# Smooth Band Termination in the Mass A=110 Region

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**Abstract.**
The systematics of smoothly terminating rotational bands based on proton $2p-2h$ excitations in the $A = 110$ mass region are presented. Terminating bands (or nearly so) based on this proton excitation have been found in nuclei ranging from $^{107}$In, up to $^{114}$Te, and possibly extending to $_{54}$Xe nuclei. The impressive agreement between experimental data and theoretical calculations is also presented. However, recently discovered structures based on proton $1p-1h$ excitations begin to show disagreement with theoretical calculations. These new bands are also discussed. The current and future directions of research into smooth band termination will be presented.

## INTRODUCTION

As far back as 1975, Bohr and Mottelson described a process in which the deformation of a rotational band decreases as the valence nucleons align their angular momenta with the rotation axis [1]. Once all the available valence nucleons have aligned, the rotational band terminates leaving a nucleus with a noncollective oblate shape. For some time the textbook case was $^{20}$Ne, in which the ground state sequence terminates in a fully aligned state at a relatively low spin of $I = 8\hbar$.

In order to observe this phenomenon in heavier nuclei, where a single deformed configuration may be followed over a large range of spin, one should look in nuclei near doubly-closed shells. This is due to several factors. First of all, such nuclei have small numbers of valence nucleons, resulting in terminating spins attainable in heavy-ion fusion-evaporation reactions. A second consideration is the number of available deformed configurations near the yrast line. A nucleus having a large number of valence nucleons will also have a large number of deformed configurations. This makes it unlikely that a single configuration can be observed over a large range of spin; rather, one configuration will be crossed by another and the non-yrast terminating states will not be observed. Near closed shells, however, particle-hole excitations across the shell gap are typically necessary to provide deformation. This tends to limit the number of near-yrast deformed configurations. Finally, nuclei having too few valence nucleons are not likely to possess stable deformed configurations. Calculations suggest that in the $A = 110$ mass region nuclei having 10-15 valence nucleons outside of the $^{100}$Sn doubly-closed shell are ideal candidates.

The classic doubly-magic nucleus is $^{100}$Sn, and third-generation $\gamma$-ray detector arrays have made it possible to study nuclei near $Z = N = 50$ to high spin. As a result, a number of spectacular examples of smooth band termination have been found in the $A = 110$ mass region. The first bands described as smoothly terminating were found in $^{109}$Sb [2] using data from the $8\pi$ array at Chalk River Laboratories; these results were later confirmed and improved [3] employing data from the Early Implementation of Gammasphere. Since then,

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many examples of smooth band termination have been documented in the mass $A = 110$ region, and we are now able to study systematic properties of such structures. In what follows we will present some of these systematics, and compare the results to theoretical calculations obtained with the Configuration Dependent Shell Correction method [4]. Some new results will also be presented, and some ideas for directions of future research as well.

## SMOOTH BAND TERMINATION: GENERAL CONSIDERATIONS

The experimental manifestation of smooth band termination can be seen in Fig. 1. In this figure the dynamic moments of inertia ($\mathcal{J}^{(2)}$) for three rotational bands in $^{109}$Sb are shown, as well as the energies of the levels versus spin, where the energies are plotted relative to a rotating liquid drop reference (henceforth this will be referred to as an $E - E_{LD}$ curve). Smooth band termination is evident as a decreasing $\mathcal{J}^{(2)}$ at the highest rotational frequencies seen in the top part of the figure, or as a characteristic parabolic shape of the $E - E_{LD}$ curve in the bottom half. Both demonstrate a loss of collectivity as the rotational bands reach the highest observed spins.

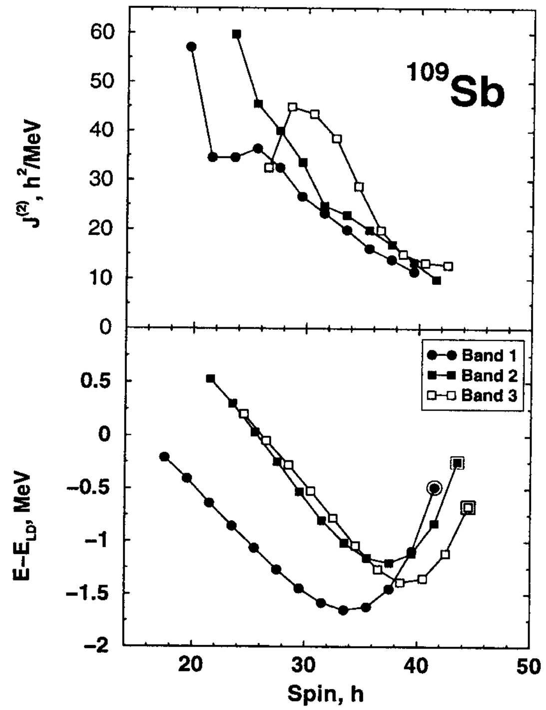


**FIGURE 1.** Top: The dynamic moment of inertia for three of the known terminating bands in $^{109}$Sb. Bottom: Level energies minus a rigid rotor reference for the same bands in $^{109}$Sb. In the bottom panel, the terminating states of the rotational bands are depicted by open symbols.

The theoretical interpretation of this phenomenon is provided by the Configuration Dependent Shell Correction (CDSC) method [4]. Calculations based on this method show that the band initially undergoes a collective (macroscopic) rotation, but microscopic considerations become important as the the valence nucleons gradually align their spins with the rotation axis. The shape of the nucleus changes from prolate to triaxial (and less deformed) as a result. Increasingly more energy is required to gain an additional $2\hbar$ alignment, resulting in the decrease in the $\mathcal{J}^{(2)}$, and the characteristic minimum in the $E - E_{LD}$ curve. Eventually all available valence particles outside the $^{100}$Sn core have aligned their spins. At this point the nucleus attains a noncollective oblate shape, and the rotational band must end, further rotation and higher spin being impossible without a change in configuration.

In general, the calculations do a remarkable job of reproducing the experimental data. An example is shown in Fig. 2. Here the $E - E_{LD}$ curves are shown for 3 different structures, the yrast bands in $^{108}$Sn, $^{109}$Sb, and $^{110}$Sb. The configurations shown are the [21,2] configurations in $^{108}$Sn, $^{109}$Sb, and the [21,3] configuration for $^{110}$Sb which have been assigned to these bands. (The notation $[p_1p_2,n]$ can be understood as follows: $p_1$ refers to the number of $g_{9/2}$ holes in the configuration, and $p_2$, $n$ refer to the number of occupied $h_{11/2}$ protons and neutrons, respectively. If one allows that the $d_{5/2}$ and $g_{7/2}$ orbitals are indistinguishable in these calculations, this notation is sufficient to completely describe the configuration of the system.) Note that these calculations do not predict the absolute energies of states, and pairing is not included in the calculations. It is therefore the shape of the curves above spin $I \sim 20\hbar$ which is important. In all cases, the calculations accurately reproduce the shapes and positions of the minima in the $E - E_{LD}$ curves. The calculations also reproduce the terminating spins, shown as open circles in the figure.

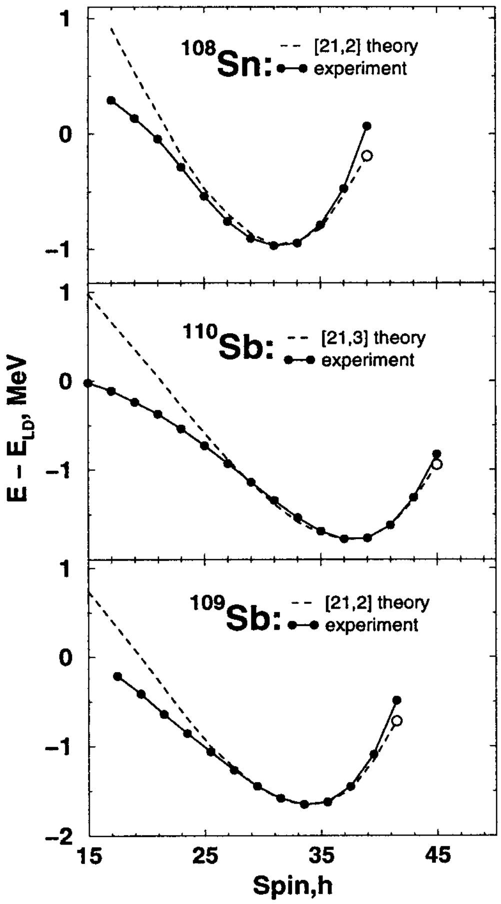


**FIGURE 2.** $E - E_{LD}$ curves for the yrast rotational bands in $^{108}$Sn, $^{109}$Sb, and $^{110}$Sb.

It is important to recognize that the decision that a band has reached termination can usually be made without recourse to the CDSC calculations. This is due to the fact that nuclei near the $^{100}$Sn doubly-closed shell have a limited number of valence nucleons. The configuration of a band can be determined by its properties at low spin, for example by the presence or absence of pair alignments, the spin sequence, and parity of the band. However, once the configuration of a band is known it is a simple application of the Pauli Principle and shell model to determine the terminating spin. For example, the $\mathcal{J}^{(2)}$ for Band 1 in $^{109}$Sb is shown in Fig. 1. The high value of the $\mathcal{J}^{(2)}$ at the lowest frequencies is suggestive of an $h_{11/2}$ neutron crossing; the peak at $I \approx 26\hbar$ can be ascribed to a $g_{7/2}$ proton crossing. Other structure information comes from the need for a $2p$–$2h$ proton excitation to supply a modest deformation of $\beta \approx 0.25$. Finally, the band is known from the

DCO ratios of the linking transitions to have negative parity and spins from $35/2\hbar$ to $83/2\hbar$. This information suggests the following complete configuration for the band: $\pi[(g_{9/2})^{-2}(g_{7/2},d_{5/2})^{2}h_{11/2}]\ \nu[(g_{7/2},d_{5/2})^{6}(h_{11/2})^{2}]$, or in our notation, [21,2]. Then, the terminating spin is determined through the Pauli Principle to be $83/2\hbar$: $\pi[(g_{9/2})_{8}^{-2}(g_{7/2},d_{5/2})_{6}^{2}(h_{11/2})_{5.5}]_{19.5}\ \nu[(g_{7/2},d_{5/2})_{12}^{6}(h_{11/2})_{10}^{2}]_{22}$. It can be seen from the bottom panel of Figs. 1 and 2 that this band extends to $83/2\hbar$ and has therefore reached the terminating state for this configuration.

## SYSTEMATICS OF 2p-2h BANDS

A series of experiments has uncovered numerous rotational bands which show characteristics of termination. The majority of these structures are based on $2p$–$2h$ proton excitations across the $Z = 50$ shell gap. Terminating bands with such $[2p_1,n]$ configurations have been identified in nuclei as light as $Z = 49$ $^{107}$In [5], up to $Z = 53$ $^{113}$I [6] and $Z = 52$ $^{114}$Te [7] and possibly beyond. In this section some systematics are presented, as well as a discussion of lifetime measurements which convincingly demonstrate the validity of the termination interpretation.

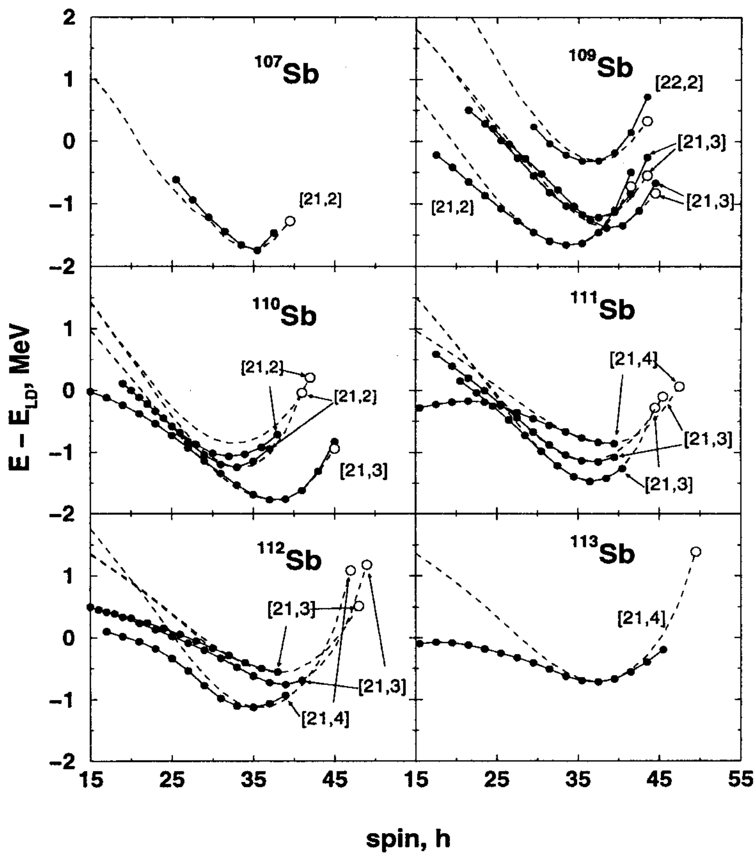


**FIGURE 3.** $E - E_{LD}$ curves extracted from several terminating bands in the $Z = 51$ Sb isotope chain. Solid lines connecting data points indicate the experimental measurements, and dashed lines indicate the theoretical calculations. The predicted terminating states are depicted as open circles at the ends of the dashed lines.

## A The Z=51 isotopes

Some of the best examples of smooth band termination can be found in the $_{51}$Sb nuclei. Rotational bands based on $3p2h$ proton configurations are known from $^{107}$Sb up to $^{119}$Sb. (Only in $^{108}$Sb have such structures not yet been identified.) In the lighter isotopes, several bands are observed to termination, or observed only 1 or 2 transitions shy of the expected terminating spin. In Fig. 3, several of the $E - E_{LD}$ curves for these bands are shown for nuclei from $^{107}$Sb to $^{113}$Sb. In addition, CDSC calculations for the relevant configurations are shown.

In all cases, the theoretical curves match the experimental measurements extremely well. The shapes and positions of the minima are well reproduced, and one can see that the bands in $^{109}$Sb and a single band in $^{110}$Sb are observed to their predicted terminating states. It is worth pointing out that a wide range of configurations are covered by these structures. Some, such as the band shown from $^{107}$Sb possess only a single $h_{11/2}$ proton in their configurations ([21,2]). Others possess two $h_{11/2}$ protons, for example the [22,2] band in $^{109}$Sb. A wide range of neutron configurations where two to four $h_{11/2}$ neutrons are occupied are also well reproduced by the calculations.

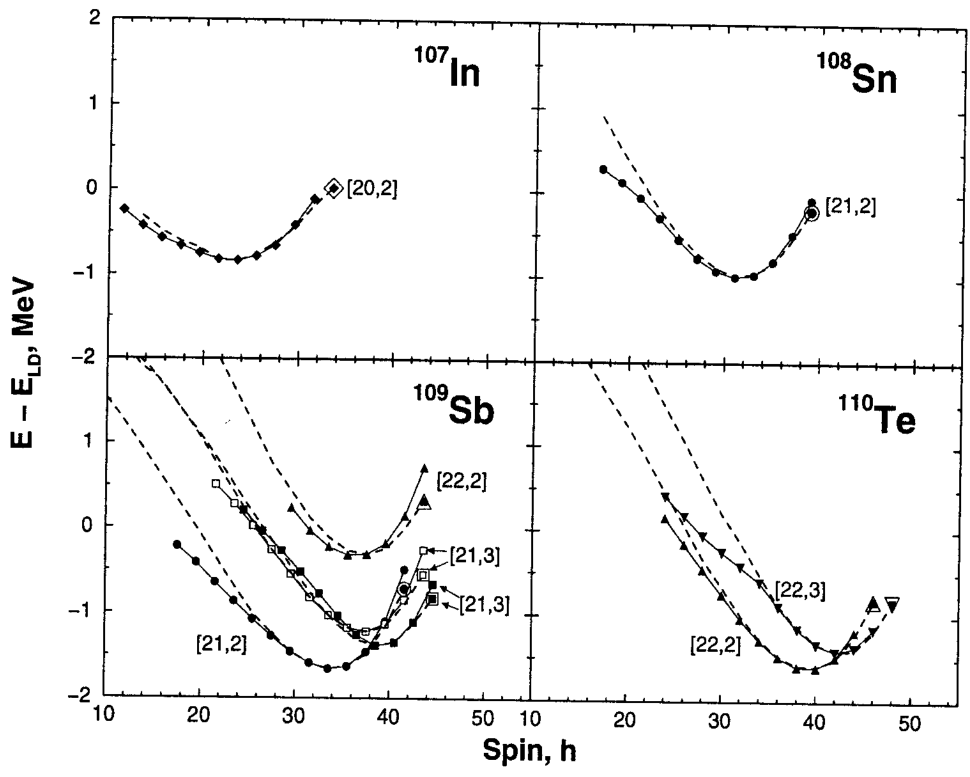


**FIGURE 4.** $E - E_{LD}$ curves extracted from several terminating bands in the $N = 58$ isotone chain. Solid lines connecting data points indicate the experimental measurements, and dashed lines indicate the theoretical calculations. The predicted terminating states are depicted as open circles at the ends of the dashed lines.

## B The N=58 isotones

The systematics of terminating structures as a function of $Z$ can also be studied. For this we choose the $N = 58$ isotones, where a number of nuclei have been found to possess terminating bands. The lightest of these is $^{107}$In, the heaviest $^{110}$Te. Both theoretical calculations and experimental data can be seen in Fig. 4.

As was the case for the $Z = 51$ isotopes, the calculations are very successful in reproducing the experimental results. In this case, a wide variety of proton configurations are accurately explained by the theory. In $^{107}$In, there are no $h_{11/2}$ proton orbitals occupied, implying a fairly modest deformation. In $^{109}$Sb, there are configurations involving one or two occupied $h_{11/2}$ proton orbitals. The results presented in this and the previous section demonstrate that the calculations are exceedingly successful in explaining the experimental results. This is true even though a large range of both proton and neutron configurations is represented. It is worth noting, however, that all the above results involve bands having a $2p{-}2h$ proton excitation. This will be discussed further in a subsequent section of this report.

## C Lifetime measurements in $^{108}$Sn and $^{109}$Sb

The agreement between theory and experiment presented in the previous sections is very impressive. But a definitive demonstration of band termination is provided by lifetime measurements performed on the nuclei $^{108}$Sn and $^{109}$Sb. If the band termination picture is in fact correct, the quadrupole moments of the bands should decrease steadily with increasing spin. The results of the lifetime measurements for the [21,2] band in $^{109}$Sb are shown in Fig. 5, taken from Ref. [8]. The transition quadrupole moments do in fact decrease with increasing spin. In addition, the theory nearly perfectly agrees with the experimental measurements. Measurements performed for two bands in $^{108}$Sn show similar agreement with theory [8].

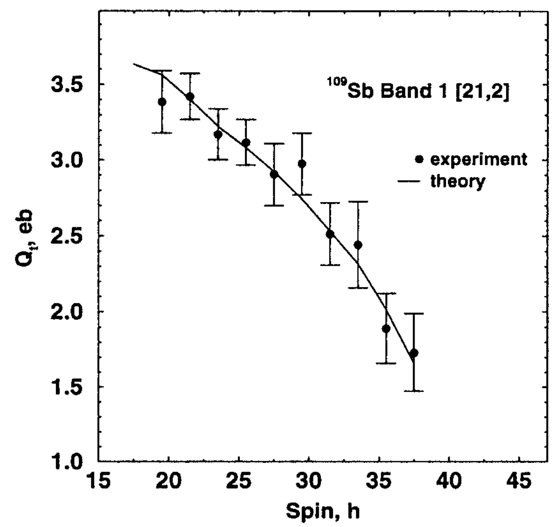


**FIGURE 5.** The measured and calculated transition quadrupole moments versus spin for the [21,2] band in $^{109}$Sb.

## NEW STRUCTURES BASED ON [11,n] CONFIGURATIONS

The terminating structures discussed so far all have a $2p{-}2h$ excitation across the $Z = 50$ shell gap in common. Since these nuclei are heavily influenced by the $Z = 50$ major shell gap, this excitation is necessary to supply deformation. This is accomplished by the $\beta$-driving influence of the $\pi g_{9/2}$ hole orbitals. However, it would be a good test of the theoretical calculations to compare them to rotational structures based on a different number of $g_{9/2}$ proton holes.

For some time it has been known that $1p{-}1h$ excitations can also provide deformation in the $_{51}$Sb nuclei [9]. For example, the odd-mass $^{107-119}_{51}$Sb nuclei are all known to possess strongly coupled rotational bands based on a proton $(g_{9/2})^{-1}(g_{7/2},d_{5/2})^2$ high-$K$ configuration. In our notation, these bands would be designated as [10,$n$], having a single $g_{9/2}$ proton hole, zero $h_{11/2}$ protons, and some number $n$ of $h_{11/2}$ neutrons. These bands are only modestly deformed however, and are typically not observed beyond $I = 20\hbar$, and hence not observed to termination. Also, since the CDSC calculations do not include pairing, they are not expected to be able to adequately describe the bands at these low spins.

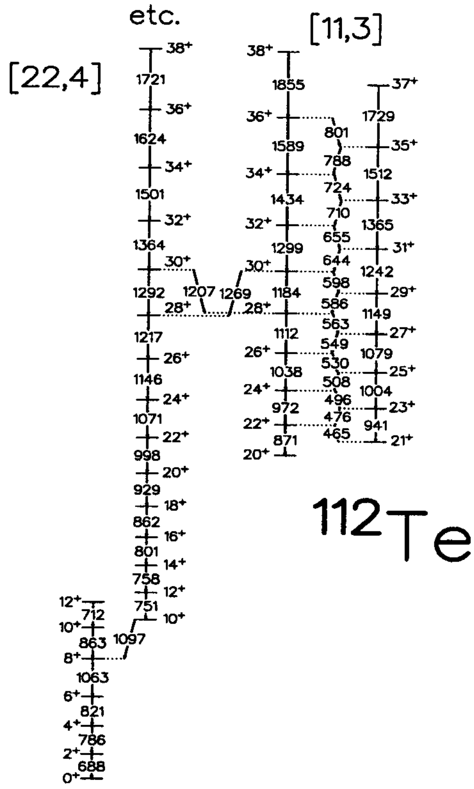


**FIGURE 6.** A partial level scheme of $^{112}$Te, showing the [22,4] band and the [11,3] signature partner bands. The [22,4] band extends to spins higher than shown in the figure.

Recently however, new structures have been observed in $^{110,112}$Te. These nuclei were produced in a Gammasphere/Microball experiment, using the $^{58}$Ni($^{58}$Ni,$\alpha$2p;4p) reactions, respectively. In $^{112}$Te, a pair of signature partner bands with strong $M1$ connecting transitions was found. This structure has been linked to low-spin states, and thus the spin sequences of these bands are known. Figure 6 shows the pair of bands, and the rather unconventional manner in which they have been linked to the low-spin states. As can be seen in the figure the two bands show a very small signature splitting, and strong $M1$ transitions linking the two signatures. The observed properties of these bands are consistent with the presence of a single high-$K$ proton $g_{9/2}$ hole in their configuration, and in fact the CDSC calculations predict a low-lying [11,3] configuration.

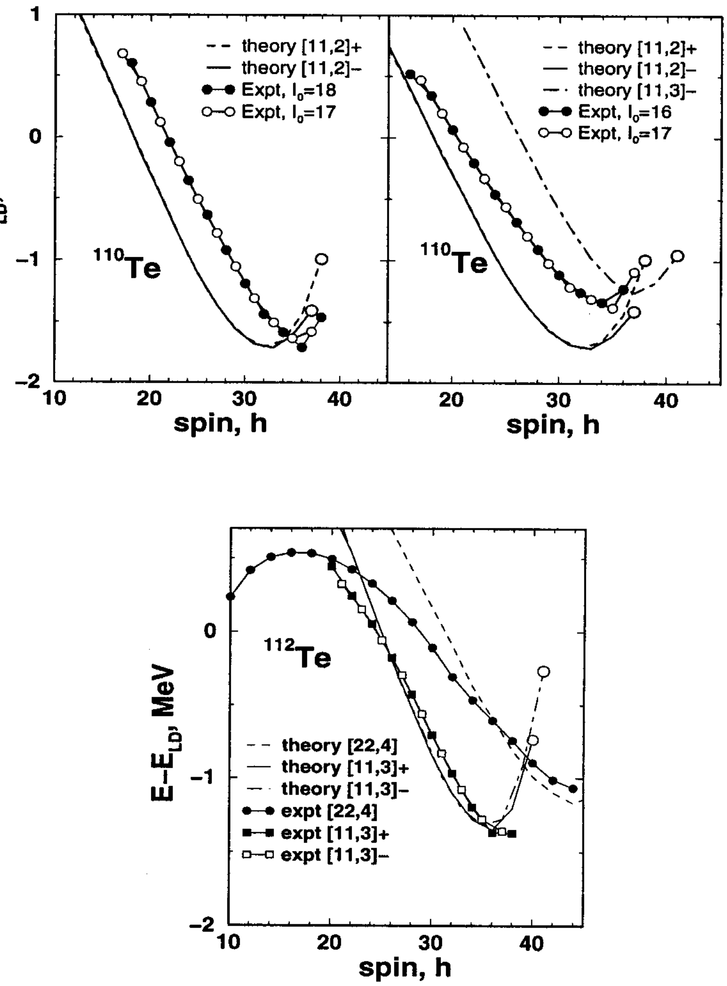


**FIGURE 7.** Plots of the energy minus a rigid rotor reference for the linked coupled bands in $^{110,112}$Te. Both possible spin assignments are shown for $^{110}$Te, as well as the possible configurations for each assignment.

In $^{110}$Te, two such pairs of signature partner bands were identified. One pair has been linked to the low-spin states, but the spins are uncertain by $1\hbar$ since the DCO ratios of the two linking transitions could not be reliably measured. Therefore the parity of the bands is uncertain as well. Interestingly, the CDSC calculations predict a single low-lying [11,2] and two [11,3] configurations in $^{110}$Te, all of which show a small signature splitting due to the single hole in the proton $g_{9/2}$ orbital.

The $E - E_{LD}$ plots for the linked coupled bands are shown in Fig. 7. For $^{110}$Te, both possible spin sequences are shown together with possible configuration assignments. The top left panel shows the band assuming the two linking transitions have $E2$ character. This fixes the parity of the band, and only the [11,2] configuration is possible. The top right panel of the figure shows the band assuming the linking transitions are dipoles. In this case positive or negative parity is possible, depending on whether the linking transitions are $M1$ or $E1$. The [11,2] and one of the [11,3] configurations are shown in the figure. Finally, the bottom panel shows the pair of coupled bands in $^{112}$Te along with calculations for the [11,3] configuration, which is the only plausible assignment. It should be pointed out that there is an additional [11,3] configuration in $^{110}$Te which is not shown in the figure. Also, the analysis and interpretation of $^{110}$Te is not finished at this date and should be considered preliminary.

In Fig. 7 one can see that the agreement between theory and experiment is not as good as it is for the bands having two holes in the proton $g_{9/2}$ orbital. In $^{112}$Te the calculations predict the minimum in the $E - E_{LD}$ plot to occur at least $2\hbar$ lower than is experimentally observed. The situation in $^{110}$Te is similar, the disparity between theory and experiment again being at least $2\hbar$. The disparity may be considerably higher however, depending on the true nature of the linking transitions.

The main difference between these bands and the low spin [10,$n$] bands known in $_{51}$Sb and $_{53}$I nuclei is the occupation of the proton $h_{11/2}$ orbital. This orbital drives the nucleus to larger deformations, allowing the [11,$n$] rotational bands to be observed to high spins and possibly all the way to termination.

## FUTURE DIRECTIONS AND CURRENT ACTIVITIES

The disagreement between theory and experiment with regard to the [11,$n$] bands known in $^{110,112}$Te points to one area where our knowledge is limited. It is worth pointing out that these are the only terminating (or nearly terminating) structures in this mass region which are not based on a $2p$–$2h$ excitation. Thus it is worthwhile to study further this discrepancy both theoretically and experimentally. In general, studies of bands having configurations [$p_1p_2$,$n$], where $p_1 \neq 2$ should be undertaken. At present a search for predicted [31,$n$] bands in $^{109}$Sb is currently underway [10] using data from a recent Gammasphere/Microball/FMA experiment.

Preliminary results in $^{117,119}$Xe [11] suggest the possibility of terminating bands having [02,$n$] configurations. However, the current evidence does not allow a firm assignment, since there are competing [22,$n$] configurations which must be considered as well. Planned lifetime experiments will be necessary to distinguish between the [22,$n$] and [02,$n$] configurations. Finally, searches continue for additional [$2p_1$,$n$] bands, mainly at the extremes of the mass region. Recent data are being searched for the presence of such bands in $^{116,118}$Xe, where particle-hole excitations across the $Z = 50$ shell are not necessary for the presence of significant deformations. Searches closer to $^{100}$Sn continue as well, with recent data expected to show deformed structures in $^{108}$Sb and $_{49}$In isotopes, among others.

Another area of future research is the linking of the terminating bands to states of known spin. The three bands shown in Fig. 2 are actually the only bands which are known to terminate *and* which are linked to the lower-spin states. (The linked signature partner bands in $^{110}$Te may represent a fourth, depending on the multipolarity of the linking transitions.) Clearly it is difficult to search for discrepancies with theory when spin assignments are uncertain, and this situation should be remedied.

Finally, recent theoretical work has suggested that the neutron $d_{3/2}$ and $s_{1/2}$ orbitals may also be important in determining the terminating spin of many of these bands [12]. However, the energies of these orbitals are not known very precisely. It is likely that they will become more important in nuclei having a relatively large number of neutrons ($^{113}$Sb, for example) near the terminating spin. Thus it is also important to observe the rotational bands to (near) termination in nuclei closer to the neutron midshell.

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