect

Review of Economic Dynamics

www.elsevier.com/locate/red

Anatomy of a credit crunch: From capital to labor markets ☆

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ARTICLE INFO

Article history:
Received 2 March 2014
Received in revised form 31 October 2014
Available online 11 November 2014

JEL classification:
E24
E44
L25

Keywords:
Financial frictions
Unemployment

ABSTRACT

Why are financial crises associated with a sustained rise in unemployment? We develop a tractable model with frictions in both credit and labor markets to study the aggregate and micro-level implications of a credit crunch—i.e., a sudden tightening of collateral constraints. When we simulate a credit crunch calibrated to match the observed decline in the ratio of debt to non-financial assets of the United States business sector following the 2007–2008 crisis, our model generates a sharp decline in output—explained by a drop in aggregate total factor productivity and investment—and a protracted increase in unemployment. We then explore the micro-level impact by tracking the employment dynamics for firms of different sizes and ages. The credit crunch causes a much larger reduction in the net employment growth rate of small, young establishments relative to that of large, old producers, consistent with the recent empirical findings in the literature.

1. Introduction

Financial crises are associated with severe economic contractions and lasting deteriorations in labor market conditions. The experiences of the Great Depression and the 2007–2008 financial crisis are dramatic examples. The evidence in Reinhart and Rogoff (2009) provides a broader picture of such phenomena. Despite the close connection between financial crises and sustained rises in unemployment, few models incorporate both credit market and labor market frictions. The goal of our paper is to build one.

We incorporate financial and labor market frictions into a tractable quantitative framework with heterogeneous agents. This allows us to study the joint dynamics of aggregate productivity, total credit in the economy and unemployment, which show more persistence than the underlying economic shocks owing to the interplay of the frictions in the financial and the labor markets over time. At the same time, we can trace out the differential impact of the shocks and market frictions on producers of different productivity, size and age.

In our model, depending on their wealth and entrepreneurial productivity, individuals choose to be entrepreneurs or (prospective) workers in each period. As workers, everyone is assumed to provide the same labor service. Entrepreneurs rent capital subject to a collateral constraint that limits the amount of capital input as a function of their financial wealth. This is the financial frictions. There is a centralized, competitive labor market where entrepreneurs hire available workers.

☆ We thank the editors and an anonymous referee for helpful comments. We benefited from many constructive suggestions by Gian Luca Clementi and Aubhik Khan. The usual disclaimer applies.
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http://dx.doi.org/10.1016/j.red.2014.11.001
1094-2025/

The arrival of unemployed workers into this centralized hiring market is where the labor market frictions are. This friction is modeled in the form of a simple matching function that dictates how many currently unemployed workers enter the hiring market. While this assumption of centralized hiring market for homogeneous labor may be simplistic, it enables a tractable and transparent analysis of an economy with both financial and labor market frictions.

We use a quantitative version of our theory to analyze the effects of a credit crunch—i.e., a sudden, unanticipated tightening of the collateral constraint on capital input. In the model, a credit crunch leads to a sharp decline in output—explained by a large drop in aggregate total factor productivity (TFP) and a relatively small decline in capital stock—and a sustained increase in unemployment.

The tightening collateral constraint reallocates capital from entrepreneurs with low net worth toward unconstrained entrepreneurs who expand their production in response to lower factor prices. The reallocation of capital is accompanied by the reallocation of complementary labor across entrepreneurs. Essentially, production factors are reallocated away from productive but constrained entrepreneurs toward those who are relatively unproductive but unconstrained. As a result, the aggregate TFP suffers.

At the same time, labor reallocation, especially the scaling down of constrained entrepreneurs' labor input due to deleveraging, entails an excess job destruction. Although unconstrained entrepreneurs expand in response to lower wages, laid-off workers must re-enter the hiring market subject to the matching friction and hence the unemployment rate abates only gradually. As the tightening of the collateral constraint recedes in the subsequent periods, capital and labor are reallocated back toward productive entrepreneurs with low net worth. This process generates a second phase of excess job destruction, which is again gradually mediated by the frictional labor market and further prolongs the higher-than-normal unemployment rates. Indeed, we find that even a very short-lived credit crunch leaves persistent adverse effects on aggregate output, total credit and unemployment.

We also explore the behavior of the economy in response to an aggregate TFP shock. We find that the implications for the dynamics of unemployment with the TFP shock are starkly different from those with the credit shock. With the TFP shock, the unemployment rate does not change at all in our model. The reason is that a decline in the aggregate TFP affects the capital and labor demands of all firms symmetrically, and flexible wages and interest rates fully offset the contractionary effect of the lower TFP on employment. On the other hand, an economy-wide credit shock has differential effects across firms depending on entrepreneurs' productivity and wealth, and such heterogeneous responses of individual firms make it impossible for a low wage—that must be equalized across firms—alone to maintain firm-level or aggregate employment. In a nutshell, it is the reallocative nature of credit shocks that is essential for realistic unemployment dynamics in the model.

In two simple extensions, we also consider how the main mechanisms of the model interact with (i) downward rigidity in wages and (ii) variable capital utilization at the aggregate level.

The remainder of our quantitative analysis explores the implications of the credit crunch for employment dynamics at a more disaggregate level. A large empirical literature has documented that credit shocks affect firms of different sizes differently. The working hypothesis of these studies is that small businesses are more heavily reliant on credit to finance their production and capital expenditures, and are hence more susceptible to recessions caused by negative credit shocks. Gertler and Gilchrist (1994) found evidence supporting this claim and, more recently, Fort et al. (2012) extended the analysis to highlight the role of firms' age as well as size: During a credit crunch, the employment growth rate of small, young firms declines by more relative to that of large, old firms.

Our model predictions are in line with the findings of these empirical studies. We show quantitatively that net employment growth rates fall by more for small, young firms relative to large, old firms. This result reflects the reallocation of labor and capital from constrained to unconstrained entrepreneurs: In the initial stationary equilibrium of the model, we discover that more than 90 percent of small, young firms are financially constrained, while less than 10 percent of old, large firms are. This information is affirmed by the distribution of marginal returns to capital across firms in the model, which shows an excess return of 10 percentage points for the former group of firms relative to the latter, a significant deviation from the equalization of rates of return across firms in the frictionless allocation.

In this context, our paper provides a theoretical underpinning for the working hypothesis of the empirical literature and explains how the aggregate behavior of the economy is shaped by the heterogeneous responses to credit shocks at the firm level.

The rest of the paper is organized as follows. We provide a review of the literature below and describe the model in Section 2. We present the results of our quantitative analysis in Section 3. We first explain how parameter values and time series for the collateral constraint are calibrated (Section 3.1). We then analyze the macroeconomic implications of a credit crunch and compare them to those of a negative TFP shock, followed by the two extensions with wage rigidity and a notion of capital utilization (Section 3.2). We also examine the impact of the credit crunch at the individual firm level and report how it varies across firms of different ages and sizes (Section 3.3). We conclude in Section 4.

Related literature Our paper is related to several strands of the literature. Our modeling of financial frictions closely follows the work of Kiyotaki and Moore (1997), in which credit is limited by a collateral constraint arising from a limited enforceability problem between creditors and debtors. However, we abstract from feedback effects going from asset prices to collateral constraints. Jermann and Quadrini (2009) adopt the same modeling strategy for financial frictions to study the role of credit as a driver of business cycles. The most salient difference between our work and theirs is that we introduce credit shocks in an economy where entrepreneurs are heterogeneous and, hence, the tightness of credit at any given point in time is different across producers. We show that this heterogeneity generates novel implications for how credit shocks affect the behavior of the aggregate economy, over and beyond the rich microeconomic implications that can be compared with the firm-level data.

Producer heterogeneity is also a feature of Khan and Thomas (2013), Buera and Moll (2012), and Shourideh and Zetlin-Jones (2012). Our contribution relative to their work is that we bring unemployment to the front and center of the analysis and study the interaction between credit and labor market frictions. We accomplish such an interaction by incorporating matching frictions and a notion of Walrasian labor markets—as in Veracierto (2009)—into Buera and Shin (2013) model of occupational choices, production, saving, and financial frictions.[1]

Our exploration of the micro-level implications of a credit crunch is motivated by a number of empirical studies in the literature. Gertler and Gilchrist (1994) was the first to document the differential dynamics of small vs. large firms during recession dates associated with monetary contractions. This finding has been validated by Chari et al. (2013), although they also find the reverse to be true during recessions not associated with monetary contractions. Moscarini and Postel-Vinay (2012) study the unconditional cyclical behavior of small and large firms and document that it is the large businesses whose employment declines the most during economic contractions.

More recently, Fort et al. (2012) re-visit the cyclical behavior of firms and highlight the role of age, together with size, in predicting the responsiveness of firms to business cycles. With respect to the small vs. large debate, their results favor the previous evidence that small firms' employment growth rates decline more during recessions than large firms'. However, they emphasize that a joint consideration of age and size yields the clearest pattern of firms' response to business cycles: Net employment growth rates of small, young firms decline by substantially more than those of large, old firms during recessions.

Our paper is related to this empirical literature in two ways. First, we take the data as a test for the firm-level implications of our model that underlie the macro-level dynamics. Second, we use the model to better understand why age and size would explain how different firms react to credit shocks. We provide a theoretical foundation for the conjecture of the empirical literature that small, young firms face the most difficulty gaining access to credit in times of tight overall credit availability.

2. Model

We model an economy populated by a continuum of individuals, who are heterogeneous with respect to their wealth, entrepreneurial productivity, and access to employment opportunity. In each period, an individual with an employment opportunity chooses whether to work for a wage or to operate an individual-specific technology (entrepreneurship). Those without an employment opportunity choose between searching for a job and operating their individual-specific technology.

Access to capital is determined by entrepreneurs' wealth through a simple collateral constraint, motivated by the imperfect enforceability of capital rental contracts. One entrepreneur can operate only one production unit (establishment) in a given period. Entrepreneurial ideas are inalienable, and there is no market for managers or entrepreneurial talent.

We assume that there is a centralized labor market where hiring entrepreneurs compete for available workers. The arrival of unemployed workers to the centralized hiring market is modeled with a simple matching function. We restrict wage contracts to be the same across workers and entrepreneurs. In the benchmark exercise, we assume that workers are paid in each period the wage that clears the current hiring market and entrepreneurs may terminate the employment relationship at any time.

Heterogeneity and demographics Individuals live indefinitely and are heterogeneous in their wealth a, entrepreneurial productivity $z \in \mathcal{Z}$, and employment opportunity. Their wealth is chosen endogenously by forward-looking saving decisions and their entrepreneurial productivity follows a stochastic process. In particular, an individual retains his entrepreneurial productivity from one period to the next with probability ψ. With probability $1-\psi$, he loses the current productivity and has to draw a new entrepreneurial productivity. The new draw is from a time-invariant distribution with a cumulative density $\mu(z)$ and is independent of his previous productivity level.[2]

As for individuals' access to employment opportunities, throughout the paper, we maintain the assumption that unemployed workers receive unemployment benefits that are equal to the market wage in each period, and that leisure does not enter the utility function. As a result, individuals are indifferent between being employed and unemployed. However, the unemployment rate is an important variable for the equilibrium definition and the aggregate dynamics of the model.

The population size of the economy is normalized to one, and there is no population growth.

Preferences Individual preferences are described by the following expected utility function over sequences of consumption, c_t:

[1] Petrosky-Nadeau (2013) also considers credit and labor market frictions in a standard search-theoretic model.

[2] We use this specification rather than the more standard AR(1) specification to help better match the Pareto-like establishment size distribution in the data. Given the persistence built into ψ, however, our results are qualitatively similar to those obtained with an AR(1) process for $\log z$.

$$U(c) = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right], \quad u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$$

where β is the discount factor and σ is the coefficient of relative risk aversion. The expectation is taken over the realizations of the entrepreneurial productivity z.

Technology At the beginning of each period, an individual chooses whether to operate his own business or not. An entrepreneur with talent z produces using capital k and labor l according to:

$$Azf(k,l) = Azk^{\alpha} l^{\theta}, \tag{1}$$

where α and θ are the elasticities of output with respect to capital and labor with $\alpha + \theta < 1$, implying diminishing returns to scale in variable factors at the establishment level. Notice that firm-level productivity consists of an aggregate component A, which acts as a source of business cycles in one of our experiments, and the idiosyncratic entrepreneurial productivity z.

Taxes and unemployment benefits We assume that unemployed workers receive a transfer equal to the period wage, which is financed with a lump-sum tax τ_t on all individuals. Given this assumption, from an individual's point of view, there is no difference between being a wage earner and being an unemployed worker. This allows us to formulate the individual problem (Section 2.1) as if they are in one of two mutually exclusive states: a worker (employed/unemployed) or an entrepreneur.[3]

Financial markets Productive capital is the only asset in the economy. There is a perfectly-competitive financial intermediary that receives deposits and rents out capital to entrepreneurs. The return on deposited assets—i.e. the interest rate in the economy—is r_t. The zero-profit condition of the intermediary implies that the rental price of capital is $r_t + \delta$, where δ is the depreciation rate.

We assume that entrepreneurs' capital rental k is limited by a collateral constraint $k \le \lambda a$, where $a \ge 0$ is individual financial wealth and λ measures the degree of credit frictions, with $\lambda = +\infty$ corresponding to perfect credit markets and $\lambda = 1$ to financial autarky where all capital has to be self-financed by entrepreneurs. The same λ applies to everyone in a given economy.

Our specification captures the common prediction from models of limited contract enforcement: The amount of credit is limited by an individual's wealth. At the same time, its parsimoniousness—the fact that financial frictions are captured by one single parameter, λ—enables us to analyze the quantitative effects of financial frictions on aggregate transitional dynamics without losing tractability.[4]

Labor markets Entrepreneurs hire workers in a centralized and competitive hiring market. We restrict labor contracts that entrepreneurs can offer to have the following properties: (i) all workers must be paid the wage that clears the hiring market in each period and (ii) employers may terminate the employment relationship at any time. In particular, all entrepreneurs, irrespective of their current state, are restricted to offer the same labor contract. A worker whose employment is terminated becomes unemployed and must re-enter the hiring market before finding another job.

We make the labor market frictional by introducing a matching friction that interferes with the (re-)entry of unemployed workers into the centralized hiring market. More specifically, we assume that instead of matching unemployed workers directly with firms, a matching function determines the fraction of the currently unemployed that can enter the centralized hiring market. (All unemployed workers face the same probability of entering the hiring market.) For those in the centralized hiring market, wages adjust to make supply meet demand.[5]

More formally, letting M_t denote the number of unemployed workers that enter the hiring market in period t, our assumptions about the matching function can be written as:

$$M_t = \gamma(U_t + JD_t) \tag{2}$$

where U_t is the number of unemployed workers at the end of the previous period and JD_t stands for the job destruction at the beginning of the current period. To be more specific,

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[3] This assumption thereby lightens the computational burden somewhat. If we were to assume that the unemployment benefit is strictly less than the market wage, holding other things equal, an additional set of unemployed individuals whose productivity is marginal will turn into entrepreneurship because their opportunity cost of entrepreneurship (i.e., forgone unemployment benefits) is now lower.

[4] Our collateral constraint can be derived from the following limited enforcement problem. Consider an individual with financial wealth $a \ge 0$ deposited in the financial intermediary at the beginning of a period. Assume that he rents k units of capital and then he can abscond with fraction $1/\lambda$ of the rented capital. The only punishment is that he will lose his financial wealth a deposited in the intermediary. In particular, he will not be excluded from any economic activity in the future. In fact, he is allowed to instantaneously deposit the stolen capital k/λ and continue on as a worker or an entrepreneur. Note that λ in this context measures the degree of capital rental contract enforcement, with $\lambda = +\infty$ corresponding to perfect enforcement and $\lambda = 1$ to no enforcement. In the equilibrium, the financial intermediary will rent capital only to the extent that no individual will renege on the rental contract, which implies a collateral constraint $k/\lambda \le a$ or $k \le \lambda a$.

[5] Our modeling of the labor market closely follows Alvarez and Veracierto (2001). Our model can also be interpreted as a simplified version of the Walrasian equilibrium theory of establishment dynamics and matching frictions in Veracierto (2009).

$$JD_t = \int \big[\max\{l_{-1} - l_t(a,z), 0\} + \mathbb{I}\{l_{-1} > 0\}\mathbb{I}\{l_t(a,z) = 0\}\big] G_t(da, dl_{-1}, dz)$$

where l_t is labor demand of an individual (positive only for entrepreneurs and a function of one's financial wealth, a, and entrepreneurial productivity, z) and G_t is the joint cumulative distribution function of wealth (a), previous period employment (l_{-1}), and current entrepreneurial productivity (z). With $\mathbb{I}$ denoting the indicator function, the second term in the integral captures exiting entrepreneurs, who enter the pool of unemployed workers. (The employees of exiting entrepreneurs are accounted for by the first term of the integral.)

It is critical for the dynamic stability of the U_t series that a fraction of the laid-off workers and exiting entrepreneurs can enter the hiring market and be employed within the period, as implied by the JD_t term appearing in the right-hand side of Eq. (2).

The evolution of unemployment is governed by the following law of motion:

$$U_{t+1} = U_t + JD_t - M_t - UB_t, \tag{3}$$

where the last term UB_t is the number of new entrepreneurs in period t who were unemployed workers at the end of period $t-1$.

2.1. Individuals' problem

At the beginning of a period, an individual's state is summarized by his financial wealth a and entrepreneurial productivity z. To be precise, the state of an individual also includes his access to an employment opportunity. However, because we assume that unemployed workers receive a transfer equal to the market wage, this information is irrelevant for an individual's problem. The value for him at this stage, $v_t(a,z)$, is the larger of the value of being an employed/unemployed worker, $v_t^W(a,z)$, and the value of being an entrepreneur, $v_t^E(a,z)$:

$$v_t(a,z) = \max\{v_t^W(a,z), v_t^E(a,z)\} \tag{4}$$

As an employed or unemployed worker, an individual chooses consumption c and next period's asset a' to maximize his utility, subject to the period budget constraint.

$$\begin{aligned} v_t^W(a,z) &= \max_{c,a'} u(c) + \beta\mathbb{E}\big[v_{t+1}(a',z')\big] \\ \text{s.t.}\quad & c + a' = w_t + (1+r_t)a - \tau_t \end{aligned} \tag{5}$$

Alternatively, individuals can choose to be entrepreneurs. The value function of being an entrepreneur is as follows.

$$v_t^E(a,z) = \max_{c,k,l,a'} u(c) + \beta\mathbb{E}\big[v_{t+1}(a',z')\big] \tag{6}$$

$$\text{s.t.}\quad c + a' = A_t z k^{\alpha} l^{\theta} - w_t l - (r_t+\delta)k + (1+r_t)a - \tau_t$$

$$k \le \lambda_t a \qquad \text{(Collateral constraint)}$$

The occupation choice of an individual is denoted by $o_t(a,z) \in \{W, E\}$. The labor and capital demands of an entrepreneur are denoted by $l_t(a,z)$ and $k_t(a,z)$, both of which take on the value of zero for employed/unemployed workers.

2.2. Competitive equilibrium

Given an initial distribution of individual wealth, previous period's labor input, and entrepreneurial productivity $G_0(a, l_{-1}, z)$ and a sequence of collateral constraint parameters $\{\lambda_t\}_{t=0}^{\infty}$, a competitive equilibrium comprises prices $\{w_t, r_t\}_{t=0}^{\infty}$, the number of unemployed workers $\{U_t\}_{t=0}^{\infty}$, allocations $\{c_t(a,z), a_{t+1}(a,z), k_t(a,z), l_t(a,z), o_t(a,z)\}_{t=0}^{\infty}$, and lump-sum taxes $\{\tau_t\}_{t=0}^{\infty}$ such that[6]:

1. Given prices $\{w_t, r_t\}_{t=0}^{\infty}$, the allocations are solutions to the individual problems (4), (5), and (6) for all $t \ge 0$;
2. The number of unemployed workers follows the equilibrium law of motion (3);
3. The government budget is balanced for all $t \ge 0$

$$\tau_t = w_t U_{t+1};$$

[6] To be absolutely precise, one needs to define a binary variable j which takes the value of one if the individual is an unemployed worker and zero otherwise. The proper cumulative distribution function is then $G_t(a, l_{-1}, z, j)$. We can then define $G_t(a, l_{-1}, z) \equiv \sum_{j=0}^{1} G_t(a, l_{-1}, z, j)$. This new variable j and the associated c.d.f. $G_t(a, l_{-1}, z, j)$ are necessary for spelling out equation (3) but $G_t(a, l_{-1}, z)$ suffices for the other elements of the equilibrium definition. In our numerical analysis, we do keep track of $G_t(a, l_{-1}, z, j)$.

Table 1
Calibration.

	US data	Model	Parameter
Top 10% employment	0.69	0.69	$\eta = 5.25$
Top 5% earnings share	0.30	0.30	$\alpha + \theta = 0.79$
Establishment exit rate (annual)	0.10	0.10	$\psi = 0.89$
Real interest rate (annual)	0.02	0.02	$\beta = 0.93$
Credit market instruments to non-financial assets	0.70	0.70	$\lambda = 7.5$

4. Capital markets clear for all $t \geq 0$:

$$K_t \equiv \int k_t(a,z) G_t(da, dl_{-1}, dz) = \int a G_t(da, dl_{-1}, dz); \tag{8}$$

5. Labor markets clear for all $t \geq 0$:

$$\int l_t(a,z) G_t(da, dl_{-1}, dz) = 1 - \int \mathbb{I}\{o_t = E\} G_t(da, dl_{-1}, dz) - U_{t+1}, \tag{9}$$

where the left-hand side is the demand for labor and the right-hand side is the number of eligible workers, which is the total population minus the number of entrepreneurs and unemployed workers;

6. The joint distribution of wealth, previous period's labor input, and entrepreneurial productivity $\{G_t(a, l_{-1}, z)\}_{t=0}^{\infty}$ evolves according to the following equilibrium mapping:

$$G_{t+1}(a, l_{-1}, z) = \psi \int\limits_{a_{t+1}(\tilde{a},z) \leq a, l_t(\tilde{a},z) \leq l_{-1}} G_t(d\tilde{a}, d\tilde{l}_{-1}, z) + (1-\psi)\mu(z) \int\limits_{a_{t+1}(\tilde{a},\tilde{z}) \leq a, l_t(\tilde{a},\tilde{z}) \leq l_{-1}} G_t(d\tilde{a}, d\tilde{l}_{-1}, d\tilde{z}).$$

3. Quantitative exploration

We now build a quantitative version of our framework and investigate the interaction between a credit crunch—modeled as an unexpected, one-time tightening of the collateral constraint—and labor market frictions. We will explore both the aggregate and the micro-level implications. To hit the economy with a credit crunch of a plausible magnitude, we calibrate the time-series of the collateral constraint parameter to match the contraction in aggregate business credit observed during the 2007–2008 financial crisis.

3.1. Calibration

Our model is parameterized so that the stationary equilibrium matches relevant aggregate and establishment-level moments in the United States (US) economy. We assume a time period in the model to be one year.

Following the standard practices, we set the coefficient of relative risk aversion σ to 1.5, the annual depreciation rate δ to 0.06, and the ratio $\alpha/(\alpha+\theta)$ to 0.33 (to match the aggregate capital income share). In terms of the parameter for the hiring market matching function, we set $\gamma = 0.667$ so as to obtain an unemployment rate of 5 percent in the stationary equilibrium.

Entrepreneurial productivity is assumed to follow a Pareto distribution, with cumulative density given by $\mu(z) = 1 - z^{-\eta}$ for $z \geq 1$. Each period, an individual retains his z with probability ψ, while a new entrepreneurial productivity should be drawn with the complementary probability $1 - \psi$.

The remaining parameters to be calibrated are $\alpha + \theta$, η, ψ, β and the collateral constraint λ of the initial stationary equilibrium. To do so, we target the following moments in the US data: employment share of the top decile of establishments, the share of earnings generated by the top 5 percent of the population, the annual exit rate of establishments, the real interest rate, and the ratio of external finance to total non-financial assets of the non-financial business sector.

Table 1 shows the moments in the US data and their counterparts in the calibrated model. The decile of the largest establishments (in terms of employment) accounts for 69 percent of total employment in 2000. The earnings share of the top 5 percentiles is 30 percent in 1998. The annual establishment exit rate is 10 percent in the Business Dynamics Statistics from the US Census. We assume that the annual interest rate is 2 percent. Lastly, we target the ratio of credit market instruments to total non-financial fixed assets in the non-financial business sector of 0.7, a level attained one year before the 2008 financial crisis.

Although all parameters are jointly pinned down in the model equilibrium, we can identify which objects in the data are mostly related to which parameters. For instance, the tail parameter of the Pareto distribution of entrepreneurial productivity, holding other values constant, controls the fraction of employment accounted for by the decile of largest establishments. Similarly, $\alpha + \theta$ can be mapped into the earnings share of the top 5 percent of the population, who, as in the data, are mostly entrepreneurs in the model. There is also a direct link from the persistence of the ability process ψ to the probability that an entrepreneur exits from production and hence the annual establishment exit rate in the data. The discount

Fig. 1. Ratio of external finance to capital stock. This figure shows the evolution of the ratio of external finance to capital stock in the data (dotted line) and the model (solid line). We report the percentage deviations from a HP-filtered series for the US data and the percentage deviations from its steady state value for the model. See footnote 7 for a description of the construction of the external finance to capital stock ratio in the data.

factor, unsurprisingly, is closely tied to the target interest rate. The collateral constraint parameter λ is primarily responsible for the ratio of external finance to capital,

$$\frac{\int \max\{k_t(a,z)-a,0\}G_t(da,dl_{-1},dz)}{K_t}, \tag{10}$$

which is the model equivalent of the ratio of credit market instruments to total non-financial assets in the non-financial business sector in the Flow of Funds data.

3.2. Aggregate dynamics of a credit crunch

We simulate the aggregate dynamics of the model following a tightening of the collateral constraint—i.e., a lower λ_t. We choose the λ_t series to generate a decline in the ratio of external finance to capital stock in the model that is comparable in magnitude to the observed decline in the stock of credit market liabilities to non-financial assets of the non-financial business sector in the US from the fourth quarter of 2008 to the first quarter of 2010.[7] We assume that, following its initial plunge, λ_t gradually recovers and eventually converges to its pre-crisis level.[8] (Following our benchmark crunch exercise, we will consider a more transitory λ shock to explore how the financial and labor market frictions make the effects of the shock last longer.) The initial drop in λ_t is a completely unexpected event—up to that point everyone in the economy expects λ_t to be constant over time—but, once this shock hits the economy, its deterministic path afterwards is perfectly known. Aggregate productivity A_t remains constant at the normalized value of 1.

Fig. 1 shows the evolution of the ratio of external finance to capital stock in the data (dotted line) and the model (solid line) since 2000. For the US data (quarterly series) we report the percentage deviations from the trend of the HP-filtered series with smoothing parameter 1600. For the model (annual series) we plot the percentage deviations from the stationary equilibrium value. The vertical line in the middle is the third quarter of 2008. In the model, this is the first period with a lower than normal λ, although the shock to the λ_t series arrives and becomes perfectly known in 2007—see footnote 8. Following the first quarter of 2008, there is a sharp decline in the ratio of credit market liabilities to non-financial assets in the data, in the order of 8 percentage points, with the series bottoming out during 2010. In the model, we generate a contraction of comparable magnitude—a drop in the external finance to capital ratio of about 8 percentage points by 2009—and a smooth recovery to its original steady state. The fact that the two series are parallel from 2008 to 2009 is our measure of the model experiment being similar to the real-world credit crunch.

In the model, the tightening of the collateral constraint leads to a sharp decline in output—explained by a large drop in TFP and a smaller decline in capital stock—and a protracted increase in unemployment. The dynamic of these series in the

[7] The stock of credit market instruments corresponds to line 18 in the L.101 table of the Flow of Funds. It includes the stock of bank loans of the corporate and non-corporate sectors, and the stock of commercial papers, municipal securities and corporate bonds of the corporate sector. The stock of non-financial assets is measured in historical prices to avoid valuation effects, which is absent in our theory. The stock of non-financial assets is given by the sum of the stock of the historical-cost net stock of private fixed assets of the non-financial corporate sector, sole proprietorships, and partnerships from Table 6.3 of the National Income and Product Account (NIPA). To have the correct level as of 2007, we multiply the historical-cost stock of non-financial assets of each year by the ratio of the current-cost stock of non-financial assets in 2007 to the historical-cost stock of non-financial assets in 2007.

[8] The sequence of collateral constraints that we calibrate is, with $t = 1$ corresponding to 2007, $\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\} = \{7.5, 4.5, 3.0, 3.5\}$ and $\lambda_t = 0.75\lambda_{t-1} + 0.25 \times 7.5$ for $t \geq 5$. The value of λ is 7.5 in the initial stationary equilibrium ($t = 0$).

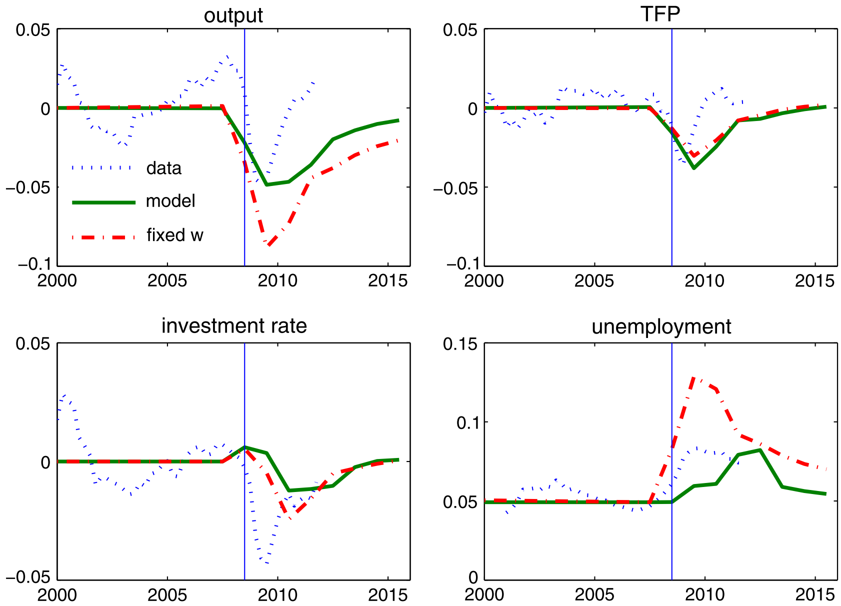

Fig. 2. Aggregate implication of a credit crunch. This figure shows the evolution output, TFP, investment and unemployment rates in the data (dotted line) and the models with flexible (solid line) and fixed wages (dashed line). For the US data we report the percentage deviations from the trend of the HP filtered series, with a smoothing parameter of 1600, for the case of output and TFP, and the simple difference with respect to the value in the second quarter of 2008 for the case of the investment rate. See footnote 9 for a description of the sources of the data.

model (solid lines) and the corresponding series around the 2008 financial crisis (dotted lines) are illustrated in Fig. 2. For output and TFP, we report percentage deviations from the steady state for the model and from the HP-trend for the US data. For investment rate, we show differences from the steady state for the model and from the value in the second quarter of 2008 for the US data. For unemployment rate, we simply plot the raw numbers from the model and the data.[9]

The credit crunch results in a reallocation of capital from entrepreneurs with little collateral, relative to their unconstrained level of capital input, toward unconstrained entrepreneurs. Undercapitalized entrepreneurs have to reduce their capital input as a direct consequence of the tightening collateral constraint (i.e., lower λ_t). As for unconstrained entrepreneurs, they increase their capital input in response to the general equilibrium effect of lower factor prices. This reallocation of credit and capital is accompanied by the reallocation of complementary labor across entrepreneurs in the same direction, subject to the matching frictions in the hiring market. In this process, production factors are on average reallocated from productive but constrained entrepreneurs to those who are unconstrained but relatively unproductive. One important outcome is the decline in aggregate TFP shown in the top right panel of Fig. 2.

The labor reallocation, especially the downsizing of constrained entrepreneurs due to deleveraging, entails an excess of job destruction. While unconstrained entrepreneurs now hire more workers in response to lower wages, laid-off workers must re-enter the hiring market subject to the matching frictions and hence the unemployment rate comes back down only gradually. In the subsequent periods, as the tightening of the collateral constraint is undone, capital and labor are reallocated back to productive entrepreneurs with little collateral. This process generates a second phase of higher-than-normal job destruction, which is again gradually mediated by the frictional hiring market and further prolongs the higher-than-normal unemployment rates.

In the left panel of Fig. 3, we illustrate the reallocation of credit across entrepreneurs with different levels of collateral or financial wealth. We plot the evolution of external finance to capital ratios of entrepreneurs whose wealth is below (solid line) and above (dashed line) the median wealth—among entrepreneurs—in each period. Entrepreneurs with low wealth are more likely to have binding collateral constraints and finance a larger fraction of their capital externally. As a result, they are hit more directly and harder by the credit crunch. Both their capital input and their external finance are sharply reduced during the credit crunch, with the external finance to capital ratio going down. To the contrary, wealthy entrepreneurs tend to have slack collateral constraints. During the credit crunch, they employ more capital and use more external finance than

[9] We use output, TFP and investment series for the US business sector. The output series corresponds to the real gross value-added of the business sector from Table 1.3.6 of the NIPA. The TFP series is constructed using an estimate of the capital stock of businesses and an index of the hours employed by the US business sector in series PRS84006033 of the Bureau of Labor Statistics (BLS). We estimate the capital stock of the US business sector by accumulating the gross domestic investment series for the business sector in Table 5.1 of the NIPA, deflated by the price indexes for private fixed investment in Table 5.3.4. In this process, we use as the initial value the current-cost net stock of private fixed assets for the US businesses in 1960 from Table 6.1 and estimate a depreciation rate using the information in Table 6.4. The unemployment rate is the rate for the total population 16 years old and over in series LNS14000000 of the BLS.

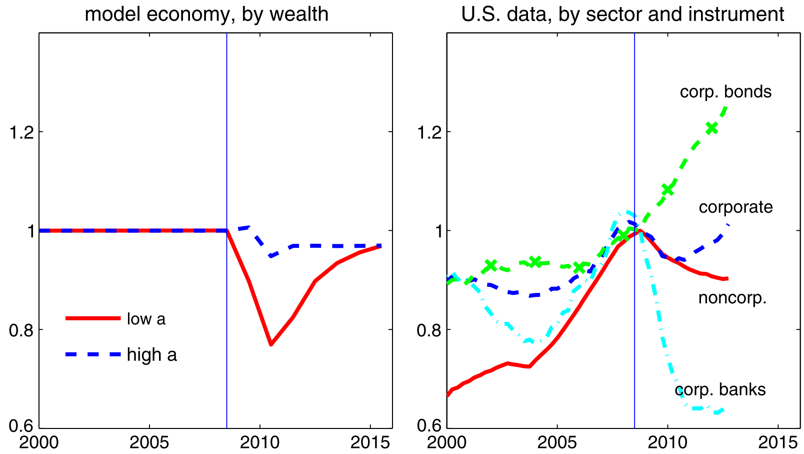

Fig. 3. External finance to capital stock in the model and the US data.

before in response to lower factor prices. Their capital input expands by more than the external finance, leading to a small decline in the external finance to capital ratio.[10]

How does the effect of the credit crunch in our model compare with the data during and after the 2007–2008 financial crisis? Fig. 2 (dotted lines) and the right panel of Fig. 3 show the relevant data. The model generates a decline in output of 5 percent, quite comparable to the 7-percent decline in output relative to trend observed during the recession that started at the end of 2007. The magnitude of the decline in aggregate TFP (4 percent) in the model is about the same as that in the data. However, the contraction in investment and the surge in unemployment in the model are only about half as large as what is in the data.

The model time series and the data exhibit very different dynamics. The output and TFP series in the model hit the trough together with the data, but the model series rebound much more slowly. As for investment rate and unemployment, the model series reach the extreme values later than the data. Below we pursue this further.

In the right panel of Fig. 3, we plot the evolution of the ratio of credit market liabilities to non-financial assets for the non-corporate and the corporate sectors, together with disaggregated data for the corporate sector's bank vs. bond liabilities. Starting in the fourth quarter of 2008, there is a sharp contraction in the stock of credit market liabilities of the non-corporate sector, together with a milder one for the corporate sector. Furthermore, at a more disaggregated level, while the corporate sector as a whole suffered a sharp reduction in their bank-related liabilities, it was partly offset by a surge in the issuance of corporate bonds, at least for those corporations that had access to the corporate bond market. To the extent that we can view corporations, and in particular those that have access to the corporate bond market, as relatively less financially-constrained, the dynamics of external credit in the data along this dimension are consistent with the dynamics of external finance for rich vs. poor entrepreneurs in the model.

Faster-reverting credit crunch One feature of the model response to the credit crunch is that the recovery is more protracted than in the US data, especially for output, TFP (Fig. 2), and the ratio of external financing to capital (Fig. 1). In order to understand how much of such sluggishness is inherited from the persistence of the λ_t process and how much is attributable to the intrinsic propagation mechanism of the model, we run an alternative exercise in which the λ_t process jumps back to its pre-crunch level immediately after reaching its lowest value. The results from this exercise are shown in Fig. 4.

The top left panel reproduces the paths of the collateral constraint parameter λ_t in the benchmark exercise (solid line, labeled "lb") and in this faster-reversal exercise (dashed line, labeled "lb-1").

One striking result is in the top right panel, which shows the dynamics of the ratio of external finance to capital stock in the two exercises. Although the credit conditions as measured by λ_t recover much more quickly in the faster-reversal exercise, the equilibrium credit-to-capital ratios are virtually indistinguishable in the two cases and far more sluggish than either λ_t process. This persistently low credit-to-capital ratio is mostly explained by entrepreneurs' saving behavior during the credit crunch. There are two separate effects on saving. First, during the credit crunch, constrained entrepreneurs have high marginal returns to saving—because wealth, as collateral, mitigates the tighter credit constraint—and accumulate wealth at a faster rate in order to self-finance a larger fraction of their capital input. Second, the credit crunch, while it lowers the aggregate income, is a "boom time" for relatively unconstrained entrepreneurs: Wages and capital rental rates are down, which boosts the scale of production and profits of unconstrained entrepreneurs. Since the credit crunch is a transitory phenomenon, these entrepreneurs correctly view the boost in their entrepreneurial profits as temporary and hence their forward-looking saving behavior dictates that they save a large fraction of their income in these periods. Once we realize

[10] The capital used by wealthy entrepreneurs, who in the initial stationary equilibrium account for 90 percent of the aggregate capital, increases by 7.4 percent, while their external finance increases by 6 percent.

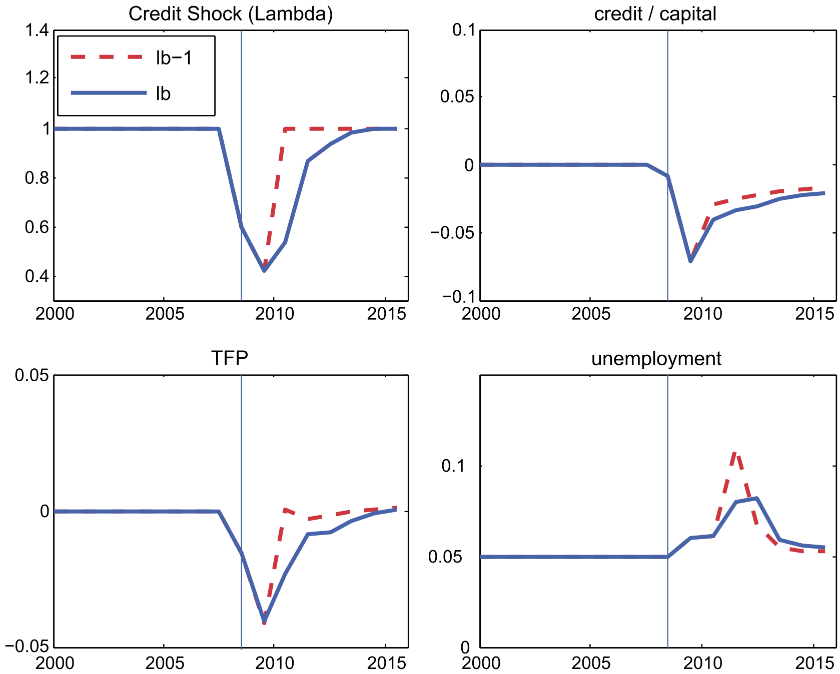

Fig. 4. Aggregate Implications of a Faster-Reverting Credit Crunch. The solid lines, labeled "lb", are for the benchmark exercise. The alternative exercise with the faster-reverting credit crunch is represented by the dashed lines and labeled "lb-1".

that higher wealth of entrepreneurs implies lower credit-to-capital ratios by construction—see Eq. (10)—and that wealth is a slow-moving variable because the source of income fluctuation, the z shock, is persistent, then it becomes clear why the credit-to-capital ratios in the economy lag behind the fast-reverting λ_t process by a wide margin.

As for TFP and unemployment, there is visible difference between the two exercises. In particular, TFP closely mimics the corresponding λ_t process (bottom left panel). Unemployment rates jump by more but then come down faster in the faster-reversal exercise than in the benchmark (bottom right panel).

The differential speed with which TFP recovers is easy to explain. A lower λ_t leads to more misallocation and hence lower TFP. A faster rebound in λ_t therefore brings about a faster recovery in TFP.

The unemployment rate dynamics merit a little more discussion. When credit constraints tighten, workers are reallocated from financially-constrained to unconstrained entrepreneurs, with the separated workers having to go through the matching friction before they can be employed again. As a result, this reallocation comes with higher unemployment rates. The immediate reversal of the tightened credit constraint, in turn, generates another round of reallocation in the opposite direction (i.e., from those who increased hiring during the credit crunch to those who had to downsize). Since these two rounds of labor reallocation are now compressed into a shorter time interval, we obtain a sharper rise and fall in unemployment rate compared with the benchmark. Still, the unemployment rate stays high for 2 to 3 additional years following the conclusion of the credit crunch. The fact that there are two rounds of labor reallocation and that the reallocation goes through frictional capital and labor markets explain why the credit crunch has a lasting impact on the labor market outcome.

3.2.1. Implication of rigid wages

While the model can account for a large fraction of the magnitude of the output and TFP decline, it underpredicts the rise in unemployment rate following the 2007–2008 recession. A simple extension of our model, in line with the arguments in Shimer (2012), is to introduce (downward) wage rigidities to the model. The effect of a credit crunch in the model with fixed wages is illustrated by the dash-dot lines in Fig. 2.[11]

A tightening of the collateral constraint in an environment with rigid wages leads to an even larger decline in output—driven by a sharper rise in unemployment than in the benchmark flexible-wage exercise. The rise in unemployment is explained by a higher flow of workers into unemployment and a relatively more protracted reallocation of these workers back into employment: A higher wage (than in the flexible wage case) inherently entails more job destruction and less

[11] The meaningful wage rigidity is the downward rigidity, since the wage would fall below its steady state level following the credit crunch if it were flexible.

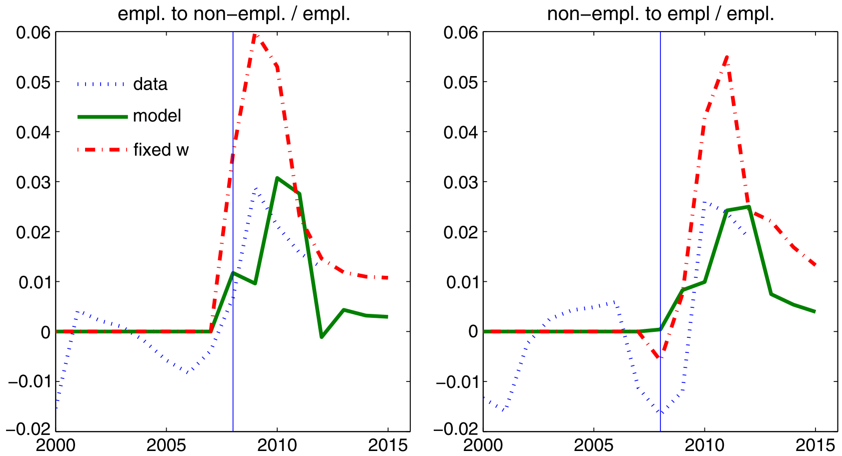

Fig. 5. Flows in and out of employment. The left panel shows the annual flow from employment to non-employment (unemployment plus out of labor force) in the data (dotted line) and the employment to unemployment flow in the models with flexible and fixed wages (solid and dash lines, respectively), all relative to the employment level at the beginning of each period. The right panel shows the corresponding flows from non-employment to employment relative to total employment. For the US data we plot the de-meaned series. For the model we show the differences from the steady state values. Footnote 12 describes the construction of these series.

hiring. The flows of workers in and out of employment, as a fraction of employment at the beginning of each period, are illustrated in Fig. 5.[12]

3.2.2. Variable capital utilization

A central implication of the model is that TFP, as measured by the Solow residual, drops during the credit crunch. As shown in Fig. 2, this is broadly consistent with the data in the beginning of the 2007–8 recession, although the recovery in the model is too protracted. Alternative measures of TFP that control for capital utilization data exhibit a smaller drop, e.g., Fernald (2012). We now provide a simple extension of the benchmark model that can address capital utilization-adjusted TFP.

Unlike the benchmark model where all capital is deposited with the financial intermediary—who then rents it out to entrepreneurs—this extension gives individuals the option to leave capital idle and have it depreciate at a rate $\underline{\delta}$, which is assumed to be less than δ. This option will be exercised whenever the market-clearing interest rate would fall below $-\underline{\delta}$, in which case the equilibrium interest rate in this extended model becomes $r_t = -\underline{\delta}$. (With $r_t = -\underline{\delta}$, individuals are indifferent between depositing the asset with the financial intermediary and leaving it idle.) Essentially, we now have a floor on the interest rate, $r_t \geq -\underline{\delta}$, which also implies a floor $-\underline{\delta} + \delta > 0$ on the rental rate of capital.[13]

With a large enough credit crunch that reduces the demand for capital by entrepreneurs to the point where this interest rate floor becomes binding, there will be surplus capital left idle in the economy. Since the surplus capital would have had a very low marginal product (less than $-\underline{\delta} + \delta$), taking it out of production will, once we correctly account for the unemployed capital in the economy, deliver a higher capital utilization-adjusted TFP than the TFP of the benchmark economy.

To illustrate this possibility, we simulate the same credit crunch as in the benchmark model but now allow individuals to leave capital idle without incurring depreciation—i.e., we assume $\underline{\delta} = 0$. In this case, about 6 percent of the capital stock is left idle in the second period of the crunch (2008), the only period in which the market-clearing interest rate would have been negative (−1.7 percent). As a result, the capital utilization-adjusted TFP drops by 1 percentage point less than the Solow residual in the benchmark: 3 percent instead of 4 percent.

3.2.3. Comparing credit crunch with exogenous TFP shock

The analysis of the previous section reveals how damaging a negative credit market shock could be to the labor market. Our goal in this section is to contrast the impact of the credit crunch to that of a more standard source of business cycle fluctuations: a negative shock to the aggregate TFP. To implement this experiment, we start the economy in its stationary equilibrium and hit it with an unanticipated drop and recovery in the aggregate TFP—A_t in Eq. (1).[14] This exogenous TFP series is constructed to mimic the endogenous TFP dynamics from the credit crunch (solid line, top right panel, Fig. 2).

Fig. 6 depicts the dynamics of aggregate quantities in response to the credit crunch (dashed lines, reproduced from Fig. 2) and to the exogenous aggregate TFP shock (solid lines). Despite the identical TFP dynamics of the two experiments

[12] The data series are annualized flow rates from the monthly flow data of the BLS research series. See http://www.bls.gov/cps/cps_flows.htm. We construct yearly flow rates by compounding the monthly transition matrices. We report the resulting series as deviations from their overall average.

[13] To be exact, the benchmark model also has an implicit floor on the interest rate, $r_t \geq -\delta$, since capital rental rate cannot be negative.

[14] Again, in 2007, everyone wakes up to a completely unanticipated shock to the future path of the exogenous aggregate TFP, which is deterministic and perfectly known at that point.

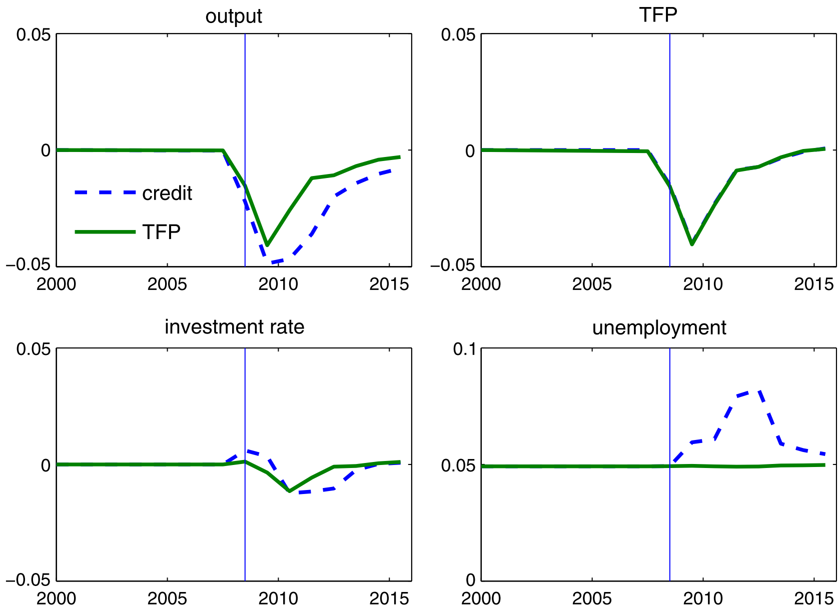

Fig. 6. Aggregate implications of credit crunch vs. exogenous TFP shock.

(engineered to be that way), the credit shock exercise shows a sharper and more protracted contraction in output. Investment rates (bottom left panel) seem comparable in both cases.[15] The bottom right panel shows that most of the differential response in output is attributable to unemployment: Unemployment is invariant to the TFP shock but increases by more than 50 percent at its peak with the credit crunch.

The defining characteristic of the credit shock that differentiates it from an exogenous decline in TFP is its reallocative nature. Even though all entrepreneurs face the same tighter collateral constraint (i.e., lower λ_t) during the credit crunch, only a subset of them—those who are productive but have little financial wealth as collateral—becomes more acutely constrained in their choices of labor and capital inputs. In contrast, an exogenous aggregate TFP shock induces a contraction in the employment of all firms symmetrically, holding factor prices constant. As a result, lower factor prices can completely offset the impact of the negative aggregate TFP shock to maintain the firm-level factor inputs, but not that of the credit crunch.[16] Once there is job destruction by now more financially-constrained entrepreneurs, laid off workers first become unemployed and then can only re-enter the hiring market subject to matching frictions, pushing up the unemployment rate.

The factor prices—wages and interest rates—also respond differently to the two shocks. The responses of interest rates are more starkly different between the two cases (right panel, Fig. 7). The credit crunch has a direct negative effect on many entrepreneurs' capital demand, forcing affected firms to scale down operations to the level allowed by the tighter collateral constraint. To bring the capital rental market back to an equilibrium, the interest rate has to fall. Because only a subset of entrepreneurs, who are unconstrained and more likely to be unproductive, can increase their capital demand in response to lower rental rates (the others are at a corner due to the tighter collateral constraint), the market clearing interest rate must fall by more (dashed line) than when all entrepreneurs can symmetrically respond to lower rental rates (i.e., the aggregate TFP shock case, solid line).

However, the same explanation does not carry over to the comparison of wage dynamics between the two experiments. In fact, wages fall by more with the exogenous TFP shock (solid line, left panel) than with the credit crunch (dashed line). We first point out that the collateral constraint does not apply directly to labor demand and as a result no active entrepreneur is at a corner when it comes to labor input choice. Unlike capital rental rates, all active entrepreneurs do respond to lower wages by increasing their labor input. However, if this were the only consideration, the wage should be still (slightly) lower with credit crunch than with the TFP shock, because the reallocation of capital toward unconstrained, unproductive entrepreneurs necessitates that the wage fall further to clear the labor market. Then, why do wages fall by less with the credit crunch in spite of the above consideration? The answer is unemployment. While the number of unemployed

[15] Buera and Moll (2012) show that the investment dynamics following a credit crunch and a TFP shock exactly coincide in a related model.

[16] While we can analytically show in our model that there exist a lower wage and capital rental rate that perfectly offset a lower A_t to support the same allocation within a period, this no-effect-on-unemployment result is by no means general. In particular, if we use a collateral constraint specification that is directly affected by firm-level productivity Az as in Buera et al. (2011) or assume that unemployment benefits do not co-move perfectly with the market wage, the exogenous TFP shock will have impact on unemployment.

Fig. 7. Factor price dynamics: credit crunch vs. exogenous TFP shock.

workers remain unchanged with the TFP shock, it increases significantly with the credit crunch. With fewer workers arriving in the competitive hiring market, the wage does not have to fall as much to clear the labor market.[17]

3.3. Micro-level implications of a credit crunch

In this section we switch the focus of our analysis to the firm-level implications of the credit crunch. A large empirical literature documents that financial crises affect firms differently depending on their size. The working hypothesis there is that small businesses are more reliant on credit to finance their production and capital expenditures than large firms and hence should be more vulnerable during recessions driven by credit contractions. Gertler and Gilchrist (1994) found evidence supporting this claim and more recently Fort et al. (2012) extended the analysis to highlight the role of firms' age, as well as size, in understanding their cyclical behavior: During credit-driven recessions, the employment growth of small, young firms declines by more relative to that of large, old firms. Our goal in this section is to provide a theoretical underpinning to the empirical literature and better understand how the aggregate behavior of the economy is shaped by the heterogeneous responses at the firm level.

3.3.1. Classifications and implementation

The dimensions of firm heterogeneity in the model are the productivity and wealth of entrepreneurs, which determine their labor and capital demand, accumulation of wealth, and the extent to which collateral constraints bind. The equilibrium of the model consists of a distribution of entrepreneurs along these two dimensions, which in turn determines the distribution of firms in terms of their size (defined as the number of employees) and age (defined as years of continuous operation, which is closely tied to the persistence of the entrepreneurial productivity process).

To examine the firm-level implications of the model, we simulate a sample of one million individuals. We first compute the invariant distribution of firms in terms of age and size. We then run the simulation for the credit crunch transition, keeping track of the evolution of the firm age-size distribution.[18]

Our classification of firms into small vs. large and young vs. old is as follows. For the size category, we first compute the median employment number $l_{m,t}$ for each period such that all firms that employ $l_{m,t}$ or fewer workers account for exactly half of the total employment in the economy. We classify a firm as small if its employment at the beginning of the period is less than or equal to $l_{m,t}$ and as large if it is above $l_{m,t}$.[19] With respect to age, we follow Fort et al. (2012) and consider firms aged 5 years or less as young and the rest as old.

Lastly, our method of aggregating job flows across firms within each age-size category is similar to that of Davis et al. (1998). We define the aggregate job creation rate ($JC_{s,t}$) and destruction rate ($JD_{s,t}$) for a given age-size category s in period t as:

$$JC_{s,t} = \sum_{i \in s} \frac{\max\{l_{t+1}(i) - l_t(i), 0\}}{0.5[L_{t+1}(s) + L_t(s)]}$$

[17] A telltale sign of this effect is the high wage in 2012 in the credit crunch transition, which coincides with the peak of the unemployment shown in the bottom right panel of Fig. 6 (dashed line).

[18] Our computations in the previous sections do not involve simulation methods: We work directly with the distribution G_t. We opt for simulation in this section to more easily keep track of firms' ages.

[19] In our model stationary equilibrium, l_m is 119. In the 2007 US data on establishment size distribution (Business Dynamics Statistics), l_m falls in the 56–99 employment bin. If the unit of analysis were firms, l_m in the US data would fall in the 500–999 employment bin.

Table 2
Properties of firm age-size distribution in steady state.

	Fraction not constrained	Share of total employment	Fraction of firms	Average TFP	Average wealth	Average rate of return	Net employment growth
Small-young	0.057	0.133	0.405	0.60	1.8	0.122	0.42
Large-young	0.111	0.045	0.004	1.15	42.5	0.074	0.25
Small-old	0.421	0.315	0.549	0.56	5.3	0.028	−0.07
Large-old	0.934	0.507	0.042	1.16	157.0	0.022	−0.09

$$JD_{s,t} = \sum_{i \in s} \frac{\max\{l_t(i) - l_{t+1}(i), 0\}}{0.5[L_{t+1}(s) + L_t(s)]}$$
$$Net_{s,t} = JC_{s,t} - JD_{s,t},$$

where $l_t(i)$ is the employment of firm i and $L_t(s)$ is the total employment of firms belonging to category s in period t. This methodology aggregates gross and net flows while keeping constant the set of firms that belong to each category between periods t and $t+1$, thus eschewing the "re-classification bias" discussed in Davis et al. (1998).

3.3.2. Disaggregate-level results: stationary equilibrium

We start by documenting the properties of each age-size group of firms in the stationary equilibrium. Table 2 reports the fraction of unconstrained firms (i.e., entrepreneurs) in each group, each group's share of aggregate employment, and the fraction of firms belonging to each group, along with some average characteristics of firms in each group.

Some numbers in the table are a direct consequence of our calibration strategy. For instance, it should not come as a surprise that a small number of large firms account for a disproportionately large fraction of aggregate employment in the model, consistent with the skewness of the empirical firm size distribution in the US: It is one of the target moments in our calibration (Section 3.1).

Other statistics in the table are predictions of the model that have not been specifically targeted. Consider the first column of the table, where we report the fraction of unconstrained firms in each age-size category in the model. According to our theory, the likelihood of becoming financially constrained is a function of the underlying state variables of the firm: entrepreneurial productivity and wealth. Holding other things equal, the unconstrained profit-maximizing capital input of those with higher productivity is higher and hence they are more likely to be constrained by the collateral constraint. Also, all else equal, the collateral constraint will more likely bind for those with low wealth/collateral. The table shows that age and size are good predictors of such likelihood: The fraction of unconstrained entrepreneurs is lowest for the small-young group and highest for the large-old group. There exists, then, a mapping from the productivity-wealth space to the firm age-size space that provides a justification for the conjecture that small, young firms are more sensitive than large, old ones to aggregate credit shocks.

The average entrepreneurial productivity and wealth across the age-size groups help clarify why such a mapping exists. In the fourth column, we see that the average entrepreneurial productivity is correlated with size but not with age: Entrepreneurial productivity is higher for large firms than for small firms but, conditional on size, they are about the same between age groups. Thus, if firms' credit conditions and production decisions were dependent only on entrepreneurial productivity, age would be irrelevant. However, wealth does matter for firms' credit conditions and production decisions and is correlated with age conditional on size. As the fifth column shows, in either size category, entrepreneurs running older firms are wealthier on average than those running younger ones: An old business implies that the managing entrepreneur has had enough time to accumulate wealth/collateral and overcome the collateral constraint.

The sixth column shows a tight connection between credit conditions and rates of return on capital. In a frictionless economy where all firms can operate at their respective unconstrained profit-maximizing scale, rates of return on capital—i.e., marginal products of capital net of depreciation—are equalized across all firms to the real interest rate. A binding collateral constraint, on the other hand, drives a wedge between the two rates: The rate of return on capital for constrained entrepreneurs is higher than the equilibrium interest rate in the economy. This is why we observe a difference of 10 percentage points between the small, young group and the large, old group, consistent with the larger fraction of constrained entrepreneurs in the former.

3.3.3. Disaggregate-level results: credit crunch

We now explore the dynamics of net employment growth rates during the credit crunch for the four age-size groups. They are plotted in Fig. 8. The vertical axis is differences in net employment growth rates from the respective steady state levels (last column, Table 2). Period 1 on the horizontal axis is 2008.

The behavior of employment growth rates is consistent with the information conveyed in Table 2. Young firms, both small and large, have higher rates of return on capital and are more likely to be financially constrained to begin with. They experience the sharpest decrease in net employment growth rates during the credit crunch. This finding suggests that the direction of factor reallocation from constrained to unconstrained entrepreneurs during the credit crunch can be represented by reallocation among age-size groups. Since it is difficult to determine in the available data whether a firm is financially constrained or not, the fact that there is a clean mapping from the entrepreneurial productivity-wealth space to the firm

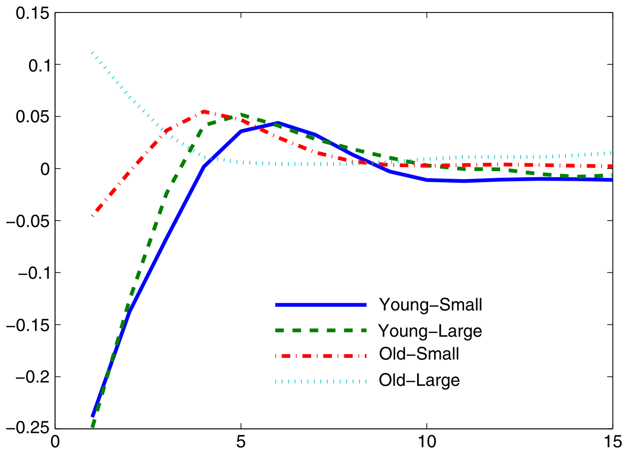

Fig. 8. Net employment growth by firm age-size during credit crunch.

age-size space allows us to interpret the dynamics of employment growth rates across firm-age size groups in the data using our theory.

4. Conclusions

In this paper we propose a heterogeneous-agent model that integrates credit and labor market frictions to study macro- and microeconomic implications of a credit crunch. We find that a salient ramification of a sudden drop in available credit is reallocation of production factors: Firms that become more financially constrained reduce their labor and capital demand and the surplus production factors get reallocated to unconstrained producers via the general equilibrium effect of lower factor prices. However, frictions in the labor market interfere with the labor reallocation. It takes time for the economy to absorb idled workers and, as a result, unemployment rates increase and remain high for a prolonged period. Moreover, the reallocation of production factors generated by the credit shock leads to an endogenous reduction in aggregate TFP, as high-productivity but low-wealth entrepreneurs downsize and low-productivity but high-wealth ones expand. Together with a contraction in investment, these effects push the macroeconomy into a recession.

Another finding of the paper is that credit and aggregate productivity shocks have different implications on both macro- and microeconomic variables. Even in the presence of matching frictions in the hiring market, the unemployment rate remains unchanged with the negative TFP shock, as the adjustment in wages and interest rates fully offset the decline in firms' factor demand. The key is that the aggregate TFP shock affects all firms' incentives to reduce labor demand symmetrically, which is not the case when the shock is credit-driven and hence has differential effects on different firms.

Another contribution of our work is that we provide a theory for the empirical finding that firms of different ages and sizes respond differently to credit-driven business cycles. We show that the combination of firm age and size is a good proxy for the combination of entrepreneurial productivity and net worth that ultimately determines the likelihood of being constrained and, hence, firms' employment changes in response to credit shocks.

Our theory emphasizes the importance of credit markets for efficient reallocation of resources across heterogeneous producers. From the perspective of our theory, financial crises are episodes where such reallocation of resources is distorted. The immediate impact is that capital is reallocated away from productive but constrained producers who must downsize their production toward unconstrained producers. Over a little longer horizon, the exacerbated credit friction implies that less capital flows into young, growing producers and, commensurately, less capital flows out of old, declining producers. The overall impact over the course of the credit crunch on the volume of capital reallocation depends on the relative strengths of these effects. Our calibrated model predicts an increase in gross capital reallocation during the credit crunch. This may appear to be at odds with the finding in Eisfeldt and Rampini (2006) that gross capital reallocation is procyclical. However, in relating our paper to theirs, one needs to realize that (i) they measure reallocation of capital *ownership* rather than reallocation of actual capital use and (ii) they do not separately identify recessions driven by credit shocks vs. other sources. With these caveats in mind, we think that a fruitful avenue of empirical research would be to construct better measures of capital reallocation and study how they correlate with different sources of aggregate fluctuations, e.g., credit vs. TFP shocks. We also conjecture that extensions of our model with capital adjustment costs can be useful for addressing these observations, as shown by Cui (2013). We leave a detailed analysis of this extension to future research.

Appendix A. Computational procedure

Discretizing the entrepreneurial productivity distribution We discretize the support of the entrepreneurial productivity distribution into 40 grid points: $\mathcal{Z} = \{\bar{z}_1, \ldots, \bar{z}_{40}\}$. Denoting the cumulative distribution function of the original Pareto distribution by $\mu(z) = 1 - z^{-\eta}$, we choose $\bar{z}_1$ and $\bar{z}_{38}$ such that $\mu(\bar{z}_1) = 0.633$ and $\mu(\bar{z}_{38}) = 0.998$. Indexing the grid points by j, we construct $\bar{z}_j$ to be equidistant from $j = 1$ through 38. The largest two values on the grid are given by $\bar{z}_{39}$ and $\bar{z}_{40}$, which satisfy $\mu(\bar{z}_{39}) = 0.999$ and $\mu(\bar{z}_{40}) = 0.9995$. Finally, the corresponding probability mass for $2 \le j \le 40$ is given by $[\mu(\bar{z}_j) - \mu(\bar{z}_{j-1})]/\mu(\bar{z}_{40})$ and $\mu(\bar{z}_1)/\mu(\bar{z}_{40})$ for $j = 1$.

Computing the stationary equilibrium We solve for the stationary equilibrium of this economy using the nested fixed-point algorithm of Aiyagari (1994). The difference is that we have to iterate on the wage w, interest rate r, and lump-sum tax τ, until the labor and capital markets clear and the government budget is balanced.

1. Guess the interest rate in the stationary equilibrium, r^i.
2. Guess the lump-sum tax in the stationary equilibrium, $\tau^{i,j}$.
3. Guess the wage in the stationary equilibrium, $w^{i,j,h}$.
4. Given the interest rate, tax, and wage, solve the individuals' problem. Given the optimal decision rules, the stochastic process for entrepreneurial productivity, and an arbitrary initial joint distribution of wealth and productivity, iterate on the laws of motion for the joint distribution until time-invariance is achieved.
5. Check the labor market clearing condition, aggregating labor demand and supply using the stationary distribution of wealth and entrepreneurial productivity. If there is excess labor demand (supply), choose a new wage $w^{i,j,h+1}$ that is greater (smaller) than $w^{i,j,h}$. Use bisection.
6. Repeat steps 4–5 until the labor market clears under the invariant distribution.
7. Compute the budget-balancing lump-sum tax $\widetilde{\tau}^{i,j}$ and compare it to $\tau^{i,j}$. If they differ, update the lump-sum tax with $\tau^{i,j+1} = \widetilde{\tau}^{i,j}$ and repeat steps 4–6. Stop when the τ^i sequence (indexed by j) converges.
8. Check the capital market clearing condition, aggregating capital demand and supply using the stationary distribution of wealth and entrepreneurial productivity. If there is excess capital demand (supply), choose a new interest rate r^{i+1} that is greater (smaller) than r^i.
9. Repeat steps 4–8 until the capital market also clears under the invariant distribution.

Computing the transition dynamics To compute the entire transition dynamics during the credit crunch and the recovery (i.e., the exogenous λ_t sequence), we have to iterate on the wage, interest rate and lump-sum tax sequences. Taking these sequences as given, we solve for the individuals' problem and then check whether the labor and capital markets clear and the government budget balances for all periods. We fix T, the period by which all transitions are completed, at 70. In our exercise, the new stationary equilibrium is the same as the initial stationary equilibrium, since there is no exogenous permanent change. We numerically verify that increasing T to 100 has no effect.

1. Guess an interest rate sequence $\{r_t^i\}_{t=0}^{\infty}$, with r_t^i equal to the initial stationary-equilibrium interest rate for $t \ge T$.
2. Guess a sequence of lump-sum tax $\{\tau_t^{i,j}\}_{t=0}^{\infty}$, with $\tau_t^{i,j}$ equal to the initial stationary-equilibrium lump-sum tax for $t \ge T$.
3. Guess a wage sequence $\{w_t^{i,j,h}\}_{t=0}^{\infty}$, with $w_t^{i,j,h}$ equal to the initial stationary-equilibrium wage for $t \ge T$.
4. Let $v_T(a,z) = v(a,z)$, where $v(\cdot)$ is the individual value function in the initial stationary equilibrium. By backward induction, taking the wage, interest rate and tax sequences as given, compute the value function $v_t(a,z)$ for $t = T - 1, \dots, 0$.
5. Using the optimal decision rules, the stochastic process for entrepreneurial productivity, and the initial joint distribution of wealth and entrepreneurial productivity, iterate forward the joint distribution over t. Check whether the labor market clears in every period. If not, construct a sequence $\{\widetilde{w}_t^{i,j,h}\}_{t=0}^{T}$ that clears the labor market period by period. Update the wage sequence: $w_t^{i,j,h+1} = \eta_w \widetilde{w}_t^{i,j,h} + (1 - \eta_w) w_t^{i,j,h}$ for all t, with $\eta_w \in (0,1)$.
6. Once the wage sequence converges, compute the budget-balancing lump-sum tax sequence $\{\widetilde{\tau}_t^{i,j}\}_{t=0}^{T}$. Update the tax sequence (from j to $j+1$) in a similar manner and iterate on the inner loop of wage sequences.
7. Once the wage and tax sequences converge, check whether the capital market clears in all periods. If not, compute a sequence $\{\widetilde{r}_t^i\}_{t=0}^{T}$ that clears the capital market period by period. Update the interest rate sequence with $\eta_r \widetilde{r}_t^i + (1 - \eta_r) r_t^i$, $\forall t$, with $\eta_r \in (0,1)$.
8. Repeat steps 4–7 until the interest rate sequence also converges.

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