Labor Unions and the Labor Wedge: A Macroeconomic Perspective

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Abstract
The measured labor wedge, defined as the difference between marginal product of labor and marginal rate of substitution, is relatively stable from 1940s to 70s, and declines secularly from 1980s onwards. This paper aims to investigate the effect of a particular labor market institution, labor union, on labor wedge. Labor unions command a wage premium, which invites job application queues and job rationing in the unionized sector. This waiting values of unionized jobs creates a wedge between wages and households’ willing to work. We provide sectoral evidence that supports a union-wedge connection in the manufacturing sector. A quantitative model which features two labor market, one competitive and the other unionized, is developed to estimate the effect of union power on labor wedge. Based on our quantitative results, approximately 20% of the decline in labor wedge from 1970s to 2000s is accounted for by the decrease in union densities.

Keywords: Labor unions, labor wedge
JEL code: E02, J42, J51

1 Introduction

General equilibrium based macroeconomic models build on two pillars: household’s present value utility maximization, and firm’s profit maximization. Taken prices as given, the household and

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firm optimally make consumption/production and work/hiring decisions. In equilibrium, demand equates supply, i.e. the regular marginal condition in each market holds, and markets clear. The deviation from equilibrium conditions, i.e. the discrepancy between the marginal condition between households and firms, wedges as labeled in the business cycle literature (Chari, Kehoe, and Mcgrattan, 2007), provides a natural metric to investigate market (in-)efficiency.

In this paper, we focus on labor wedge, the wedge between the marginal rate of substitution of consumption for leisure \(MRS_{c,n}\), and marginal productivity of labor \(MP_n\). Without distortions, labor market efficiency requires that

\[ MRS_{c,n} = MP_n \]

The labor wedge measures the violation of this condition, which could be a result of inefficiency in either the labor market or other markets. Our paper investigates the effect of a specific form of labor market institution, labor unions, on the labor wedge. The main intuition lies in the following formula

\[ MRS_{c,n} = W^c < W^u \]

with \(W^c\) and \(W^u\) denoting wages in the competitive market and wages controlled by labor unions. From a macro perspective, the marginal rate of substitution, or people’s willingness to work, equals to the wage level in the competitive market. Labor unions, however, demand a wage premium and creates a wedge between unionized wages and the marginal rate of substitution\(^1\).

Figure 1 presents the long run trend of the labor wedge in U.S. from the third quarter of 1947, the earliest date relevant data is available, to the third quarter of 2007. The series is truncated at 2007 to avoid the effect of the Great Recession.\(^2\) The labor wedge is relatively stable from 1940s to 70s, and has declined continuously since around 1980. The Wald test for structural break with unknown break point shows that 1977-Q3 is the break point\(^3\). On the other hand, the union density in U.S., measured as the percentage of union members in total employment, follows a similar trend: relatively stable until around 1980, and declines after that\(^4\). The correlation coefficient between the

\(^1\)The labor wedge \(\frac{MP}{MRS} = \frac{MP}{W} \cdot \frac{W}{MRS}\). In this paper, there is no wedge between \(MP\) and \(W\). In Cobb-Douglas production function, Marginal productivity is proportional to the average productivity, \(AP\), and the trend of \(\frac{AP}{W}\) reflects the behavior of the labor share. While the latter only declines at a small magnitude after 1980s, the relatively large decline in the labor wedge comes mainly from the household side wedge.

\(^2\)See section II for details of measurement. It is well known that the labor wedge has a countercyclical pattern, and rises in recessions. The measured labor wedge increases from 2007 to 2010 (see Appendix for the overall pattern.

\(^3\)See appendix for details. The time series trend for labor wedge is sightly increasing before 1977-Q3 and decreasing after that.

\(^4\)See appendix Figure 11 for the time series pattern of union density
two series, measured in a yearly frequency, from 1947 to 2007 is 0.75 and statistically significant.

The labor wedge has been a focus of research especially in one branch of the business cycle literature. Rotemberg and Woodford (1991) defines markup as the wedge between marginal product of labor and wage, which is the first component of labor wedge, with the second the gap between wage and marginal rate of substitution, and documents that the markup has a countercyclical pattern. Hall (1997) decomposes the labor market equilibrium condition, i.e. marginal product of labor equals marginal rate of substitution in order to investigate the sources of fluctuation of hours. The residual in their decomposition equation\(^5\) is essentially labor wedge. Gali, Gertler, and Lopez-Salido (2006) uses the gap between marginal product of labor and the marginal rate of substitution\(^6\) as a measure of economic efficiency, and uses it to calculate the efficiency cost of business fluctuations. In an influential paper, Chari, Kehoe, and McGrattan (2007) builds four reduced-form wedges, corresponding to productivity, labor, investment, and government expenditure, into the stochastic Neoclassical growth model, and finds that efficiency and labor wedges account for most of the fluctuations over business cycles.

Chari, Kehoe, and McGrattan (2007) has generated a series of research that has the labor wedge as the central focus. Several papers, in particular, examined the effect of labor market frictions on the labor wedge. Shimer (2009) reviews the labor wedge literature and suggests that search friction combined with real wage rigidities are promising explanations for endogenous cyclical labor wedge. Pescatori and Tasci (2001) augments the benchmark RBC model with a labor market featuring search frictions, in which employed workers and firms bargain over both wage and

\(^5\)Equation 3.2 in paper
\(^6\)The inefficiency gap defined this way equals to the negative labor wedge
working hours. According to their results, the search friction itself doesn’t not cause variation in labor wedge over the business cycle since the effect of searching friction is completely absorbed by wage instead of working hours. Cheremukhin and Restrepo-Echavarria (2014) decomposes the labor wedge, associated with search frictions, and finds that fluctuations in matching efficiency account for 90% of variations in the labor wedge. We share the same focus of labor wedge as this line of literature. Our paper, however, concerns the long run trend, instead of cyclical patterns, of the labor wedge.

This paper is also closely related to a small but growing set of papers that incorporate union into macroeconomic general equilibrium models. Observing the inverted-U-shape of union density and U-shape of inequality in U.S. over the 20th century, Dinlersoz and Greenwood (2013) develops a model of union in a general equilibrium framework. In their model, A continuum of firms with varied productivities hire skilled and unskilled labor in a competitive labor market. Only unskilled labor can be unionized. Bearing an organizing cost, unions target firms with relatively high productivities. In that framework, skilled-biased technological change (or unskilled-biased in early 20th century) generates simultaneously the inverted-U-shape of Union density and U-shape of inequality in U.S. in the 20th century. Rudanko and Krusell (2016) models a monopoly union in an economy with search frictions. Rather than determined by bargaining between firms and workers, the wage is set unilaterally by a universal-coverage union which values the welfare of both employed and unemployed workers. While making wage proposals, the Union takes their effect on job creation into account. Efficiency is achieved in all but the initial periods in the case the union fully commits to proposed wages. Without full commitment, employment is lower than efficient levels in both the short and long run. Taschereau-Dumouchel (2015) investigates the effect of Unions and the threat of unionization on wage distribution. Non-unionized firms respond to the Union threat by hiring more anti-union high skilled workers, and less low-skill ones who supports unionization. That strategy endogenously compresses the wage distribution in non-unionized firms under the assumption of decreasing return to scale for each level of skills.

Perhaps the closest paper to ours is Cole and Ohanian (2004). That paper is motivated by the fact that, during the Great Depression, consumption is significantly below trend in 1939, comparing to its 1929 level, while leisure time (non-working time) and wage are much above trend. These combined, don’t satisfy the marginal condition of labor supply, \( \frac{c}{\ell} = w \) in the case of log utilities\(^7\). Cole and Ohanian (2004) argues that the increasing influence of labor unions in 1930s is responsible for this divergence. In their framework, there are two sectors, one competitive and the other unionized.

\(^7\)Cole and Ohanian (2004) doesn’t use the term ‘labor wedge’ explicitly in their paper. However, the gap between the left and right hand side of Household’s marginal condition is the first component of labor wedge
Insiders in the unionized sector determine the size of union and the wage premium each period. Outside workers has to wait to be rationed a position in order to enter the unionized sector. The rationing creates a wedge between wage and household’s marginal rate of substitution, with the gap reflecting the value of waiting. Different from Cole and Ohanian (2004), in our paper where unions control both wage and employment, in our model, Unions decides wages, and unionized firms optimally post vacancies, taking wages as given.

The rest of paper is organized as following: section 2 provides a detailed description of the measurement of labor wedge; sector level evidence that supports the connection between union power and labor wedge is presented in section 3. Section 4 lays out the full model. The quantitative results of model are explored in section 5. Section 6 concludes.

2 Measurement of the Labor Wedge

The section details on the measurement of the labor wedge. The procedure here follows Shimer(2009) which itself adopts the approach commonly used in the labor wedge literature. The economy features a representative household and firm. Time is discrete and infinite. The representative household’s problem is to maximize lifetime utility given by

$$\sum_{t=1}^{\infty} \beta^t (\log c_t - \frac{\gamma \epsilon}{1 + \epsilon} n_t^{\frac{1}{1+\epsilon}})$$

where $\beta$ is the discount rate, and $c_t$ and $n_t$ denotes consumption and working hours respectively. $\gamma$ measures the disutility of working, while $\epsilon > 0$ is the Frisch elasticity of labor supply. The household respects its period budget constraint

$$c_t + k_{t+1} - (1 - \delta)k_t \leq r_t k_t + w_t n_t$$

Denote $\lambda_t$ the Lagrange multiplier for the budget constraint. First order conditions for consumption and labor supply are given by

$$\frac{1}{c_t} = \lambda_t$$

$$\gamma n_t^{\frac{1}{\epsilon}} = \lambda_t w_t$$

---

8It is assumed that the (dis)utilities from consumption and working are separable, with the former in the form of log, and the latter CRRA. This specific functional form is to ensure the existence of a balanced growth path.
A combination of the two first order conditions above leads to

\[ w_t = \gamma c_t n_t^{\frac{1}{2}} \]  \hspace{1cm} (1)

Assume a Cobb-Douglas production technology. The representative firm’s problem is standard, rent capital and hire labor in spot markets and maximizes period profit

\[
\max_{k_t, n_t} A_t k_t^\alpha n_t^{1-\alpha} - r_t k_t - w_t n_t
\]

The firm’s optimal condition corresponding to its choice of labor reads

\[ w_t = (1 - \alpha) \frac{y_t}{n_t} \]  \hspace{1cm} (2)

where \( y_t = A_t k_t^\alpha n_t^{1-\alpha} \) denotes total products.

Combining the labor supply and demand, i.e. (1) and (2), yields the standard labor market equilibrium condition. Define the labor wedge as \( \tau_t \equiv \log\left( \frac{MP_n}{MRS_{c,n}} \right)^9 \), and it satisfies

\[
\tau_t \equiv \log\left( \frac{MP_n}{MRS_{c,n}} \right) = \log \frac{1 - \alpha}{\gamma} + \log \frac{y_t}{c_t} - (1 + \frac{1}{\epsilon}) \log n_t \]  \hspace{1cm} (3)

Data from the following sources are utilized to measure the labor wedge

- \( n_t \), hours time employment-population ratio, both taken from Cociuba, Prescott and Ueberfeldt (2012)\(^{10}\)

- \( c_t \), nominal personal consumption expenditure, and \( y(t) \), nominal GDP, from NIPA; used to produce consumption-income ratio on the right hand side of equation (3).

- The value of \( \frac{\gamma}{1-\alpha} \), acting as a shift coefficient, does not affect the trend over time, which is the focus of the current paper. This value is chosen such that the average labor wedge equals 0.4.

For the baseline case, we pick 1 as the value for the Frisch elasticity of labor supply. The measured labor wedge is presented in Figure 1. Note that from the definition here, a deviation from the Cobb-Douglas technology, e.g. a decreasing-return-to-scale production function, does not change

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\(^9\)Note that the labor wedge can also be measured as \( \tau_t = 1 - \frac{MRS_{c,n}}{MP_n} \). The two measures give very similar long run trends. We choose the measurement above since it is in logarithm and unit free.

\(^{10}\)We have tried to use instead average hours time employment-labour force ratio and average hours only. The decrease from 1970s onwards is smaller in both case. In the paper, however, we follow the literature (Shimer(2009) etc.) and use average hours times employment-population ratio.
the trend of the labor wedge.

The representative household’s marginal condition reads

\[ \gamma cn^\epsilon = w \]

or equivalently,

\[ \gamma \frac{cn^{\epsilon+1}}{y} = \frac{wn}{y} \]

The right hand side is the labor share, which declines but at a relatively small magnitude. In this paper, we take it as constant, and focus on the behavior of the left hand side. Figures 5 and 6 in appendix provide the trend of working hours and Consumption-Income ratios in U.S. from 1947 onwards. Both demonstrate an increasing trend from 1970s to 2000s. Table 1 lists their values for 1970s and 2000s.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours-1</td>
<td>24.56</td>
<td>28.04</td>
</tr>
<tr>
<td>Hours-2</td>
<td>39.83</td>
<td>42.29</td>
</tr>
<tr>
<td>C-Y ratio-1</td>
<td>60.46</td>
<td>67.08</td>
</tr>
<tr>
<td>C-Y ratio-2</td>
<td>77.32</td>
<td>81.89</td>
</tr>
</tbody>
</table>

Both increases in hours and the consumption income ratio increases the measured MRS, i.e. \( \gamma \frac{cn^{\epsilon+1}}{y} \). Table 2 shows the relative change in MRS from 1970s to 2000s, for different values of \( \epsilon \), and under different measures. We use 0.17 as the benchmark.

<table>
<thead>
<tr>
<th>( \epsilon )</th>
<th>Measure-I</th>
<th>Measure-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.34</td>
<td>0.17</td>
</tr>
<tr>
<td>0.5</td>
<td>0.44</td>
<td>0.24</td>
</tr>
<tr>
<td>2</td>
<td>0.29</td>
<td>0.15</td>
</tr>
</tbody>
</table>

3 Sectoral Level Evidence

This section provides sectoral level evidence for the union-wedge connection. We have constructed a database which covers 75 manufacturing sectors from 2005 to 2014\(^{11}\). Data on union density

\(^{11}\)We choose this interval since Annual Survey of Manufactures only provide publicly accessible data from 2005 onwards.
comes from the Current Population Survey (i.e. CPS). Union density is measured as the fraction of union members in total wage and salary earners in each sector. The industrial classification in CPS is based on 2000 Census Industry Code. One industry in 2000 Census Code might correspond to one or several 3, 4, 5 or 6 digit NAICS sectors. In total, there are 75 Manufacturing sectors according to 2000 Census Code. Table 3 presents the relevant summary statistics.

Table 3: Summary statistics: Union density

<table>
<thead>
<tr>
<th>Year</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>13.4%</td>
<td>7.8</td>
<td>1.3</td>
<td>35.2</td>
<td>75</td>
</tr>
<tr>
<td>2009</td>
<td>11.3</td>
<td>8.1</td>
<td>0</td>
<td>32.5</td>
<td>75</td>
</tr>
<tr>
<td>2014</td>
<td>10</td>
<td>8.2</td>
<td>0</td>
<td>42.3</td>
<td>75</td>
</tr>
</tbody>
</table>

Data Source: CPS and Annual Survey of Manufactures.

The union density is in decline in the U.S. As can be seen from the table above, over the last decade and across manufacturing sectors, union density decreases from 13.4% to 10%. On the other hand, there are still relatively big variations in union density across sectors. The standard deviation of union density is stable at around 8, a relatively big number. Even in 2014, union members in the highest manufacturing sector accounts for over 40% of total employment. The lower bound of union density reaches its lowest possible value, 0%, in 2014. The relatively large variation in union density provides us the opportunity to investigate its effect on labor wedge.

To calculate sectoral labor wedge, we extracts data on value added and hours from Annual Survey of Manufactures (i.e. ASM). ASM uses NAICS codes to define sectors (3, 4, 5, and 6 digit). We merge the two data sets using the industry crosswalk tables from the census website. To incorporate multiple sectors, we adopt the following utility function.

\[
U(c_t, n_{i,t}) = \log(c_t) - \gamma \frac{\epsilon}{1 + \epsilon} \sum_{i=1}^{N} \frac{n_{i,t}}{1 + \epsilon}
\]

where \(c_t\) denotes aggregate consumption\(^{12}\). Consumption for disaggregated sectors would be very difficult, if not impossible, to obtain. The advantage of using the utility function above is that, only aggregate consumption is needed to calculate sectoral level labor wedge. Labor wedge for sector \(i\) in year \(t\) is then measured as

\[
\tau_{i,t} \equiv \log \frac{MP_{n_{i,t}}}{MRS_{c,n_{i,t}}} = \log \frac{\alpha y_{i,t}}{n_{i,t}} - \log \frac{\gamma n_{i,t}^{1/\epsilon}}{1/c_t}
\]

\[
= \log y_{i,t} - (1 + 1/\epsilon) \log n_{i,t} - \log(c_t) + constant
\]

\(^{12}\)See appendix for the derivation of the utility function specified here.
$y_{i,t}$ and $n_{i,t}$ denote sectoral level value added (deflated by GDP deflator) and employee hours respectively\textsuperscript{13}, both from Annual Survey of Manufactures, 2005-2014. $c_t$ denotes personal consumption expenditure in 2009 dollar.

Merging the two datasets produces a panel dataset for 75 sectors and over 10 years\textsuperscript{14}. To test whether a higher union density leads to a larger labor wedge, we run the following regression

$$\tau_{i,t} = \beta_0 + \beta_1 \times \text{union}_{i,t} + \beta_2 \times X_i + \beta_3 \times D_t + \beta_4 \times Z_{i,t} + \epsilon_{i,t}$$

- $D_t$: year dummies, 2005-2014;
- $X_i$: sectoral controls, including mean and standard deviation of annual growth rates (in value added and employment) from 2005 to 2014 for each sector;
- $Z_{i,t}$: ratio of production to non-production employees.

The baseline value of the Frisch elasticity of labor supply is set to be 1. Table 4 presents the baseline regression results.

### Table 4: Dependent var.: Labor wedge ($\epsilon = 1$, in log)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>union density</td>
<td>0.011**</td>
<td>0.027***</td>
<td>0.030***</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.006)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>$\frac{\text{prod.emp.}}{\text{non-prod.emp.}}$</td>
<td>-0.109***</td>
<td>-0.111***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.017 )</td>
<td>(0.030)</td>
<td></td>
</tr>
<tr>
<td>$\Delta y - \text{mean}$</td>
<td>-0.067***</td>
<td>-0.068***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>$\Delta y - \text{st.dev.}$</td>
<td>0.025***</td>
<td>0.026***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.004)</td>
<td></td>
</tr>
<tr>
<td>Year Dummy</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.006</td>
<td>0.08</td>
<td>0.10</td>
</tr>
<tr>
<td>Obs.</td>
<td>753</td>
<td>753</td>
<td>753</td>
</tr>
</tbody>
</table>

Note: $^* p < 0.1; ^{**} p < 0.05; ^{***} p < 0.01$. Data Source:

\textsuperscript{13}calculated as $\frac{\text{total employment}}{\text{production workers}} \times \text{production worker hours}$

\textsuperscript{14}For 2012, 2013, and 2014, one new sector, 1190, is added into the census code. That is why in later regressions, there are 753 instead of 750 observations.
Column (1) shows the univariate regression result, with sectoral labor wedge and union density as the dependent and independent variables, respectively. It can be seen that the effect of union density on labor wedge is positive and significant at 5% confidence level. The coefficient values at 0.011, which means that a 1% increase in union density leads to about 1% increase in labor wedge.

In the second column, several variables are added to the regression to control for sectoral level heterogeneities. In particular, we add the ratio of production to non-production workers, average growth rate of sectoral value added, and standard deviation of growth rate of value added. The fraction of production workers is correlated with union density if production workers are more likely to be unionized. It might also affect the aggregate efficiency as represented by the marginal conditions if the two set of workers are subject to different compensation rules in reality. Whether a sector is high-growth or low-growth, and whether the growth rates are relatively stable over time, are also taken as controls.

The positive and significant effect of the union density is robust to additional controls, and the coefficient increases to 0.027. This effect is stronger than that in the single variable regression. In addition, labor wedge tends to be lower in sectors with higher fraction of production workers, higher average growth rate, and where growth rates are relatively stable over time. It is well known that labor wedge has a strong cyclical pattern. To control for cyclical fluctuations, we add year dummies into the regression. The results are presented in the third regression. All coefficients from column (2) are robust.

The value of the Frisch elasticity of labor supply, $\epsilon$, is critical to determine the response of labor supply to the change in wage rate. In our baseline regression, we set that $\epsilon = 1$. In the RBC literature, a value as large as 4 has been employed to match the relatively large effect of wage changes on aggregate labor supply. Micro evidence, however, usually supports a value of $\epsilon$ smaller than 1. To verify the robustness of the baseline results, we vary the value of $\epsilon$, and present regression results for $\epsilon = 0.5$ and $\epsilon = 2$ in Table 5. It shows very similar results as baseline cases (i.e. $\epsilon = 1$), with the only exception that for $\epsilon = 0.5$ and single variable regression, the effect of union density becomes insignificant.

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15Note that labor wedge is measured in log. We don’t use log for union density since it is already in percentage and unit-free.

16We have tried to measure the union density as percentage of wage earners whose wage contract are covered by unions’ collective bargaining. The results presented above are robust to this alternative measure. We have also tried to use unemployment rate, instead of year dummies, to control for cyclic fluctuations, and found similar patterns. The qualitative results remain if mean and standard deviation of growth rates in employment (instead of valued added as in the baseline case) are used as sectoral level controls.
Table 5: Dependent var.: Labor wedge (in log)

<table>
<thead>
<tr>
<th></th>
<th>$\epsilon = 0.5$</th>
<th>$\epsilon = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I-(1)</td>
<td>I-(2)</td>
</tr>
<tr>
<td>union density</td>
<td>0.005</td>
<td>0.035***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>prod.emp.</td>
<td>-0.032</td>
<td>-0.034</td>
</tr>
<tr>
<td>non-prod.emp.</td>
<td>(0.010)</td>
<td>(0.053)</td>
</tr>
<tr>
<td>$\Delta y - \text{mean}$</td>
<td>-0.15***</td>
<td>-0.15***</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>$\Delta y - \text{st.dev.}$</td>
<td>0.056***</td>
<td>0.057***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Year Dummy</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.0004</td>
<td>0.10</td>
</tr>
<tr>
<td>Obs.</td>
<td>753</td>
<td>753</td>
</tr>
</tbody>
</table>

Note: *p < 0.1; **p < 0.05; ***p < 0.01. Data Source: CPS and Annual Survey of Manufactures, 2005-2014.

4 The Model

This section presents the model, which incorporates union into an otherwise standard Neoclassical growth model. The economy features two sectors, one competitive and the other unionized. In the competitive sector, wage and employment are determined by the usual supply and demand. Wages in the unionized sector are controlled by a monopoly union. The union values both the wage premium and membership size. The higher wage in the unionized sector invites an application queue, and jobs are rationed by the labor union. The labor rationing process and associated waiting value create a wedge between wage and worker’s willing to work.

Time is discrete and infinite. Within the representative household, some household members work in the competitive sector, and some search and work for unionized firms. As standard, household owns capital and makes investment decision. The objective of the representative household is to maximize lifetime utility, i.e.

$$
\max \sum_{t=0}^{\infty} \beta^t (u(c_t) - \nu(n_t))
$$

(4)

c_t and n_t are consumption and non-leisure time, respectively. There are a continuum of firms, with index $i \in [0, 1]$ in the economy. Firms locate in $[0, \phi], 0 < \phi < 1$, are unionized, and the $(\phi, 1]$
range behave competitively. Denotes $n_t(i)$ the employment of firm $i$. Since there would be wage premiums for unionized works, outsiders have to queue, and wait to be rationed a union position. The rationing process models the practice of membership restrictions and organization costs (such as certification elections) of labor unions in the real world. Denote $q_t(i)$, which is determined in equilibrium, the probability of getting a union job, total working hours is given by

$$n_t = \int_0^\phi n_t(i) \frac{q_t(i)}{q_t(i)} di + \int_\phi^1 n_t(i) di$$

(5)

Household income includes wage income of workers in both competitive and unionized sectors, capital rental income, and firms’ profits in both sectors. The budget constraint for the household is

$$c_t + i_t = \int_0^\phi (w_t^u n_t(i) + \Pi_t(i)) di + \int_\phi^1 (w_t^c n_t(i) + \Pi_t(i)) di + r_t \int_0^1 k_t(i) di$$

(6)

where $w_t^u$ and $w_t^c$ are wages in the unionized and competitive sector, and $\Pi_t(i)$ profits. Denote $k_t = \int_0^1 k_t(i) di$ the aggregate capital stock, and $i_t$ investment, the following law of motion for capital holds

$$k_{t+1} = (1 - \delta) k_t + i_t$$

(7)

The goal of the representative household is then to maximize (4), subject to constraints (5), (6), and (7).

The final goods is produced by combining the composite goods of the competitive and unionized sectors, $y_t^c$ and $y_t^u$, according to

$$Y_t = (y_t^u \rho + y_t^c \rho)^\frac{1}{\rho}$$

$\rho$ governs the elasticity of substitution between sectoral goods, which itself is an aggregation of intermediated goods

$$y_t^u = (\int_0^\phi y_t(i) \zeta di)^\frac{1}{\zeta}; \quad y_t^c = (\int_\phi^1 y_t(i) \zeta di)^\frac{1}{\zeta}.$$  

$\zeta$ determines elasticities of substitution among firms within each sector. We use the final goods as numeraire and normalize its price to 1. Denote $p_t(i)$ the price of the intermediate goods produced by firm $i$, the final goods producers’ problem is

$$\max Y_t - \int_0^\phi p_t(i) y_t(i) di - \int_\phi^1 p_t(i) y_t(i) di$$

(8)

In both sectors, intermediate goods producing firms take factor prices as given, and optimally make
hiring decisions. Firms in both sectors solve the following standard optimization problem

\[ \Pi_t(i) \equiv \max_{k_t(i), n_t(i)} p_t(i) F^i(k_t(i), n_t(i)) - r_t k_t(i) - w_t n_t(i) \]  

(9)

Note that \( w(t) = w^u(t) \) for \( 0 < i < \phi \), and \( w(t) = w^c(t) \) if \( \phi < i < 1 \). The differences between the competitive and unionized sectors lie in how wages are determined. In the competitive sector, wages are determined by marginal conditions. Unionized wage is controlled by a monopoly union, whose objective is to

\[ \max_{w^u_t} \sum_{t=0}^{\infty} \beta^t G(w^u_t - w^c_t, \int_0^\phi n_t(i) di) \]  

(10)

That is, the union values both the wage premium, \( w^u_t - w^c_t \), and the size of union members. The union takes into account the hiring behavior of firms while making wage proposals. Denote \( D(w^u_t, \bullet) \) the labor demand function of unionized firms, and the union respects the following constraint

\[ n^u_t = D(w^u_t, \bullet), \quad \text{for } 0 < i < \phi \]  

(11)

In addition, firms in the unionized sector should maintain a nonnegative profit

\[ \Pi_t(i, w^u_t, \bullet) \geq 0, \quad \text{for } 0 < i < \phi \]  

The dynamic competitive equilibrium of the economy consists of a sequence of wages in both sectors, \( \{w^u_t\}_{t=0}^{\infty} \) and \( \{w^c_t\}_{t=0}^{\infty} \), interest rates, \( \{r_t\}_{t=0}^{\infty} \), prices of intermediate goods, \( \{p_t(i)\}_{t=0}^{\infty} \), employment in both sectors, \( \{n^c_t\}_{t=0}^{\infty} \) and \( \{n^u_t\}_{t=0}^{\infty} \), job rationing probability, \( \{p_t\}_{t=0}^{\infty} \), capital employed in both sectors, \( \{k^c_t\}_{t=0}^{\infty} \) and \( \{k^u_t\}_{t=0}^{\infty} \), intermediate and final goods, \( \{y_t(i)\}_{t=0}^{\infty} \) and \( \{Y_t\}_{t=0}^{\infty} \), consumption \( \{c_t\}_{t=0}^{\infty} \), investment \( \{i_t\}_{t=0}^{\infty} \), and capital stock \( \{k_t\}_{t=0}^{\infty} \), such that

1. Given wages and interest rates, the representative households maximizes lifetime utilities, (4), subject to (5) – (7), final goods producers maximize profits in (8), and intermediate goods producing firms in both sectors maximize profits and solve (9);

2. The monopoly union maximizes its objective, (9), subject to (10) and (11);

3. Markets Clear\(^{17}\)

   - Capital Market
     \[ \int_0^\phi k_t(i) di + \int_0^1 k_t(i) = k_t, \quad \forall t. \]

\(^{17}\)Note that the labor market clearing condition is implicitly assumed by employing the same notation for labor demand and supply.
4.1 A Static Case

In this subsection, we present a static version of the full model and focus on the symmetric equilibrium, which illustrates the main mechanism at work. All notations remain the same as the previous section. The representative household’s problem is

\[
\max_{n^c,n^u} u(c) - \nu(n)
\]

s.t. \[
c = \phi w^u n^u + (1 - \phi) w^c n^c + \phi \Pi^u + (1 - \phi) \Pi^c
\]

\[
n = \frac{\phi n^u}{q} + (1 - \phi) n^c
\]

Firms in both the competitive and unionized sectors take wage as given and optimally make hiring decisions

\[
\Pi^i = \max_{n^i} p(i) F(n^i) - w^i n^i
\]

Assume the union’ objective function is of Cobb-Douglas\(^{18}\). The union proposes wages while taking account for the fact that firms in the unionized sector hire workers optimally. Denote \(D(w^u)\) firms’ demand for unionized workers at the wage rate \(w^u\). The problem of unions is

\[
\max_{w^u} (w^u - w^c)^\eta n^u^{1-\eta}
\]

s.t. \(n^u = D(w^u, \cdot)\)

To see the effect of unions on the labor wedge, note that the two marginal conditions, w.r.t. working time in competitive \((n^c)\) and unionized \((n^u)\) sectors, are

\[
u' * w^c = \nu'
\]

\[
u' * w^u = \nu \frac{1}{q}
\]

It is frictionless in the competitive sector. There exists a labor wedge, given by \(\frac{1}{q}\), in the unionized sector. Note that the labor wedge is created by the wage premium and its associated labor rationing process in the unionized sector. Denote \(\phi = \frac{\phi n^u}{\phi n^u + (1 - \phi) n^c}\) the fraction of employees that work in

\(^{18}\text{Dinlersoz and Greenwood (2003) share the identical objective function of labor unions.}\)
the unionized sector, and $1 - \tilde{\phi}$ in the competitive sector. We have the following relation

$$W = \tilde{\phi}w^u + (1 - \tilde{\phi})w^c = \left[\tilde{\phi} \frac{1}{q} + (1 - \tilde{\phi})\right] * MRS$$

The size of the wedge is determined by the exogenous parameter $\phi$, endogenous variables $n^u, n^c,$ and $q$.

We parameterize the economy by choosing the following functional forms

$$u(c) = \log(c); \quad \nu(n) = \gamma \frac{\epsilon}{1 + \epsilon} n^{1+\epsilon}; \quad F(n) = n^\alpha$$

The labor demand function for unionized firms, $D(n^u)$, is

$$D(n^u) = \left(\frac{w^u}{\alpha p}\right)^{\frac{1}{\alpha - 1}}$$

The union’s problem becomes

$$\max_{w^u} \quad (w^u - w^c)^\eta n^{u1-\eta}$$

$$\text{s.t.} \quad n^u = \left(\frac{w^u}{\alpha p}\right)^{\frac{1}{\eta - 1}}$$

Note that the union takes wage in the competitive sector as given. It is straightforward to solve the optimization above and obtain

$$w^u = \frac{1 - \eta}{1 - \eta + (\alpha - 1)\eta}w^c$$

We restrict attention to the case $1 - \eta + (\alpha - 1)\eta > 0$. Denote $\Delta \equiv \frac{1 - \eta}{1 - \eta + (\alpha - 1)\eta}$. Since $\alpha - 1 < 0$, it follows that $\Delta > 1$, and $w^u > w^c$. Combining this relation with the marginal conditions on the household side leads to

$$\frac{1}{q} = \frac{1 - \eta}{1 - \eta + (\alpha - 1)\eta}$$

The intuition for this equation is, if the wage premium of unions is higher, then the queue is longer outside of the labor union, which implies a lower matching probability.

The union density, i.e. the fraction of union members in total employment, is

$$\tilde{\phi} = \frac{\phi n^u}{\phi n^u + (1 - \phi)n^c} = \frac{\phi}{\phi + (1 - \phi)\Delta^{\frac{1}{1-\alpha}}}$$

and the wedge between the average wage, $W \equiv \tilde{\phi}w^u + (1 - \tilde{\phi})w^c$, and the marginal rate of substitution, is $\tilde{\phi}\Delta + (1 - \tilde{\phi})$. 
Note that the way we calculate the marginal product of labor is
\[ MPL = \frac{\alpha Y}{N} \equiv \frac{\alpha \phi p^u y^u + (1 - \phi)p^c y^c}{\phi n^u + (1 - \phi)n^c} \]
\[ = \tilde{\phi}\alpha \frac{p^u y^u}{n^u} + (1 - \tilde{\phi})\alpha \frac{p^c y^c}{n^c} \]
\[ = \tilde{\phi} w^u + (1 - \tilde{\phi})w^c \equiv W \]

The size of the labor wedge, defined as \( \log_{\frac{MP}{MRS}} \) is therefore proportional to \( \tilde{\phi}\Delta + (1 - \tilde{\phi}) \).

### 5 Quantitative Analysis

This section implements quantitative analyses. We choose the following functional forms
\[ u(c) = \log(c); \quad \nu(n) = \gamma \frac{\epsilon}{1 + \epsilon} n^{\frac{1 + \epsilon}{\epsilon}}; \quad F^j(k, n) = A^j(k^{1-\alpha}n^{\alpha})^\chi, j = u, c. \]

In principle, productivities in the unionized and competitive sectors can be different from each other\(^{19}\).

The objective function of the union is chosen to be\(^{20}\)
\[ G(w^u_t - w^c_t, n_t(i)) = (w^u_t^{\theta_1} - w^c_t^{\theta_2})^\eta \left( \int_0^\phi n_t(i) di \right)^{1 - \eta} \]

The union values the wage premium, \( w^u_t - w^c_t \), but might have different elasticities towards increases in \( w^u_t \) and decreases in \( w^c_t \).

Several parameters are standard especially in the business cycle literature, and we choose widely used values for these parameters. One period in the model corresponds to one year in the data. We choose the annual depreciation rate \( \delta \) to be 10\%. The value of the discount rate is set as \( \beta = 0.96 \) such that the annual net return of investment equals 4\%. In the baseline calibration, we set the Frisch elasticity of labor supply as \( \epsilon = 1 \). The value of \( \gamma \), the weight on working disutility in household’s preference, is chosen such that the household spend one third of hours on working\(^{21}\).

\(^{19}\)Dinlersoz, Greenwood, and Hyatt (2016) documents that unions generally target more productive firms.

\(^{20}\)A more general utility function is used since the wage premium is constant under a Cobb-Douglas objective function.

\(^{21}\)This is for the year 1970. Total working hours are affected by the size of the unionized sector, as measured by \( \phi \).
On the production side, we follow Buera, Kaboski and Shin (2011), and choose the span of control parameter in production function $\chi = 0.79^{22}$. The value of $\alpha$ is calibrated to be 0.81 to match a labor share of 64%, as in Kydland and Prescott (1982).

Without loss of generality, we normalize productivity in the competitive sector, $A^i, \phi < i < 1$ (or simply $A^c$), to be 1. Due to the existence of decreasing return to scale, a small difference in wages translates into a relatively large gap in employment. That is, employment in firms in the competitive sector is significantly higher than unionized employment even if the union wage premium is moderate$^{23}$. Table 6 lists the distribution of firm size groups among union members and regular workers. The median union worker works in firms with more than 1000 persons in both 1992 and 2007, while the total number of persons in the firm the median non-union worker works in is between 100 and 500.

<table>
<thead>
<tr>
<th>Table 6: Distribution of workers among firm size groups</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>&lt;10</td>
</tr>
<tr>
<td>--------------------------------</td>
</tr>
<tr>
<td>1992</td>
</tr>
<tr>
<td>Non-Union</td>
</tr>
<tr>
<td>Union Member</td>
</tr>
<tr>
<td>Union Coverage</td>
</tr>
<tr>
<td>2007</td>
</tr>
<tr>
<td>Non-Union</td>
</tr>
<tr>
<td>Union Member</td>
</tr>
<tr>
<td>Union Coverage</td>
</tr>
</tbody>
</table>

Data Source: CPS. Firm size indicates the total number of persons who work in the firm. The universe is workers who work for wage and salaries in the private sector.

We increase the productivity of the unionized firms, $A^i, 0 < i < \phi$ (or $A^u$), to be consistent with the fact that union members, on average, command a wage premium and as well work in larger firms than non-union workers. In the baseline calibration, we set $A^u = 1.23$ such that, under a 20% of union wage premium, the average employment size of unionized firms is 20% higher than that in the competitive sector. A 26% of union density, i.e. fraction of union members in total employment, requires 22.6% of firms to be unionized$^{24}$.

---

$^{22}$There are different calibrations for this span of control parameter. For example, the parameter is set as 0.85 in Midrigan and Xu (2014).

$^{23}$See Appendix for a detailed derivation. It can be seen there that the employment ratio of competitive to unionized firms is a function of $\frac{A^u}{A^c}$ and the wage ratio $\frac{w^u}{w^c}$.

$^{24}$Results under different values of $A^u$ are presented in appendix.
The parameter $\zeta$ determines the elasticity of substitution among firms within each sector. This parameter is widely used in the New Keynesian business cycle literature. We pick the value of $\zeta = 0.83$, the benchmark value used in Christiano, Eichenbaum, and Evans (CEE, 2005). For the parameter $\rho$, which governs the elasticity of substitution between sectoral goods, note that the following relation holds under the aggregate production function, $Y_t = (y^{u\rho}_t + y^{c\rho}_t)^{\frac{1}{\rho}}$:

$$\log \frac{p^u_t y^u_t}{p^c_t y^c_t} = \frac{\rho}{\rho - 1} \log \frac{p^u_t}{p^c_t}.$$ 

The relation between relative expenditure and relative price between the competitive and unionized sectors from data can therefore be used to estimate the parameter $\rho$. Table 11 in appendix lists the average union density, ranked from low to high, for 2 digit sectors in NAICS. Sectors that have a union density higher than 15% are labeled as unionized sectors, and the rest competitive sectors. We then run a single variable regression with relative expenditure share and relative price of unionized sectors, comparing to competitive sectors, as dependent and independent variables, respectively. Each year is one observation, and there are 34 years (1983-2017). The regression results yields a value of $\rho = 0.5$ for all economy.

We normalize $\theta_2 = 1$. The Union’s optimization problem implies a relation between wages in competitive and unionized sectors, $\varphi(w^u, w^c, \theta, \cdot)$. We use this relation from data to estimate $\theta_1$, and obtain the value $\theta_1 = 1.26$. We jointly calibrate parameters $\gamma, \phi,$ and $\eta$, to target a working hour of about $1/3$, a wage premium of 20%, and a union density of 21%. All moments refer to their 1977 values. Table 7 lists the results.

<table>
<thead>
<tr>
<th>Table 7: Calibration moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moments</td>
</tr>
<tr>
<td>working hour</td>
</tr>
<tr>
<td>wage premium</td>
</tr>
<tr>
<td>union density</td>
</tr>
</tbody>
</table>

Table 8 summarizes the calibration results.

---

**Footnotes:**

25 Similar methods has been employed in Cole and Ohanian (2004), and Acemoglu and Guerrier (2008), to estimate the parameter governing the elasticity of substitution between sectors.

26 Defined this way, the value added share of unionized sectors is 43.1%, and 33.6% in the private economy in 1987.

27 We calculate price index for each sector as the weighted average of prices of industries within that sector, with value added used as weights.

28 As robustness check, we have tried to use $\rho = 0.1$ and found robust the main results regarding changes of the labor wedge.

29 See Table 12 in appendix.
Table 8: Summary of Calibration

<table>
<thead>
<tr>
<th>Para.</th>
<th>Description</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.96</td>
<td>Annual 4% net return of inv.</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate of K</td>
<td>0.1</td>
<td>Annual 10% depreciation</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Disutility of work</td>
<td>4.04</td>
<td>$\frac{1}{3}$ working time</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Elasticity of labor supply</td>
<td>1</td>
<td>Standard</td>
</tr>
<tr>
<td>$A^c$</td>
<td>Productivity in competitive sector</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$A^u$</td>
<td>Productivity in unionized sector</td>
<td>1.23</td>
<td>Relative employment size</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Span-of-control in prod. fun.</td>
<td>0.79</td>
<td>Buera et al. (2011)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Labor’s share</td>
<td>0.81</td>
<td>Labor share in NIPA</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Elasticity of sub. within sectors</td>
<td>0.83</td>
<td>CEE(2005)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Elasticity of sub. btw. sectors</td>
<td>0.5</td>
<td>Expenditure-price elasticity</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>Union’s elasticity w.r.t. $w^c$</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>Union’s elasticity w.r.t. $w^u$</td>
<td>1.26</td>
<td>See text</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Union’s weight on wages</td>
<td>0.38</td>
<td>Union wage premium</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Fraction of unionized firms</td>
<td>0.18</td>
<td>Union density in 1977</td>
</tr>
</tbody>
</table>

Note: see text for details.

To see the implications on the labor wedge, note that, the labor wedge is originally from the gap between average wage and the marginal rate of substitution, due to the wage premium commanded by the union and the associated job rationing. The fraction of employment in the unionized sector is measured as $\tilde{\phi} \equiv \frac{\phi u}{\phi u + (1 - \phi) c}$, and the economy wide wage is given by

$$W = \tilde{\phi} w^u + (1 - \tilde{\phi}) w^c$$

Denote $MPL \equiv \alpha \chi \frac{Y}{N}$ the economy wide marginal productivity of labor. As in the static case, it follows that

$$MPL = W = lw * MRS$$

where

$$lw = \frac{1}{q} \frac{1}{\phi} + (1 - \tilde{\phi})$$

$q$ is the probability of a successful job rationing in the unionized sector, which is determined endogenously. We treat each year as a steady state, and vary the value of $\phi$ across years, by targeting union densities in data. Table 9 lists results for several selected years.

Table 9: Model moments
Based on previous calculations, the labor wedge in data has declined 0.17 log points from 1970s to 2000s. For the same period, our model, by matching the magnitude of union density, implies a decrease of labor wedge by 0.024 log points, which accounts for 14% of the overall decline in data.

Union premiums means a higher wage in the model. In reality, however, a large proportion of union premium does not come in the form of wages, but in things such as a flexible working hours, and larger retirement benefits. To capture these non-wage benefits, we have tried an alternative calibration to target a wage premium of 30%, instead of 20%. Table 10 lists main results under these alternative calibration.

<table>
<thead>
<tr>
<th>Period</th>
<th>φ</th>
<th>w^u</th>
<th>w^c</th>
<th>n^u</th>
<th>n^c</th>
<th>Union D.</th>
<th>LW (in log)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970-79</td>
<td>0.21</td>
<td>1.05</td>
<td>0.86</td>
<td>0.38</td>
<td>0.32</td>
<td>23.95%</td>
<td>5.13%</td>
</tr>
<tr>
<td>1980-89</td>
<td>0.16</td>
<td>1.04</td>
<td>0.85</td>
<td>0.40</td>
<td>0.33</td>
<td>18.22%</td>
<td>4.01%</td>
</tr>
<tr>
<td>1990-99</td>
<td>0.13</td>
<td>1.03</td>
<td>0.84</td>
<td>0.41</td>
<td>0.35</td>
<td>14.49%</td>
<td>3.23%</td>
</tr>
<tr>
<td>2000-07</td>
<td>0.11</td>
<td>1.02</td>
<td>0.83</td>
<td>0.43</td>
<td>0.37</td>
<td>12.26%</td>
<td>2.76%</td>
</tr>
</tbody>
</table>

Under this alternative calibration, the model generates 3.73% log points of decrease in the labor wedge, which accounts for 22% of the decline observed in data.

6 Conclusion

Labor wedge, the difference between marginal product of labor and marginal rate of substitution, is a reduced form representation of deviations from competitive market and allocation efficiency. The measured labor wedge in U.S. since world war II has been relatively stable until early to middle 1970s, and steadily declined for the recent 3-4 decades. In this paper, we propose that the existence of labor unions, and their power to influence wages and employment in the labor market affect the behavior of labor wedge. Overall, union density follows a similar stable-then-decline trend. We further assemble a panel dataset of 75 manufacturing sectors from 2005 to 2014, and provide empirical support for the union-wedge connection.
We developed a dynamic general equilibrium model to quantify the effect of union power on labor wedge. In our model economy, there are two sectors, competitive and unionized. Wages in the competitive market are determined competitively, while the unionized wages are controlled by a monopolistic union. The union commands a wage premium, which invites job application queues and job rationing in the unionized sector. This job rationing process creates a wedge between wages and households’ willing to work. According to our results, approximately 20% of the decline in labor wedge from 1970s to 2000s can be accounted for by the decreases in union density.

The long run trend of labor wedge in general, and its decline since 1970s in particular provide a summary of market efficiency and its overall change. The decline of labor wedge, or the improvement of market efficiency, might also be a result of changes beyond declining power of labor unions. One example is the decrease of tax rate, which also create a wedge between marginal conditions. We leave explorations along these directions for future research.

7 References


21. Taschereau-Dumouchel, Mathieu, ”The Union Threat”, *working paper*, 2015

8 Appendix

Union density, 1930-2007

Figure 2: Union Density

Labor Wedge, 1947-2011

Figure 3: Labor Wedge, 1947-2011
Structural break for the labor wedge series. The break point: 1977-Q3

Figure 4: Structural break test for the labor wedge series
**Working hours** are measured in 2 ways. In the baseline measure, it equals to average weekly working hours (from CPS) times the ratio of employment to working age population (i.e. population from 16 to 64 years old). The ratio of employment to labor force is used in the second measure.

**Figure 5: Working Hours**

![Graph showing working hours over time](image)

Note: weekly hours equal average weekly hours of workers times the ratio of employment to working age population, whereas Weekly Hours−2 employs the ratio of employment to labor force

**C-Y ratio** is also measured in 2 ways. It is the share of Personal Consumption Expenditure of GDP in the baseline measure. As an alternative, consumption is measured as personal consumption expenditure plus government consumption expenditure.

**Figure 6: Consumption-Income Ratio**

![Graph showing consumption-income ratio over time](image)

Aggregate consumption is the sum of personal consumption and government consumption, the latter is government expenditure in consumption and investment times the ratio of private investment in total private expenditure in consumption and investment.
Labor wedge with multiple sectors

Consider a more general utility function with different sectors. Let utility from consumption be

$$\max \Sigma_{i=1}^{N} \theta_i \ln c_i$$

s.t.

$$\Sigma_{i=1}^{N} p_i c_i = C$$

$$\Sigma_{i=1}^{N} \theta_i = 1$$

where $C$ denotes total expenditure on all consumption goods. Denote $\lambda$ the Lagrangian multiplier for budget constraint. The first order conditions are given be

$$\frac{\theta_i}{c_i} = \lambda p_i \quad i = 1, ..., N$$

$$\Sigma_{i=1}^{N} p_i c_i = C$$

It follows that $\lambda = C$. Then aggregate utility from consumption is

$$\Sigma_{i=1}^{N} \theta_i \ln c_i = \Sigma_{i=1}^{N} \theta_i \ln \left( \frac{\theta_i C}{p_i} \right)$$

$$= \ln C + \text{const.}$$

Though widely used in economics, we should caution that these results follow directly from the Cobb-Douglas utility function across different consumption goods. Deviation from C-D utility function might generate an aggregate utility function where $\frac{\partial U}{\partial C}$ depends on the vector of prices, $\{p_i\}_{i=1}^{N}$, which complicates the calculation of sectoral labor wedge.
Employment in competitive and unionized sector  

Labor and capital demand in unionized sector is

\[ p^u (1 - \alpha) x * A^u * (k^u)^{(1-\alpha)x-1} * (n^u)^{\alpha x} = r \]

\[ p^u x^* A^u * (k^u)^{(1-\alpha)x} * (n^u)^{\alpha x - 1} = w^u \]

It follows from the two optimality conditions that labor demand is given by

\[ n^u = (\chi A^u)^{\frac{1}{1-x}} * \left( \frac{1 - \alpha}{r} \right)^{\frac{(1-\alpha)x}{1-x}} * \left( \frac{\alpha}{w^u} \right)^{1/(1-x)} \]

Similarly, we can solve the labor demand in the competitive sector, which is

\[ n^c = (\chi A^c)^{\frac{1}{1-x}} * \left( \frac{1 - \alpha}{r} \right)^{\frac{(1-\alpha)x}{1-x}} * \left( \frac{\alpha}{w^c} \right)^{1/(1-x)} \]

The ratio of employment in the unionized sector to that in the competitive sector is

\[ \frac{n^u}{n^c} = \frac{(A^u)^{\frac{1}{1-x}} (w^c)^{1/(1-x)}}{(A^c)^{\frac{1}{1-x}} (w^u)^{1/(1-x)}} \]

Note that the exponent \(\frac{1-(1-\alpha)x}{1-x} > 1\), if \(A^u = A^c\), a relatively moderate difference in \(w^c\) and \(w^u\) translates into a large difference in the \(n^u/n^c\) ratio.
### Table 11: Union density across sectors

<table>
<thead>
<tr>
<th>Naics</th>
<th>Industry</th>
<th>Union density, %</th>
<th>Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>52</td>
<td>Finance and Insurance</td>
<td>2.0</td>
<td>c</td>
</tr>
<tr>
<td>54</td>
<td>Professional, Scientific, and Technical Services</td>
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<td>11</td>
<td>Agriculture, Forestry, Fishing and Hunting</td>
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<td>81</td>
<td>Other Services (except Public Administration)</td>
<td>3.2</td>
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</tr>
<tr>
<td>72</td>
<td>Accommodation and Food Services</td>
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<td>55</td>
<td>Management of Companies and Enterprises</td>
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<td>53</td>
<td>Real Estate and Rental and Leasing</td>
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<td>42</td>
<td>Wholesale Trade</td>
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<td>Administrative Support and Waste Management ...</td>
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<td>Retail Trade</td>
<td>7.5</td>
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<td>71</td>
<td>Arts, Entertainment, and Recreation</td>
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<td>Health Care and Social Assistance</td>
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<td>21</td>
<td>Mining, Quarrying, and Oil and Gas Extraction</td>
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<td>Information</td>
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<tr>
<td>31-33</td>
<td>Manufacturing</td>
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<tr>
<td>23</td>
<td>Construction</td>
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<td>Public Administration</td>
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<td>Utilities</td>
<td>33.3</td>
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<td>61</td>
<td>Education Services</td>
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<td>48-49</td>
<td>Transportation and Warehousing</td>
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Note: Union density is the average from 1983 and 2007.

### Table 12: Union Wage Premium

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<td>2.87</td>
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<td>Premium</td>
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Data Source: CPS, Western and Rosenfeld (2011). Listed are hourly wages in log.