Problem 1

Are the following functions differentiable? Justify your answer.

(a) $f(x) = x^2$

(b) $f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$

(c) $f(x) = \sqrt{x}$

Problem 2

Compute eigenvalues and eigenvectors of the following matrices.

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \mathrm{e} & 0 \\ 0 & 0 & \pi \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ -3 & 3 & 0 \\ 3 & 2 & 2 \end{bmatrix}$

Problem 3

Define $f : \mathbb{R}^2 \to \mathbb{R}^3$ by $f(x, y) = (x^2 y, x + 2y^2, xy)$ and $h : \mathbb{R}^3 \to \mathbb{R}$ by $h(u, v, w) = \mathrm{e}^{2u-v+w}$. Compute the derivative of the composite function $h \circ f$ at $(x_0, y_0) = (1, 1)$.

Problem 4

Generate the matrix $\boldsymbol{A} \in \mathbb{C}^{32\times 32}$ using the following Matlab command

```
A = fft(eye(32))/sqrt(32);
```

(a) Write a Matlab script that generates $\boldsymbol{A}$ and verifies that it is an orthonormal matrix. Submit your script by printing it out and attaching to the rest of your solutions.

(b) Consider the system of equations $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, where $\boldsymbol{A}$ is in (a) and $\boldsymbol{b} \in \mathbb{C}^{32}$ is generated as follows

```
b = ones(32, 1);
```

Write a Matlab script that generates $\boldsymbol{A}$ and $\boldsymbol{b}$, and then solves this system. What is the solution? Submit your script by printing it out and attaching to the rest of your solutions.