Endogenous Growth Theory

Ping Wang
Department of Economics
Washington University in St. Louis

February 2019
I. Introduction

A. Stylized Facts and Growth Empirics

(i) Basic Stylized Facts:

- on-going increasing K/L and Y/L ratios
- increasing wage rates with stationary interest rates
- stationary wage and capital income shares
- widened cross-country disparities

(ii) Sources of Data: Summers-Heston, Barro-Lee, Barro-Sala-i-Martin, others

(iii) Growth Accounting: Denison, Jorgenson, Tallman-Wang
### (iv) Factor Accumulation and Growth

<table>
<thead>
<tr>
<th>Country</th>
<th>I/Y (%)</th>
<th>$Y_i/Y_{US} \times 100$ (1990)</th>
<th>$\Delta Y_i/Y_i$ (%)</th>
<th>Country</th>
<th>H index</th>
<th>$Y_i/Y_{US} \times 100$ (1990)</th>
<th>$\Delta Y_i/Y_i$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>24.0</td>
<td>100</td>
<td>2.1</td>
<td>U.S.</td>
<td>11.8</td>
<td>100</td>
<td>2.1</td>
</tr>
<tr>
<td>Algeria</td>
<td>23.3</td>
<td>14</td>
<td>2.2</td>
<td>Argentina</td>
<td>6.7</td>
<td>19</td>
<td>0.7</td>
</tr>
<tr>
<td>Zambia</td>
<td>27.9</td>
<td>4</td>
<td>-0.8</td>
<td>Philippines</td>
<td>6.7</td>
<td>14</td>
<td>1.3</td>
</tr>
<tr>
<td>Guyana</td>
<td>25.1</td>
<td>7</td>
<td>-0.9</td>
<td>Korea</td>
<td>9.2</td>
<td>45</td>
<td>6.3</td>
</tr>
<tr>
<td>Japan</td>
<td>36.6</td>
<td>80</td>
<td>5.6</td>
<td>New Zealand</td>
<td>12.3</td>
<td>63</td>
<td>1.4</td>
</tr>
<tr>
<td>Singap.</td>
<td>32.6</td>
<td>60</td>
<td>6.4</td>
<td>Norway</td>
<td>10.6</td>
<td>81</td>
<td>3.7</td>
</tr>
</tbody>
</table>
(v) TFP Growth and Long-Run Development

<table>
<thead>
<tr>
<th>Country</th>
<th>TFP Growth (%)</th>
<th>Contribution to Output Growth (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>0.4</td>
<td>13</td>
</tr>
<tr>
<td>Germany</td>
<td>1.6</td>
<td>49</td>
</tr>
<tr>
<td>U.K.</td>
<td>1.3</td>
<td>52</td>
</tr>
<tr>
<td>Chile</td>
<td>1.5</td>
<td>40</td>
</tr>
<tr>
<td>Mexico</td>
<td>2.3</td>
<td>37</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>2.2</td>
<td>30</td>
</tr>
<tr>
<td>Singapore</td>
<td>-0.4</td>
<td>-5</td>
</tr>
<tr>
<td>Korea</td>
<td>1.2</td>
<td>12</td>
</tr>
<tr>
<td>Taiwan</td>
<td>1.8</td>
<td>20</td>
</tr>
</tbody>
</table>
(vi) Convergence:

Convergence in per capita real GDP (Baumol, Barro, Barro-Sala-i-Martin):

- $\beta$-convergence - $\frac{\dot{y}_i}{y_i} = \theta_i = \beta_0 + \beta y_i(0) + ...$

- $\sigma$-convergence - $\frac{d}{dt}[Var(y_i)] < 0$

Problems:
- Galton Fallacy (Quah 1993)
- Twin-peak hypothesis (Quah 1996)
- Endogeneity problems; measurement errors

(vii) Sources of Growth:

- Theory ahead of empirics
- Kitchen sink regressions
(viii) Open Issues:

- Robustness (Levine-Renelt 1992, de la Fuente 1997)
- Test of endogenous vs. exogenous growth (Quah 1993, Jones 1995)
- Cross-country vs. country studies (Tallman-Wang 1994)
- Identification of endogenous threshold

B. Neoclassical Exogenous Growth Theory

(i) Solow-Swan: use variable $k/y$ to resolve the Harrod knife-edge problem

(ii) Allais-Phelps golden rule: $S-S$ per capita consumption maximization

\[
\max_k \quad C/L = f(k) - (n + \delta)k
\]

\[
\implies f_k = n + \delta \implies k^g
\]
(iii) Ramsey-Cass-Koopman optimal exogenous growth:

a. Continuous-time (optimal control):

\[
\max U = \int_0^{\infty} u(c) e^{-pt} \, dt
\]

s.t. \( \dot{k} = f(k) - (n+\delta)k - c, \quad k(0) = k_0 > 0 \).

- Transversality condition TVC (\( \lambda = \) co-state): \( \lim_{t \to \infty} \lambda k e^{-pt} = 0 \)
- Keynes-Ramsey equation:

\[
\dot{c} = \sigma(c)c[f_k - p - (n+\delta)], \quad \sigma(c) = -\frac{u_c}{u_{cc}}
\]

- Steady-state: golden rule is dynamically inefficient (over-accumulation)
- Dynamic equilibrium path: unique stable saddle
b. Discrete-time (dynamic programming)

\[ v(k) = \max_c u(c) + \beta V(k') \]

subject to \[ k' = f(k) + (1 - \delta)k - c, \quad k(0) = k_0 > 0, \quad n = 0. \]

- FOC: \[ u_c(c) = \beta V_k(k') \]
- Benveniste-Scheinkman: \[ V_k(k) = \beta V_k(k')[f_k(k) + (1 - \delta)] \]
- Euler equation: \[ u_c(c) = \beta u_c(c')[f_k(f(k) + (1 - \delta)k - c) + (1 - \delta)] \]

(iv) Problems of the exogenous growth models:

a. long-run growth exogenously determined by exogenous technical progress
b. lack of strong evidence in global convergence
c. failure to explain widened growth disparities
d. policy does not matter unless it can affect the rate of technical progress
C. Development of Endogenous Growth Theory

- Is the endogenous growth theory new?
- Seminal work before 1980s:
  - Solow (1956, at the end of his seminal paper): IRS & sustained growth
  - Pitchford (1960 ER): DRS with sustained growth
  - Shell (1966 AERP&P): inventive activity & growth
  - Wan (1970 REStud) - learning by doing & growth

- Creators of the new waves:
  - Romer (1986 JPE): general/knowledge capital
  - Stokey (1988 JPE): learning-by-doing
  - Rebelo (1991 JPE): basic one & two sector models
(i) New stylized facts:

(S1) club-convergence: bi-mode (Quah 1996)

(S2) divergent paths for physical and human capital:

<table>
<thead>
<tr>
<th>Country</th>
<th>Physical capital (K)</th>
<th>Human capital (H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>high</td>
<td>high</td>
</tr>
<tr>
<td>Macao</td>
<td>low</td>
<td>high H</td>
</tr>
<tr>
<td>Congo</td>
<td>high K</td>
<td>low H</td>
</tr>
<tr>
<td>Zaire</td>
<td>low K</td>
<td>low H</td>
</tr>
</tbody>
</table>

(S3) increasing rates of growth for the leading economy:

<table>
<thead>
<tr>
<th>Period</th>
<th>Country</th>
<th>Rate of growth (θ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1700-1785</td>
<td>Netherlands</td>
<td>θ = -0.07%</td>
</tr>
<tr>
<td>1785-1820</td>
<td>UK</td>
<td>θ = 0.5%</td>
</tr>
<tr>
<td>1820-1890</td>
<td>UK</td>
<td>θ = 1.4%</td>
</tr>
<tr>
<td>1890-1970</td>
<td>US</td>
<td>θ = 2.3%</td>
</tr>
</tbody>
</table>
(S4) both skilled/unskilled migrate to rich countries: why do the unskilled want to do so if paid by marginal products?

(S5) over-taking/lagging behind development experiences: why did Korea/Taiwan by-pass the Phillipines? why did Argentina dropped from the top?

(S6) capital deepening, product broadening, financial deepening, and world productivity and income distribution widening

(ii) How to design plausible models to endogenously determine the rate of economic growth and to match with these stylized facts?
<table>
<thead>
<tr>
<th></th>
<th>One-Sector</th>
<th>Two-Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Remarks: Other areas recently developed include,

a. fiscal policy (government spending and finance)
b. money (inflation and growth)
c. trade (goods flows, factor movements, technology transfer)
d. demography (fertility, aging, gender gap, social security)
e. labor market (education, learning, training, unemployment, sectoral shift)
f. political economics (government infrastructure, size of nation, voting, institutional/organizational design)

II. One-Sector Models

A. General Methodology

(i) Key:

Marginal products of reproducible factors are bounded below by a constant, thus requiring the production function be CRS or IRS in reproducible factors
(ii) Trick:

In perfectly competitive equilibrium, the model must be consistent with zero profit conditions, thus requiring CRS in privately provided factors

(iii) Decentralized problem:

a. consumer optimization:

\[
\text{max} \quad U = \int_0^\infty u(c) e^{-pt} dt, \quad u(c) = \frac{c^{1-\alpha} - 1}{1-\alpha}, \quad \alpha = \sigma^{-1}
\]

s.t. \[\dot{\Omega} = r\Omega - c, \quad \Omega(0) = \Omega_0 > 0.\]

Transversality condition (\(\lambda = \text{co-state}\)):\[\lim_{t \to \infty} \lambda \Omega e^{-pt} = 0.\]
b. producer optimization:

\[
\max \quad V = \int_0^\infty [f(k) - qx] e^{-\int_0^t r(s) ds} \, dt
\]

s.t. \( \dot{k} = x - \delta k \), \( k(0) = k_0 > 0 \).

Transversality condition (\( \mu = \) co-state): \( \lim_{t \to \infty} \mu k e^{-\int_0^t r(s) ds} = 0 \).

c. Key relationships:

- Keynes-Ramsey equation:
  \[
  \theta = \frac{\dot{c}}{c} = \frac{r - \rho}{\alpha} \quad \Rightarrow \quad r = \rho + \alpha \theta \quad \text{(UU)}
  \]

- Firm’s Euler equation:
  \[
  \frac{\dot{q}}{q} = (r + \delta) - \frac{f_k}{q}
  \]
• Balanced Growth Path (BGP):
\[ \frac{\dot{c}}{c} = \text{const.}, \quad \frac{\dot{k}}{k} = \text{const.} \implies \frac{\dot{q}}{q} = 0 \implies r = \frac{f_k}{q} - \delta \quad (YY) \]

d. Determining the BGP endogenous growth rate:

\[ \theta = \frac{\left( f_k/q \right) - (\rho + \delta)}{\alpha} \]

- diagram: \( f_k = \text{constant} \)
- exogenous growth: given \( \theta, UU \) and \( YY \) pin down \( r \)
- endogenous growth: given \( f_k/q \), \( UU \) and \( YY \) pin down \( \theta \)
e. Key conditions:

Condition U: (Bounded Utility) \( \rho > (1 - \alpha) \theta^{\max} \) (Brock-Gale)

Condition G: (Positive Growth) \( \min_k \{f_k/q\} > \rho + \delta \) (Jones-Manuelli)

(iv) Social planner problem:

- This is the same as the decentralized one if there are no externalities nor distortionary taxes

- In this case, the optimization can be simplified as:

\[
\max U = \int_0^\infty \frac{c^{1-\alpha} - 1}{1 - \alpha} e^{-\rho t} dt \quad \text{s.t. } \dot{k} = \frac{f(k)}{q} - \delta k - \frac{c}{q}, \quad k(0) = k_0, \quad q(0) = 1
\]
• Remarks: $f_k(k)$ must be bounded below by a positive constant in order to support sustained growth, which includes three possible cases: (1) CRS, (2) asymptotic CRS, and (3) IRS. The second case includes the short-run diminishing return cases in Jones-Manuelli (1990) and in Pitchford (1960), where $MPK = A + \gamma Bk^{\gamma - 1}$ and $[B_1^\varepsilon + B_2(L/k)^{1-1/\varepsilon}]^{1/(\varepsilon-1)}$, respectively. The third case can be found in Solow (1956).

B. Ak Model: Rebelo (1988) I

(i) Setup: $y = f(k) = Ak$; $q = 1$

(ii) Endogenous growth rate: $\theta = \frac{A - (\rho + \delta)}{\alpha} > 0$

(iii) Main features:
   a. common growth (for c, k, y): $\theta$ is increasing in $(A, \sigma)$, decreasing in $(\rho, \delta)$
   b. non-convergence (S1)
   c. any policy affecting the level of $A$ has a growth effect
   d. no transitional dynamics
   e. CE $\iff$ PO
C. CRS Model with Uncompensated Positive Spillover: Romer (1986) I

(i) Setup: \( y = f(k) = Ak^{1-\beta} K^\beta \); \( q = 1 \) (aggregate capital \( K = k \) in equilibrium)

\[ \Rightarrow r = A(1-\beta) \left( \frac{K}{k} \right)^\beta - \delta = A(1-\beta) - \delta \text{ in equilibrium} \]

(ii) Competitive common growth rate:

\[ \theta = \frac{r - (\rho + \delta)}{\alpha} = \frac{A(1-\beta) - (\rho + \delta)}{\alpha} > 0 \]

(iii) Main features:

a. \( \theta \) is increasing in \( A \) and \( \sigma \); decreasing in \( \rho, \delta \) and \( \beta \)

b. non-convergence (S1)

c. any policy affecting \( A \) has a growth effect

d. no transitional dynamics

e. CE is not the same as PO
(iv) Pareto-optimal endogenous growth rate:

- Substitute in $K = k$ before differentiation and $y = Ak$

$$\Rightarrow \quad \theta^* = \frac{A - (\rho + \delta)}{\alpha} > 0$$

$$\Rightarrow \quad r < r^*; \quad \theta < \theta^*$$

- Under-investment in equilibrium is due to the free-rider problem, which is absent iff $\beta = 0$.

(v) Remedy inefficiency using a Pigovian policy: Barro and Sala-i-Martin (1992)

a. production subsidy: $\frac{\beta}{1-\beta} (r + \delta) k$  \quad (\tau^* = \frac{\beta}{1-\beta})

b. factor price subsidy: $\frac{f_k}{1-\beta} - \delta$  \quad (q^* = 1-\beta)
D. IRS Model with Uncompensated Positive Spillover: Romer (1986) II

(i) Setup: \( y = f(k) = Ak^{1-\beta} K^{\beta+\gamma}; \ q = 1; \ \gamma \geq 0 \) (aggregate K=k in equilibrium)

(ii) Equilibrium paths:

no longer common growth ==> need to distinguish different rates of growth

\[
\theta_c = \frac{\dot{c}}{c} = \left(\frac{1-\beta}{\alpha}\right) Ak^\gamma - \frac{p}{\alpha}
\]

\[
\theta_k = \frac{\dot{k}}{k} = Ak^\gamma - \frac{c}{k}
\]

(iii) Transformation - Xie (1991): \( z = k/c \) with \( 1-\beta = \alpha \)

\[
\theta_z = \frac{\dot{z}}{z} = -\frac{1}{z} + \frac{p}{\alpha} \quad \text{(a Bernoulli equation for } z)\]
(iv) Solution: \( z(t) = \frac{\alpha}{\rho} + (a - \frac{\alpha}{\rho}) e^{(\rho/a)t} \) where \( a \) is an integration constant

- **TVC:** \( \lim_{t \to \infty} \lambda k(t) e^{-\rho t} = \lim_{t \to \infty} c^{-\alpha} c z e^{-\rho t} = 0 \)

  \[ \implies \lim_{t \to \infty} c^\beta \left[ \frac{\alpha}{\rho} e^{-\rho t} + \left( a - \frac{\alpha}{\rho} \right) \left( \frac{1}{1-\beta} \right)^{-(1-\beta)\rho t} \right] = 0 \]

- Thus, \( a = \frac{\alpha}{\rho} \implies z(t) = \frac{\alpha}{\rho} \) (so, \( \theta_e = \theta_k = \theta \))

(v) Competitive growth rates: \( (\theta_y) \) is strictly increasing and strictly concave

\[
\theta = Ak^\gamma - \frac{\rho}{\alpha} ; \quad \theta_y = (1+\gamma) \left[ A^{1+\gamma} y^{1+\gamma} - \frac{\rho}{\alpha} \right]
\]

(vi) Main features:
- increasing growth rate (S2), at a diminishing speed if \( \gamma < 1 \)
- well-defined transitional dynamics
- more sever free-rider problem.
E. Public Capital: Barro (1990)

(i) Setup: \( y = f(k) = Ak^{1-\beta} G^\beta \); \( q = 1 \) (public capital \( G = \tau y \) in equilibrium)

(ii) Endogenous growth rate:

\[
\theta = \frac{1}{\alpha} \left[ (1-\beta) A^{1/(1-\beta)} \tau^{\beta/(1-\beta)} (1-\tau) - (\rho + \delta) \right]
\]

which is maximized at \( \tau^* = \beta \) (optimal government size, max U too)

F. Long-Run CRS with Short-Run DRS: Jones-Manuelli (1990)

(i) Setup:

\( y = Ak + Bk^\gamma \); \( q = 1 \) (0 < \( \gamma \) < 1); \( \delta = 0 \)

\[
\Rightarrow r = A + B\gamma k^{\gamma-1} \rightarrow A \quad \text{as} \quad k \rightarrow \infty \quad \text{(thus, there is an asymptotic BGP)}
\]
(ii) Transformation: \( z_1 = c/k; z_2 = y/k \)

\[
\theta_c = \frac{1}{\alpha} \left[ A + \gamma B k^{\gamma - 1} - \rho \right] = \frac{1}{\alpha} \left[ A + \gamma(z_2 - A) - \rho \right]
\]

\[
\theta_k = \frac{y}{k} - \frac{c}{k} = z_2 - z_1
\]

\[
\theta_y = \frac{1}{z_2} [A + \gamma(z_2 - A)] \theta_k
\]

\[
\theta_{z_1} = \frac{\dot{z}_1}{z_1} = \left( \frac{\gamma}{\alpha} - 1 \right)(z_2 - A) + z_1 - \psi
\]

\[
\theta_{z_2} = \frac{\dot{z}_2}{z_2} = -\frac{1}{z_2} (1 - \gamma)(z_2 - A)(z_2 - z_1)
\]

where \( \psi = \frac{1}{\alpha} [\rho - (1 - \alpha)A] > 0 \)
(iii) The dynamical system: The case of $\alpha > \gamma$

- Well-defined transitional dynamics with a unique half saddle arm
- Along the transition, $z_1$ and $z_2$ move in the same direction toward the BGP
- Remark: for $\alpha < \gamma$, $z_1$ and $z_2$ move in the opposite direction, which is in consistent with empirical observations
III. Two-Sector Models

A. Physical and Human Capital with CRS: Lucas (1988) I

(i) Setup:

$$\max \quad U = \int_{0}^{\infty} \frac{c^{1-\alpha} - 1}{1-\alpha} e^{-\rho t} \, dt$$

s.t. \quad \dot{k} = y - \delta k - c, \quad y = F(k, uh); \quad k(0) = k_0 > 0$

$$\dot{h} = \varphi(1-u)h, \quad h(0) = h_0 > 0$$

(ii) Key: linear education evolution to simplify the analysis (simplified Uzawa)

(iii) Solution with Cobb-Douglas Production: (CE = PO)

$$\theta = \frac{\varphi - \rho}{\alpha} > 0$$

Only education matters (goods production technology does not matter)!
B. Physical and Human Capital with IRS: Lucas (1988) II

(i) Setup: \[ y = F(k, uh, H) = Ak^{1-\beta}(uh)^{\beta}H^\gamma \quad (H = h \text{ in equilibrium}; \gamma > 0) \]

H is due to uncompensated positive spillover from peers (\( \gamma = 0 \Rightarrow \text{Lucas I} \))

(ii) Key:

linear human capital evolution accepts BGP
CRS in private factors ensures consistency with CE

(iii) Competitive BGP equilibrium:

common growth for c, k, F \( \Rightarrow \) \[ \theta_c = \theta_k = \frac{\dot{F}}{F} = \theta \]

Cobb-Douglas PF \( \Rightarrow \) \[ \theta_h = \left(\frac{\beta}{\beta + \gamma}\right)\theta_k = \left(\frac{\beta}{\beta + \gamma}\right)\theta \]
a. FOCs:
\[ c^{-\alpha} = \mu \quad \implies \quad \theta = -\frac{1}{\alpha} \frac{\dot{\mu}}{\mu} \] (CC)

\[ \mu \frac{\beta F}{u} = \lambda \varphi h \quad \implies \quad \frac{\dot{\mu}}{\mu} + \frac{\dot{F}}{F} - \frac{\dot{u}}{u} = \frac{\dot{\lambda}}{\lambda} + \theta_h \] (FS)

b. Euler equations:
\[ \frac{\dot{\mu}}{\mu} = (\rho + \delta) - (1-\beta) \frac{F}{k} \quad \implies \quad \theta = \frac{1}{\alpha} \left[ (1-\beta) \frac{F}{k} - (\rho + \delta) \right] \] (UU)

\[ \frac{\dot{\lambda}}{\lambda} = \rho - \varphi \] (LL)

c. Transversality conditions: \( \lim_{t \to \infty} \mu k(t) e^{-\rho t} = 0; \quad \lim_{t \to \infty} \lambda h(t) e^{-\rho t} = 0 \)

d. Transformation: \( z_1 = \frac{c}{k}; \quad z_2 = \frac{y}{k} = A(uh)^{\beta+\gamma}/k^\beta \)

production function \( \implies \quad \frac{\dot{F}}{F} = \beta \frac{\dot{u}}{u} + \theta \) (PP)
e. BGP growth:

\[ CC,FS,UU,LL,PP \implies \theta = \frac{(\beta + \gamma)(\varphi - \rho)}{\alpha \beta + (\alpha - 1) \gamma} > \frac{\beta(\varphi - \rho)}{\alpha \beta + (\alpha - 1) \gamma} = \theta_k \]

\( \theta \) is more responsive to \( \varphi \) than in the case of \( \gamma = 0 \)

(iv) Pareto Optimum

Similar to Romer II, people under-invest in CE due to the free-rider problem; one can remedy the inefficiency by an education subsidy.

(v) Transitional dynamics:

Lucas believes that \( k \) and \( h \) should move together along the saddle path toward the BGP, which has been shown incorrect by Xie (1994) and Benhabib-Perli (1994). Specifically, there may be dynamic indeterminacy in the sense that there is a continuum of transition paths converging to the unique BGP, depending crucially on how strong the positive externality and intertemporal substitution are.

- Liberty ship (same blue print): Searle (1945) and Rapping (1965) identify 12-24% and 11-29% learning-by-doing effect in production

(i) Setup:

Final good output: \( y = F(n, z) = Anz^\xi \)

Experience accumulation: \( \dot{z} = G(n, z) = nz^\xi \)

(ii) Key:

- experiences grow over time
- more employment is better for both production and accumulating experiences
(iii) Results:

\[ z(t) = [z_0^{1-\xi} + (1-\xi) \int_0^t n(\tau) d\tau]^{1/(1-\xi)} \]

- With \( n = \bar{n} \), we have: \( y(t) = A\bar{n} [z_0^{1-\xi} + (1-\xi)\bar{n}t]^{1/(1-\xi)} \)
- Rate of productivity:

\[ \mu(t) = d\ln(z^\xi)/dt = \xi \bar{n}z^{\xi-1} \rightarrow \xi/[(1-\xi)t] \text{ if } z_0 \rightarrow 0 \]

(iv) Main Findings:

a. presence of scale effect: \( d\mu/d\bar{n} > 0 \)

b. based on Rapping (1965), \( \xi = 0.2 \); so, \( d\mu/dt = 0.25 \)

c. rapid decay in learning means that making a miracle requires continual emergence of new innovation for new products
(v) Problems:

a. scale effect not empirically supported =>
   (1) Young (1991): bounded learning
   (2) Young (1998): removal of the scale effect

b. continual emergence of new products =>
   (1) product ladder models:
       • Romer (1990): intermediate goods broadening
       • Aghion-Howitt (1992): ex ante perfect competition & ex post monopoly in R&D
       • Laing-Palivos-Wang (2002): continual development of new product in the presence of search frictions with zero profit ex ante
   (2) endogenous basket models:
       • Stokey (1988, 1995): LBD and new goods

- Why should we treat H as the main driving force?
- Why should we believe in monotone transition?
- Why should we focus on the primal instead of the dual?

(i) Setup:

Notation:  
C, K, H, X, Y all in levels; c = C/H, k = K/H, x = X/H, y = Y/H;
k_x = (sK)/(uH), k_y = [(1-s)K]/[(1-u)H];
p = p_x/p_y = \lambda/\mu \text{ (goods are numeraire)}

\[
\max \int_0^\infty \frac{c^{1-\alpha} - 1}{1 - \alpha} e^{-\rho t} \, dt
\]

s.t.  
\[
\dot{K} = xH - \delta K - C, \quad x = uf(k_x); \quad K(0) = K_0 > 0
\]
\[
\dot{H} = yH - \eta H, \quad y = (1-u)g(k_y); \quad H(0) = H_0 > 0
\]
(ii) Key:

a. General CRS technologies
b. Intertemporal no-arbitrage
c. Polarization Theorem

(iii) Competitive BGP equilibrium:

a. FOCs: \( c^{-\alpha} = \mu \implies \theta = -\frac{1}{\alpha} \frac{\dot{\mu}}{\mu} \) (CC)

\[ r = r_x(k_x) = pr_y(k_y); \quad w = w_x(k_x) = pw_y(k_y) \] (FPE)

\[ \implies \text{Stolper-Samuelson Theorem in endog. growth} \]
\[ \implies k_x(p), k_y(p), r(p), w(p) \]

Case 1: \( k_x < k_y \implies \theta_w < 0 < \theta_p < \theta_r; \quad dk_i/dp < 0 \)

Case 2: \( k_x > k_y \implies \theta_r < 0 < \theta_p < \theta_w; \quad dk_i/dp > 0 \)
b. Euler equations:

\[ \theta_p = (r - \delta) - \left( \frac{w}{p} - \eta \right) \text{ (INA)} \]

Case 1: \( k_x < k_y \)

\[ \Rightarrow \quad \frac{dr}{dp} > 0; \quad \frac{d(w/p)}{dp} < 0 \quad \Rightarrow \quad \frac{d\theta_p}{dp} > 0 \]

Case 2: \( k_x > k_y \)

\[ \Rightarrow \quad \frac{dr}{dp} < 0; \quad \frac{d(w/p)}{dp} > 0 \quad \Rightarrow \quad \frac{d\theta_p}{dp} < 0 \]

c. Transversality conditions:

\[ \lim_{t \to \infty} \mu K(t) e^{-\rho t} = 0 \]

\[ \lim_{t \to \infty} \lambda H(t) e^{-\rho t} = 0 \]
d. Factor evolution:

\[ \frac{\dot{K}}{K} = \frac{u}{k} f(k_x(p)) - \delta - \frac{c}{k} \]

\[ \frac{\dot{H}}{H} = (1 - u)g(k_y(p)) - \eta \]

\[ \implies \text{common growth ( } \theta_c = \theta_k = \theta_H = \theta_X = \theta_Y = 0 \text{ ) and } \theta_p = 0 \]

e. Full employment:

\[ uk_x(p) + (1-u)k_y(p) = k \implies u = u(p, k), \ du/dp < 0; \ du/dk < (>) 0 \text{ for 1 (2)} \]

Rybczynski Theorem in dynamic setting: ( \( dx/dp < 0, dy/dp > 0 \) )

Case 1: \( k_x < k_y \)

\[ \implies \theta_x < 0 < \theta_k < \theta_y; \ dx/dk < 0, dy/dk > 0 \]

Case 2: \( k_x > k_y \)

\[ \implies \theta_y < 0 < \theta_k < \theta_x; \ dx/dk > 0, dy/dk < 0 \]
f. Endogenous growth rate (Conditions U, G, FP):

\[ \theta = \frac{1}{\alpha} [r(p) - (\rho + \delta)] = \theta(p) \quad \text{where} \quad d\theta/dp > (<) 0 \quad \text{in Case 1 (2)} \]

(iv) Transitional Dynamics

\[ \begin{align*}
\theta_p &= (r(p) - \delta) - \left( \frac{w}{p} (p) - \eta \right) \\
\theta_c &= \frac{1}{\alpha} [r(p) - (\rho + \delta)] - y(p,k) + \eta \\
\theta_k &= \frac{1}{k} x(p,k) - y(p,k) - \frac{c}{k} + (\eta - \delta)
\end{align*} \]

Key features:

a. polarization between p and (c,k) \(\Longrightarrow\) saddle-path stability
b. distortionary taxes such that price and physical factor intensity measures are reversed \(\Longrightarrow\) instability or dynamic indeterminacy

- 3x3 dynamical system \( (c = \frac{C}{H}, k = \frac{K}{H}) \):
  \[
  \begin{align*}
  \frac{\dot{p}}{p} &= (r - \delta) - \left(\frac{w}{p} - \eta\right) \\
  \frac{\dot{c}}{c} &= \frac{1}{\alpha} (r(p) - (p + \delta)) - y(p, k) + \eta \\
  \frac{\dot{k}}{k} &= \frac{x(p, k)}{k} - y(p, k) - \frac{c}{k} - (\delta - \eta)
  \end{align*}
  \]

- Jacobean:
  \[
  J = \begin{bmatrix}
  p & 0 & 0 \\
  J_{21} & 0 & -y_k \\
  J_{31} & -1 & x_k - y_k - (\delta - \eta)
  \end{bmatrix}
  \]
  where \( J_{11} = p \theta_{p} > (<) 0 \) in case 1 (2), \( J_{23} = -y_k < (>) 0 \) in case 1 (2), and
  \[
  J_{33} = x_k - y_k - y - (\delta - \eta) = x_k - y_k - \frac{x - c}{k}
  \]
  - in case 1, \( x_k < 0 \) and \( y_k > 0 \), so
    \[
    J_{33} = x_k - y_k - \frac{x - c}{k} < 0
    \]
  - in case 2, \( x_k > \frac{c}{k} > 0 \) (Rybczynski) and \( y_k < 0 \), so
    \[
    J_{33} = \frac{c}{k} - y_k + \frac{x - c}{k} \left(\frac{k}{x} x_k - 1\right) > 0
    \]

- Define \( J_{2 \times 2} = \begin{bmatrix} 0 & -y_k \\ -1 & x_k - y_k - y - (\delta - \eta) \end{bmatrix} \), so
  \[
  \begin{align*}
  &- \text{tr}(J_{2 \times 2}) = J_{33} < (>) 0 \text{ in case 1 (2)} \\
  &- \text{det}(J_{2 \times 2}) = -y_k < (>) 0 \text{ in case 1 (2)}
  \end{align*}
  \]

- With distortionary taxes or sector-specific positive externalities, price dynamics and quantity dynamics may be reversed, leading to the possibility of unstable source and dynamic indeterminacy.
(v) Extensions:

a. factor taxation
b. dynamic Heckscher-Ohlin
c. dynamic sector shifts & economic transition
d. public vs. private sector
e. formal vs. informal sector

IV. Development of the Literature

(i) Endogenous Technological Progress & Adoption

- Rustichini-Schmitz (1991): imitation vs innovation
- Aghion-Howitt (1992): creative destruction
- Parente-Prescott (1994): adoption barriers
- Stokey (1995): R&D & endogenous basket of goods
- Jovanovic-Nyarko (1996): learning & technology choice
- Eicher-Turnovsky (1998): no-scale effect model
- Thesmar-Thoemig (2000): adoption & organization
- Aghion (2002): general purpose technology (GPT)
(ii) Endogenous Human Capital Accumulation

- Boucekkine-Licandro-Puch-del Rio (2004): vintage capital
- Boldrin-Levine (2004): competitive market for R&D

- Tssidon (1992): moral hazard trap to growth
- Acemoglu (1996): microfoundation human capital externality
- Benobou (1996): stratification and growth
- Ferschtman-Murphy-Weiss (1996): social status and human capital
- Redding (1996): human capital and R&D
- Tamura (2001): teacher and growth
- LLoyd-Ellis-Bernhart (2002): managerial capital
(iii) Endogenous Fertility Choice
- Palivos (1995): fertility & multiple growth paths
- Greenwood-Seshadri (2005): technology & baby boom
- Soares (2005): mortality & fertility

(iv) Government Spending, Taxation and Growth
- Lucas (1990): human capital accumulation & tax incidence
- Perotti (1993): distributive policy & growth
- Stokey-Rebelo (1994): flat-rate capital taxation & growth
• Krusell & Rios-Rull (1999): government size & growth
• LLoyd-Ellis (2000): public education
• Werning (2007): nonlinear tax incidence
• Acemoglu (2009): political economy & tax incidence
• Chen-Chen-Wang (2009): labor-market frictions & dynamic tax incidence

(v) Money, Inflation and Growth
• Jones-Manuelli (1995): growth effects of inflation
• Jha-Wang-Yip (2002), growth dynamics with money
• Chang-Chang-Lai-Wang (2007), growth dynamics with money & banking
• Wang-Xie (2009): labor-market frictions & welfare cost of inflation
(vi) Financial Intermediation, Credit Creation and Growth
- Greenwood-Jovanovic (1990): risk pooling and effective monitoring
- Bencivenga-Smith (1991): liquidity management
- Becsi-Wang-Wynne (1996): funds pooling and endogenous market structure
- Aghion-Dewatripont-Rey (1999): financial discipline & growth

(vii) Trade, Growth and Development
- Rivera-Batiz and Romer (1991): economic integration
- Ventura (1997): international interdependence
- Bond-Trask-Wang (2003): dynamic Heckscher-Ohlin
- Nishimura-Shimomura (2003): trade and indeterminacy
• Yi (2003): vertical specialization and growth
• Grossman (2004): allocation of talent and trade
• Farmer-Lahiri (2005): trade and indeterminacy
• Wan (2002): learning, trade and industrialization
• Bond-Jones-Wang (2004): learning from export
• Ghironi-Melitz (2005): trade & firm dynamics
• Alvareza-Lucas (2007): trade & firm growth
• Riezman-Wang (2009): outsourcing & demand-led growth

(viii) Dynamic Indeterminacy
• Benhabib-Farmer (1994): one-sector, IRS
• Boldrin-Rustichini (1994): two-sector, IRS
• Benhabib-Perli (1994): two-sector, IRS
• Xie (1994): two-sector, IRS
• Bond, Wang and Yip (1996): distortionary taxes
• Benhabib-Meng-Nishimura (2000): two-sector, CRS with sector-specific externalities
• Mino (2001): two-sector, CRS with sectoral externalities
• Nishimura-Venditti (2002): two-sector, CRS with intersectoral externalities
• Nishimura-Shimomura-Wang (2005): multi-sector, CRS with sector-specific externalities
• Mino-Shimomura-Wang (2005): multi-sector, OG with endogenous occupational choice
• Mino-Nishimura-Shimomura-Wang (2007): two-sector, CRS with sectoral externalities in discrete time

(ix) Income Distribution and Growth
• Galor-Zeira (1993): income distribution in OLG
• Glomm-Rivikumar (1992): public vs. private education and inequality
• Galor-Moav (2000): ability-biased technical progress & wage inequality
• Krusell, Ohanian, Rios-Rull & Violante (2000): technical progress, relative price change & inequality
• Ghiglino-Sorger (2002): indeterminacy & wealth distribution
• Aghion (2002): between and within-the-skilled-group inequality
• Matsuyama (2002): inequality and industrial transformation
• Eeckhout-Jovanovic (2002): production knowledge spillovers & inequality
• Fender-Wang (2003): educational choice & inequality
• Galor-Moav (2004), inequality & development
• Laing-Palivos-Wang (2003): vintage & inequality
• Foellmi-Zweimuller (2006): inequality & demand-led innovation
• Huggett-Ventura-Yaron (2006): human capital & earnings distribution dynamics
• Manuelli-Seshadri (2007): human capital & wealth distribution
• Kambourov-Manovskii (2009): occupational shifts & inequality

(x) Search, Matching, Unemployment and Growth
• Aghion and Howitt (1994): unemployment
• Laing, Palivos and Wang (1995): learning, matching and unemployment
• Acemoglu (1997): training and innovation
• Huang-Laing-Wang (2004): unemployment & crime
• Chen-Chen-Wang (2009): growth effects of human capital policy with labor-market frictions
• Chen-Mo-Wang (2009): micro-matching and turnover
• Jovanovic (2009): micro-matching and technology cycle

V. Conclusions: Perspectives of Growth Theory


