WUSTL FA2020.L24.Math.5051.01 - Measure Theory and Functional Analysis I
Fall Term 2020

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I am a professor in the Mathematics department since 2019. I was a professor at University of Virginia and a postdoc at Brown before coming to WUSTL. I got my PhD in 2012 from Indiana University. My fields of expertise are Real, Fourier and Harmonic Analysis, and Partial Differential Equations, especially those related to fluids.

Class time/place TR 8:30A-9:50A STL TIME

The course will be fully online with links from the Canvas webpage. Lectures organized as follows

- Tue 8:30A-9:50A - Online synchronous lecture on ZOOM, recorded and made available to course participants on Youtube portal after the class. During the Tuesday lecture I will introduce the main topic of the week and cover most of the theoretical aspects of that topic.
- Thu 8:30A-9:50A - Online synchronous lecture and discussion on ZOOM, recorded and made available to course participants on Youtube portal after the class. During the Thursday lecture, I will answer questions on course material and previously assigned problems. Then we will discuss examples and applications of the theory. On occasion, a problem related to current topics will be presented/assigned and the students will be encouraged to work in groups and present their solution strategies to the other groups.
- lecture notes and slides will be provided beforehand so that you can follow along the presentation.
- a course diary is mantained on the Canvas webpage, with topics and examples covered each day.

Office and Office Hours STL TIME - On ZOOM portal

- Monday and Wednesday: 4-5pm (open hours).
- Tuesday: 10:00-11:00am (open hours)
- Thursday: 10:00-11:00am (open hours), 4-5pm (help session - I discuss homework problems upon request)
- otherwise by appointment (email 15 minutes in advance)

Prerequisites Undergraduate real analysis (limits and continuity, differentiation and integration, series, uniform convergence, etc.) at the level of Browder, Pugh, Krantz, or Rudin's *Principles*. Linear Algebra (vector spaces, linear mappings, matrices, determinants, etc.), and some knowledge of set theory and topology of metric spaces. The first week of class is designed for the student to self-verify his effective ability to stay in, and benefit from, the class itself.

Course textbook and topics covered I will use my textbook

From Real to Harmonic Analysis, previous version available at
https://sites.wustl.edu/francescodiplinio/teaching/

with each Chapter being distributed to the students on Canvas, before the corresponding lectures: the content will be slightly different from the version above, which was used in past editions of my courses. The past version should only be used for reference or for a bird's eye view. This year's version integrates some of the exercises in the development of the theory.

We will cover Chapters 1 to 7, plus possibly additional topics from Chapters 8-12 according to students' suggestions. The chapter numbers here refer to the lecture notes.

0 Brief review of elementary set theory, cardinality, real numbers, topology of metric spaces. The axiom of choice.

1 Outer measures. The Lebesgue outer measure via dyadic cubes. σ-algebras, measures and measurability. The Lebesgue and Borel σ-algebras via Caratheodory's construction

2 Abstract integration. Integration of positive functions and the monotone convergence theorem, integration of complex functions and the dominated convergence theorem.
3 Elementary theory of L^p spaces: Hölder, Minkowski and Jensen inequalities
4 Elementary theory of Hilbert spaces. Representation theorems and applications to PDE. Orthonormal bases of $L^2(\mathbb{R})$.
5 Complex measures, absolute continuity, the Radon-Nykodym theorem. Conditional expectations and L^2-martingales.
6 Differentiation of measures. The dyadic Hardy-Littlewood maximal functions, L^1-martingales. Functions of bounded variation.
7 Product measures, convolution, approximation of identity.
8 Additional topics: Hausdorff measure and Hausdorff dimension. Radon measures. Fourier series. Weighted L^p spaces and Muckenhoupt weights. Hilbert transform, dyadic shifts. Other additional topics can be proposed by the students.

Other textbooks of interest which I have used in the preparation of my lecture notes include

- *Real analysis: modern techniques and applications*, by G.B. Folland
- *Real and complex analysis*, by W. Rudin
- *An Introduction to measure theory*, by T. Tao (available online for free)
- *Measure, integrals and martingales*, by R. Schilling
- *Real analysis...* by E. Stein and R. Shakarchi

Course web page **https://wustl.instructure.com/courses/49583** on Canvas. Used to gain access to syllabus, detailed course outline, assignments, homework and exam grades, lecture notes, and course announcements.

Homework There will be weekly homework assignments, collected weekly (usually on Thursdays) for grading purposes. The homework is designed to complement and enrich the theory presented in class. A large part of the homework problems will be discussed during the Thursday interactive. You will also be able to see a summary of your homework grades on the course page. Since solutions will be provided shortly afterwards the deadline, no extensions will be granted as a general policy. **Homework is submitted online through Canvas**.

Exams *Pending dispositions of the Director of Graduate Studies concerning qualifying exams for the graduate program* there will be two midterm exams and a final exam. All exams are off-line. All three exams will be composed of two portions, roughly of equal weight. The first part will consist of questions related to the theory we have seen in class, usually with short answers. The second component will consist of 2-3 problems of length comparable to the homework assignments. The midterm exams will be posted on the tentative dates below and students will have a week of time to complete the assignment. Submission will be through Canvas. The final exam will be posted at the end of the course, with due date that of the final exam, Jan 6, 2021.

Grading

MIDTERMS	35%	dates **Thu Oct 15, Thu Nov 19**
HOMEWORK	25%	(lowest hw score dropped)
FINAL	40%	TAKEHOME

Other factors such as in-class participation and improvement over time may impact positively your final grade.

Grading table

A+	A	A-	B+	B	B-	C+	C	C-	D	F
>100	[90,100]	[85,90)	[80,85)	[75,80)	[70,75)	[65,70)	[60,65)	[55,60)	[50,55)	[0,50)

Submission guidelines Homework submission is electronic on Canvas portal or email to the instructor.

(1) Write your name clearly at the top of every page. Put the problems in order, indicating clearly what you have skipped.
(2) Turn in assignments in time **ON OR BEFORE DUE DATE**. Write neatly. If your homework is too messy, the grader might decide, with the instructor's consent, not to grade it.
(3) **Sourcing solution material from the Internet without mentioning the sources or collaborating without specifying the names of the collaborators is considered blatant academic plagiarism**. Any instance of academic plagiarism will result in a 0% in the corresponding assignment/exam and discussed with the appropriate University Academic Officers. You can talk to each other about any of homework problems, but when you write up the problems to be handed in, you must work alone.

Questions and office hours Mathematical questions are appreciated and encouraged any time during the class. Please use the office hours as much as possible for additional clarifications, occasional homework help. I will also be answering email questions. Expect a 1 hour reply time on weekdays-work hours and a 12 hour reply time on nights/weekends.