

Demand-Led Growth and Accommodating Supply*

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This version May 2019

Forthcoming, *Cambridge Journal of Economics*

Abstract

We model demand-led growth with endogenous adjustment of labor supply and productivity to accommodate the demand-led path, reconciling Harrod's warranted rate of demand growth with supply. The model delivers a range of growth paths and unemployment rates rather than a single "natural rate." Theoretically, the steady-state growth path may be dynamically stable or unstable, but a detailed empirical calibration favors stability. We show analytically that if demand dynamics are stable, supply will converge to demand. While a minimum unemployment rate ultimately imposes a supply constraint on growth, empirical results show that a wide range of steady-state growth rates are feasible across different demand regimes. The results explain how economies can become trapped with low growth due to weak demand or fiscal austerity and suggest policy responses to stagnant demand.

Key words: demand-led growth, autonomous demand, super-multiplier, aggregate demand and supply reconciliation, secular stagnation.

JEL codes: E12, O40, E32

*We thank Olivier Allain, Fabio Freitas, Marc Lavoie, John McCarthy, Thomas Palley, Irina Panovska, Franklin Serrano, Mark Setterfield, Peter Skott, and Ricardo Summa along with the participants in the 2017 Berlin FMM conference, the 2018 Brazilian Keynesian Association conference, the 2019 meeting of the Eastern Economics Association, seminars at the University of Bergamo (Italy), the University of Missouri-Kansas City, and the Federal University of Rio de Janeiro, and two anonymous referees for helpful comments and discussion. We are grateful for financial support from the Institute for New Economic Thinking and the University of Bergamo. We dedicate this paper to the memory of our late colleague, co-author, and friend Edward Greenberg whose creative thinking helped establish the foundation on which the work presented here is based.

1. Introduction

The question of what determines medium-term macroeconomic growth looms large in developed countries. A decade after the financial crisis burst the US housing bubble growth has not been anywhere near sufficient to recover the pre-crisis trend. In Europe, growth has fallen short of expectations (or expectations are continually revised downward) with fiscal austerity as a likely culprit in many countries. Growth in the Japanese economy has underperformed for more than two decades.

In a market economy, profit-seeking firms hire workers and produce because they expect to sell output, an expectation grounded in actual demand. Of course, demand growth is not sufficient to generate expansion. Growth also requires the resources and technology to produce more output. In a very general sense, therefore, growth depends on the dynamics of both demand and supply.

Mainstream growth models, going back at least to Solow (1956), treat the demand and supply sides asymmetrically. Demand is assumed to accommodate to supply automatically beyond possible short-run fluctuations. Growth models based on the neoclassical synthesis perspective assume (often implicitly) that market adjustments of nominal wages and prices close any gap between aggregate demand and a supply-determined output path. More recently, in so-called “New Keynesian” models, wise monetary policy reconciles demand and supply which again implies that growth beyond the short run is driven by the supply side only, with the dynamics of aggregate demand pushed entirely offstage.¹

We challenge the view that economic growth beyond the short run can be explained by the supply side alone. The idea that nominal adjustment eliminates demand constraints has always been a

¹ In mainstream models of endogenous growth, the demand side can affect growth rate by changing conditions of supply. For example, a change in saving behavior can affect capital accumulation. But in these models, the proximate determinant of long-run output and growth is still supply.

weak link in mainstream macroeconomics.² The general ability of monetary policy to close demand gaps has also been questioned, especially in the stagnant aftermath of the Great Recession (Summers, 2014, for example). We propose a model in which demand constrains output over a medium-run to long-run horizon. This approach is consistent with heterodox research on demand-led growth models.³ That said, these models often leave the supply side implicit or assume an infinitely elastic supply of labor (see Dutt, 2012 and Freitas and Serrano, 2015 for examples). This approach may have empirical relevance for emerging-market economies that can draw large numbers of workers from subsistence agriculture into market production. But developed economies have operated for much of the time in recent decades not so far away from conventional estimates of full employment. In this situation, it is unreasonable to assume that arbitrary increases in demand growth can be accommodated without limit by the supply side. Setterfield (2013, page 24), following Cornwall (1972), describes this context as a “mature economy in which conditions of full employment may, in principle, be approached.” Skott (1989, 2010) considers the importance of labor supply constraints even when demand dynamics determine growth.

We begin with a model of aggregate demand dynamics following Harrod (1939). As in Fazzari, et al. (2013), however, the inherent instability of the Harrod model is contained from above by the supply of resources and from below by autonomous demand, that is, demand that is not induced by income or output. If autonomous demand grows at a constant rate, this growth rate determines the steady-state growth rate of demand, a result that Serrano (1995) labeled the “supermultiplier” model (also see Cesaratto, et al., 2003; Allain, 2015; Freitas and Serrano, 2015; and Serrano and Freitas,

² The conventional view that Keynesian results depend on nominal rigidity is ironic considering Keynes’s own arguments in chapter 19 of the *General Theory*. See Palley (2008) for a recent contribution and additional citations. Bhaduri (2006, p. 70) makes similar arguments in the specific context of growth theory.

³ For surveys and comparisons of heterodox research on demand-led growth, see Setterfield (2010), Keen (2012), Hein (2014), and Lavoie (2014).

2017). We confirm and extend these demand-side results here and interpret their relevance for understanding actual economic growth.

The main contribution of this paper is to introduce dynamics of supply that link both labor supply growth and labor productivity endogenously to the state of the economy determined by demand just the opposite of the mainstream perspective about economic growth. We show that demand leads supply in the sense that supply growth will converge to the demand-led growth path, just the opposite of the mainstream perspective about growth. Therefore, there is no single “natural” rate of supply growth. Instead, different levels of autonomous demand growth can generate a continuum of dynamic paths for actual output and the unemployment rate. This result proposes a solution to Harrod’s “reconciliation problem” between the growth of supply and demand.

While growth is demand led, supply constraints limit the maximum feasible rate of growth. If the unemployment rate is bounded below (even at zero) the range of demand growth rates that supply can accommodate is bounded above. Therefore, demand cannot generate arbitrarily high rates of growth. That said, calibration of the model, based on original empirical estimates of the linkages between demand and supply conditions, shows the range of feasible demand growth rates that can be accommodated by supply potentially covers a wide range of empirically relevant growth rates. We use this result to interpret how, for example, the loss of autonomous demand associated with the end of unsustainable household financial dynamics or with misguided fiscal austerity can trap an economy on a low-growth path of both demand and supply, well below what is feasible. This approach links our model to the interpretation of “secular stagnation” (see Summers 2014 and Cynamon and Fazzari, 2015) in the U.S. and other developed economies.

Section 2 of this paper presents the demand side of the growth model and basic steady-state results. The endogenous dynamics of supply are discussed in section 3 and section 4 shows how the demand and supply sides are reconciled. In section 5 we present an empirical calibration of the model.

These results show that although the model can generate both stable and unstable cyclical growth paths, convergence toward the growth path determined by autonomous demand is the most likely empirical outcome. In addition, analytical results demonstrate that if demand dynamics are stable, supply will converge to demand as long as steady-state demand growth does not require an unemployment rate below the minimum feasible level. Section 6 discusses the implications of our model for understanding how the practical dynamics of demand lead economic growth. In particular, our empirical analysis shows how changes in autonomous demand growth that induce just modest changes in the unemployment rate can cause supply to accommodate acceleration of demand from stagnant levels to growth rates associated with booming economies. Section 7 concludes and suggests where the results presented here can lead further research.

2. Aggregate Demand and Steady-State Growth

Aggregate demand in our model consists of three components: business investment that accumulates the productive capital stock, consumption spending induced by income, and an autonomous component. Autonomous demand is defined as spending with dynamics independent of the state of the economy that does not build productive capacity. It could consist of autonomous consumption spending (as in Freitas and Serrano, 2016), government spending (Allain, 2015, for example), or exports (Nah and Lavoie, 2017, for example).

We assume that firms choose investment in period t to reach a target expected capital-output ratio in $t+1$. Firms control the capital stock in $t+1$, but they do not know period $t+1$ output when they choose period t investment. Therefore, investment is based on information at the beginning of period t including expectations about period $t+1$ output (EY_{t+1}). We denote the target capital-output ratio as $\hat{v}_t = K_{t+1}/EY_{t+1}$. We treat \hat{v}_t as a behavioral choice by firms rather than as an endogenous outcome

that depends on the state of the economy, as is the case in many demand-led growth models.⁴ This point is explained by Skott (1989, p. 53): “investment is primarily induced rather than autonomous, and in steady growth the utilisation ratio of capital must be at the desired level” and by Cesaratto et al. (2003, p. 41): “capacity-creating expenditures basically depend on expectations about the evolution of effective demand over the life of the equipment.” But despite their intention to invest to reach \hat{v}_t , firms may not exactly hit the target because of uncertainty and time lags between the investment decision and the time that the capital becomes productive. For these reasons, as in Allain (2015 equation 6) and Freitas and Serrano (2016, equation 3), we assume that the target capital-output ratio (or the target rate of capacity utilization) adjusts partially in each period toward a long-run value (v^*) chosen by firms:

$$\hat{v}_t = (1 - \lambda)v_{t-1} + \lambda v^*$$

In Fazzari et al. (2013) we assumed $\hat{v}_t = v^*$, that is $\lambda = 1$. Serrano and Freitas (2017) present a strong case that immediate adjustment of the capital stock to the long-run target level may induce unrealistic instability and hence they prefer a specification with $\lambda < 1$ along the lines of the traditional “flexible accelerator.” Empirical evidence is also consistent with a value of λ substantially less than one. Indeed, in section 5 we discuss estimates of λ from U.S. data as low as 0.09.⁵

The long-run v^* depends on the technical requirements of production. It also depends on the strategic choices of firms about excess capacity. If firms wish to have less than full capacity utilization in normal times to meet unanticipated demand shocks, v^* will be higher than the technical capital-output ratio. As v_t converges to v^* capacity utilization will converge to the desired level.

Gross investment in period t is set to reach the capital stock in period $t+1$ determined by \hat{v}_t :

$$I_t = K_{t+1} - (1 - \delta)K_t = \hat{v}_t(1 + Eg_t)^2 Y_{t-1} - (1 - \delta)K_t$$

⁴ See Lavoie (2014, section 6.5) and Hein (2014, section 6.1) for an overview of the extensive literature on this issue.

⁵ Because the actual capital-output ratio is effectively the same as the capacity utilization variable studied extensively in related research, the slow adjustment of v_t to v^* is related to the idea that firms may tolerate some deviation between short-run and long-run utilization rates for an extended period. See the detailed discussion in Hein (2014, chapter 11).

where δ is the geometric depreciation rate of the capital stock and Eg_t is expected growth in output between periods $t-1$ and $t+1$.⁶ We also restrict gross investment to be non-negative ($I_t \geq 0$).

As in Fazzari, et al. (2013, also see Ferri and Variato, 2010) we use a simple adaptive rule for expected growth:

$$Eg_t = (1 - \alpha)g_{t-1} + \alpha Eg_{t-1} = (1 - \alpha) \sum_{j=0}^{\infty} \alpha^j g_{t-j-1}.$$

Expected growth is a weighted average of past actual growth rates with weights that decline geometrically over time.⁷ Furthermore, the form of the adaptive growth expectation rule is somewhat different from the partial adjustment rule for the target capital-output ratio. The dynamics of \hat{v}_t are anchored to a long-run parameter (v^*) known by firms because it is chosen by firms. There is no similar anchor for growth expectations. As we show below, the model has a steady-state growth rate, but this rate is a property of the model and does not arise directly from the agent behaviors. There is no reason that firms will know the steady-state growth rate of demand or even if a stable steady-state demand path exists in the context of uncertainty about the structure of the economy. If the dynamics of the model are stable, a learning process like the one captured in our behavioral specification for Eg_t might lead expectations to converge to estimate the level of steady-state growth, but we argue that steady-state growth itself should not appear in the behavioral specification of expectation formation which depends entirely on the actual history of growth rates.

⁶ We normalize expected growth to a single period. Without loss of generality, we assume expected growth is the same between $t-1$ and t and between t and $t+1$. All that matters for investment is the compound expected growth rate over two periods.

⁷ An anonymous referee points out that expectations may have excessively long “memory,” especially for relatively large α (which we find empirically, see section 5). We also consider an expectation specification that truncates the effect of lagged growth after 10 periods with the weights on lagged growth normalized to sum to one. Steady-state results are identical with the two specifications. The effect of the truncated expectation rule on dynamic stability is very small; see further discussion in section 5.

Induced consumption depends on income. Again, to keep the model strictly recursive, we assume that consumption plans during period t are made by projecting $t-1$ income forward one period:

$$C_t = (1 - s)(1 + E g_t)Y_{t-1}$$

where s is the constant marginal propensity to save out of expected income. Aggregate demand (Y_t^D) is the sum of the demand components:

$$Y_t^D = C_t + I_t + F_t$$

where F_t represents autonomous demand. Substituting the specifications for consumption and investment gives

$$Y_t^D = (1 - s)(1 + E g_t)Y_{t-1} + \hat{v}_t(1 + E g_t)^2 Y_{t-1} - K_t(1 - \delta) + F_t. \quad (1)$$

Suppose, initially, demand is less than supply so that demand constrains the economy in both periods t and $t-1$. In this case output and income are determined by demand ($Y_t = Y_t^D$). Divide equation 1 by Y_{t-1} to obtain the law of motion for demand-determined output growth:

$$1 + g_t = (1 - s)(1 + E g_t) + \hat{v}_t(1 + E g_t)^2 - \frac{K_t}{Y_{t-1}}(1 - \delta) + \frac{F_t}{Y_{t-1}}. \quad (2)$$

where $g_t = Y_t/Y_{t-1} - 1$.

To explore possible steady-state properties of the model, solve equation 2 for a time-dependent equilibrium growth rate g_t^* such that expectations are realized ($g_t = E g_t$). Also set the target capital-output ratio equal to its long-run desired level ($\hat{v}_{t+1} = v^*$). The growth rate g_t^* that satisfies these conditions is:

$$\begin{aligned} 1 + g_t^* &= (1 - s)(1 + g_t^*) + v^*(1 + g_t^*)^2 - \left(\frac{K_t}{Y_t}\right)\left(\frac{Y_t}{Y_{t-1}}\right)(1 - \delta) + \left(\frac{F_t}{Y_t}\right)\left(\frac{Y_t}{Y_{t-1}}\right) \\ &= (1 - s)(1 + g_t^*) + v^*(1 + g_t^*)^2 - v^*(1 + g_t^*)(1 - \delta) + f_t(1 + g_t^*) \\ g_t^* &= \frac{s - f_t}{v^*} - \delta \end{aligned} \quad (3)$$

where the autonomous demand ratio is $f_t = F_t/Y_t$. As discussed in Fazzari et al. (2013), the definition of g_t^* is closely related to Harrod's warranted rate of growth (set the autonomous demand ratio and the depreciation rate to zero).⁸ But g_t^* varies over time unless the ratio of autonomous demand to aggregate demand is constant.

Equation 3 leads to a central result: the model has a constant steady-state growth rate only if aggregate demand grows at the same rate as autonomous demand, that is, if the ratio f_t is constant. Therefore, *the steady-state rate of growth of output in a demand-constrained system equals the growth rate of autonomous demand*. Assuming a constant growth rate of g^* for autonomous demand and given the structural parameters s , v^* , and δ , output growth must also converge to grow at g^* if the system is to have a constant steady-state growth rate.

If the growth rate of autonomous demand is constant and the model is stable, convergence to steady state occurs through the endogenous adjustment of f_t .⁹ Formally whether f_t adjusts to approach a steady-state growth path depends on the stability of the full model. But the law of motion for growth in equation 2 provides some intuition about how adjustment of f_t could generate convergence. Suppose that the system begins in steady state, growing at g^* , and demand growth is exogenously shocked downward to $g' < g^*$. Endogenous dynamics lead to slower growth of both consumption and investment, but autonomous demand continues to grow at g^* . With output growing more slowly than autonomous demand, the final term in equation 2 (the ratio of autonomous demand to lagged output) increases, eventually raising the actual growth rate and pushing it back in the direction of g^* . This

⁸ See Sen (1970), Harris (1978, p. 27), and Fazzari (1985) for an interpretation of Harrod's warranted rate in terms of realized expectations of demand growth. Harrod (1939) seems to assume zero depreciation, that is, infinitely lived capital, without being explicit about this assumption. (See, in particular, his simple numerical example on page 18 that he claims "may be fairly representative of modern conditions.") Dutt (2010, p. 26) and Foley and Michl (2010, p. 52) assume zero depreciation rates for simplicity but recognize that the more realistic case of positive depreciation. Skott (2010, p. 109) includes a positive depreciation rate. Realistic empirical calibration of this and related models (see section 5) requires a positive depreciation rate.

⁹ Lavoie (2016, p. 177), following Serrano (1995) makes a similar argument.

intuition drives home the significance of “autonomous” demand in this model: it is a component of demand with dynamics that are independent of the actual evolution of the economy. It plays a fundamental stabilizing role in the dynamics of demand growth.

It is straightforward to solve for the steady-state path of demand-determined output:

$$Y_t^* = \left[\frac{1}{s - v^*(g^* + \delta)} \right] F_t \quad (4)$$

Equation 4 is familiar from static Keynesian models: equilibrium output is autonomous spending times a multiplier that reflects induced spending. The denominator of the bracketed multiplier term is the marginal propensity to save minus the marginal propensity to invest in steady state ($dl_t/dY_t = v^*(g^* + \delta)$ along the steady-state growth path). The multiplier implied by equation 4 is what Freitas and Serrano (2016, equation 5) define as the “supermultiplier.” This concept goes back to Hicks (1950, pages 61-62) and was further developed in Serrano (1995). Cesaratto et al. (2003, equation 7) present an almost identical expression. Details of the specification used by Allain (2015) are somewhat different, but the results are effectively the same. Also see Lavoie (2014), section 6.5.7.

Consider what the steady-state result tells us, and what it does not tell us, about demand dynamics. First, while the steady-state path implied by the output solution from equation 4 corresponds to a perfect foresight or a “rational expectations” dynamic equilibrium, the law of motion for demand growth (equation 2) does not impose any particular behavior on expectations. Whether reasonable dynamics of learning and expectation formation lead to convergence of the system to the steady-state path depends on the dynamic stability of the full model, which we consider in section 5. In this respect, our interpretation of these results differs somewhat from Lavoie’s summary of much of the related literature (2014, p. 408): “supporters of the supermultiplier ... refer to perfect foresight or to correct forward-looking expectations,” although Lavoie notes that recent work, especially Allain (2015), relaxes perfect foresight.

Second, on a related topic, we do not assert that the steady-state growth path is necessarily some kind of “fully adjusted” or “long run” equilibrium to which the system converges. Again, a necessary condition for such convergence is the dynamic stability of the model. Furthermore, long-run convergence would require a long-run constant rate of growth of autonomous demand, which is empirically unlikely. Indeed, there may be different sources of autonomous demand with different growth rates. Hicks (1950) thought of autonomous demand as investment spending. Allain (2015) and Ferri (2016) define autonomous demand as government spending, while Freitas and Serrano (2016) model autonomous demand as consumption. It seems reasonable that autonomous demand would consist of these components, as well as a share of imports and likely most of exports (Thirlwall, 1980; Setterfield, 1997; Lavoie and Nah, 2017). In the realistic case that different components of autonomous demand grow at different rates, total autonomous demand will never grow at a constant rate in finite time. Medium-run dynamics will reflect different growth rates of different autonomous demand components and the relative size of these components.¹⁰

Third, and perhaps most important, even if the system does not converge to steady state, the steady state can function as a “center of gravity” for actual dynamics. In Fazzari et al. (2013) we explain how the path of autonomous demand determines the dynamics of the lower bound of a cyclical path. Furthermore, if autonomous demand grows at rates less than or equal to the growth of supply (as we discuss in the next section) the average growth rate of a cyclical path is determined by the growth rate of autonomous demand even if the system does not converge to steady state.

In addition, it is important to recognize that while the steady-state *growth rate* is determined entirely by the growth rate of autonomous demand, the *level* of the steady-state path depends on all parameters of demand. This result is evident from equation 4. For example, a decline in the saving rate,

¹⁰ We thank Peter Skott for emphasizing this point in comments on an earlier draft of this paper.

other things equal, does not change the steady-state growth rate of output. But equation 4 shows that a permanently lower value of s raises the level of steady-state demand.¹¹ Therefore, the model implies a steady-state “paradox of thrift” result.

3. Endogenous Dynamics that Adapt Supply to Demand

The central contribution of this paper is to explore how supply adapts to demand conditions. In particular, we are interested in how changes in demand growth could *lead* supply expansion so that supply growth accommodates demand expansion. If this is the case, demand can be the proximate constraint on output at any point in time, but demand growth will endogenously create supply to prevent aggregate supply from limiting production.¹²

As is typical in related research, we assume that output is produced with a linear Leontief technology.¹³ The capital-output ratio v^* determines the output capacity provided by the current capital stock. The capital stock is endogenous in the model, that is, capital is produced. We assume firms target a sufficiently low capacity utilization rate that output is not constrained by an inadequate supply of capital, at least for empirically sensible aggregate fluctuations. As described by Freitas and Serrano (2016), among others, this behavior arises because production is profitable at the margin for firms and

¹¹ In steady state, a decline in s raises output so that the ratio of autonomous demand to output also declines: $f^* = s - v^*(g^* + \delta)$, from equation 3. Lavoie (2016) presents a similar result. Nah and Lavoie (2017) also derive this kind of result in a model with autonomous demand growth determined by exports. Also see the discussion in Nikiforos (2018).

¹² The results to follow imply that this outcome is theoretically possible and empirically realistic. But we cannot rule out the possibility that demand growth might be sufficiently high to exceed supply. In the simulation model discussed later in the paper actual output will be set at the level of supply if demand exceeds supply, that is, $Y_t = \min(Y_t^D, Y_t^S)$. In a more fully developed model of possible supply constraints, sectoral bottlenecks could be introduced as well as inflation and monetary policy responses as output approaches supply limits. We leave these developments for further research.

¹³ Tavani and Zamparelli (2017b) in an insightful survey of models of economic growth identify a Leontief technology as a common feature of heterodox growth models that rejects the marginal productivity theory of income distribution and allows for under-utilized capital and labor. This survey provides a useful overview of many of the linkages between the supply side and the demand side discussed in this section along with extensive references.

they wish to have enough capacity to meet unanticipated increases in sales. The assumption also accords well with empirical measures of capacity utilization that stay well below 100 percent.¹⁴

Labor supply may impose a more meaningful constraint on production over the medium run.¹⁵ Skott (1989 and subsequent work) pioneered the analysis of labor constraints in demand-led growth models. We agree that in modern developed economies, what Skott (2010, p. 109 and 119-122) calls “mature” economies, growth can be limited by the supply of labor. Even if the economy never reaches true full employment, low unemployment rates may trigger fears of runaway inflation (justified or not) and lead to restrictive monetary policy that constrains demand. Denote labor productivity by A_t so that:

$$Y_t^S = A_t L_t$$

where L_t is the number of hours the labor force is willing to work. If $Y_t^D = Y_t < Y_t^S$ then the unemployment rate $u_t = 1 - Y_t^D / Y_t^S$ is positive.

Labor supply growth is related to the unemployment rate:

$$g_t^{LS} = \theta_0 - \theta_1 u_{t-1} \tag{5}$$

The constant θ_0 captures exogenous demographic factors like the growth of the working age population as well as preferences and social norms, such as female labor force participation. The second term creates a negative effect of the unemployment rate on labor supply growth. One reason is a decline in labor force participation due to the rising difficulty of finding an acceptable job match as unemployment rises. Phillips curve effects of higher unemployment on wages could also discourage labor force participation and reduce working hours (see Delong and Summers, 2012 and *The Economist*, 2016).¹⁶ High unemployment also tends to reduce immigration (see Setterfield, 2003, who

¹⁴ Utilization averaged 79.1 percent from January, 1980 through September, 2016, exceeding 85 percent in just two months during that long period.

¹⁵ In one sense, this approach harkens back to classical political economy in which, as described by Harris (1978, p. 22) “[p]roduction is attributable to labor, that is, to current labor services and to means of production that are themselves reducible to the labor services embodied in them.”

¹⁶ These effects are evident in U.S. experience following the Great Recession, see Yagan (2017).

follows Cornwall, 1977). The empirical appendix describes findings of an economically and statistically significant θ_1 coefficient.

We model the growth rate of the labor productivity as a function of two key variables, the unemployment rate and the replacement rate of the capital stock:

$$g_t^A = \rho_0 - \rho_1 u_{t-1} + \rho_2 (g_{t-1}^K + \delta). \quad (6)$$

The constant ρ_0 captures exogenous changes in labor productivity. Labor productivity is usually strongly procyclical, a fact emphasized in the “real business cycle” literature and captured in the second term of equation 6. The incentives for labor-saving innovation also rise in a low-unemployment environment (see Storm, 2017, pp. 17-21 and Tavani and Zamparelli, 2017b, section 5 for discussion and further references). R&D expenditure also tends to be procyclical (see Brown et al. 2009).

Robinson (1962, page 51) writes “[w]hen firms can see profitable markets expanding around them but cannot get hands, they set about trying to find labour-saving devices.” Dutt (2006, p. 325) refers to Robinson and argues that labor productivity growth depends on the change, rather than the level of unemployment. He summarizes the argument with “the old adage that necessity is the mother of invention. ... The speed of technological change is essentially determined by pressures and bottlenecks in the economy” (also see Cornwall and Cornwall, 1994). While this intuition is compelling, it seems to imply that the growth rate of labor productivity depends on the *level* rather than the change in the unemployment rate, as argued persuasively in Palley (2014), and consistent with the specification in equation 6. Flaschel and Skott (2006) discuss how labor shortages (low unemployment levels) could raise incentives for labor-saving innovation (as well as greater immigration, consistent with equation 5 above). Bhaduri (2006) links labor productivity to “labor discipline” by positing that labor productivity grows faster when unemployment is lower to keep real wages from rising due to labor shortages. Hein and Tarassow (2010) and Bivens (2017) provide empirical support for this channel and for the broader implication that a stronger labor market raises productivity growth. Furthermore, as DeLong and

Summers (2012) point out, high unemployment leads to “decay” in workers skills, reducing productivity.

The final term in the productivity growth equation captures the effect of new capital. Even though the capital-output ratio is modeled as a constant, we assume that the dissemination of technical progress as well as learning-by-doing effects are embodied in the new capital stock along the lines described by Kaldor (1978, also see Palley, 1996; Setterfield, 1997; McCombie, 2002; Hein, 2014, page 315; and Bassi and Lang, 2016). McCombie and Spreafico (2015, page 2) summarize Kaldor’s perspective as “the act of investment itself generate[s] new and improved methods of production.” Palley (1996, page 124) writes “[t]echnical progress is therefore both ‘revealed’ and ‘realized’ through investment, so that investment serves simultaneously as the means of (1) expanding the capital stock, (2) feeding technical innovations into the production process, and (3) uncovering further possibilities for innovation” (also see Palley, 1997). This specification introduces a version of Verdoorn’s Law into our model (a positive relation between output growth and productivity growth). Both net and replacement investment add capital with a newer vintage that embodies improved technology, becoming productive with a one-period lag. As DeLong and Summers (2012) and Summers (2014) argue, a decline in the investment share leads to slower growth in productivity.¹⁷

These effects correspond to the concept of dynamic increasing returns in production and how they relate to the size of markets (related to the expansion of demand) summarized early on by Young (1928): unit costs decline by “finding an outlet for a potential product” (page 537) and “change becomes progressive and propagates itself in a cumulative way” (page 533). Young even attributes “the Industrial Revolution of the eighteenth century” in large part “to the enlargement of markets” (page 536) and not only exogenous technological innovation.

¹⁷ An anonymous referee points out that equation 6 effectively adds a second lag to the model’s dynamics because $g_{t-1}^K = I_{t-2}/K_{t-2} - \delta$.

We model the growth rate of aggregate supply (potential output) as the sum of labor supply growth and productivity growth. This approach ignores, for simplicity, the compound term ($g_t^{LS} * g_t^A$). But this term is trivial for reasonable values of the growth rates.

4. Demand-Led Growth of Supply in Steady State

The specifications for supply growth in equations 5 and 6 cause aggregate supply to adapt to the state of demand. Because both the unemployment rate and the gross investment rate depend on the state of the economy, and because the state of the economy depends on the level of aggregate demand, *demand leads supply*. In this section we present the implications of this demand-supply interaction for the steady-state growth path.

The growth of aggregate supply from equations 5 and 6 is:

$$g_t^{LS} + g_t^A = \theta_0 - \theta_1 u_{t-1} + \rho_0 - \rho_1 u_{t-1} + \rho_2 (g_{t-1}^K + \delta).$$

On a steady-state path, growth equals the rate of growth of autonomous demand g^* . In steady state, firms will invest so that the capital-output ratio remains at its target v^* . Therefore, in any steady state, $g_{t-1}^K = g^*$. If the growth rate of aggregate supply is in steady state and equals the growth rate of aggregate demand, then the unemployment rate must be constant. Solving the equation above for the unemployment rate that yields a constant rate of supply growth yields:

$$u^* = \frac{\theta_0 + \rho_0 - g^*(1 - \rho_2) + \rho_2 \delta}{\theta_1 + \rho_1} \quad (7)$$

Is supply growth with the unemployment rate at u^* the same as steady-state demand growth? The answer is yes. At any point in time:

$$u_t = 1 - \frac{L_t}{L_t^S} = 1 - \frac{Y_t}{A_t L_t^S} = 1 - \frac{Y_t}{Y_t^S}$$

and $u_t = u^*$ only if demand-determined output grows at the same rate as supply. This result does not necessarily imply that actual supply and demand growth will converge to each other. That result

depends on the overall dynamic stability of the model. But if the system is in steady state, demand and supply growth must equal the growth rate of autonomous demand.

What Harrod called the “natural rate” of aggregate supply growth is endogenous in our model. This feature leads to a central result in this paper: changes in the growth rate of autonomous demand can, within bounds, affect the growth rate of aggregate supply. Suppose g^* increases; equation 6 shows that u^* will decline. If the dynamics of the model are stable, supply growth will converge to the higher level of g^* driven by a lower level of u^* . The maximum value of g^* , however, is determined by the minimum feasible value for u^* . Suppose this minimum is \hat{u} . Then the maximum autonomous growth rate that can be accommodated by steady-state supply growth is:

$$\hat{g} = \frac{\theta_0 + \rho_0 + \rho_2 \delta - \hat{u}(\theta_1 + \rho_1)}{(1 - \rho_2)} \quad (8)$$

Demand growth can lead supply growth, but only within limits.¹⁸

Equations 7 and 8 provide insights into how our model can address the classic “reconciliation problem” posed first by Harrod (1939; also see Setterfield, 2003; Sawyer, 2012; and Allain, 2017) in which the steady-state growth rate of supply (Harrod’s “natural rate”) need not equal the steady-state growth rate of demand (the “warranted rate”). If $g^* < \hat{g}$ then the steady-state growth rates of supply and demand can be reconciled by the adjustment of the unemployment rate to u^* . From equation 7 a necessary condition for u^* to exist is for the growth rate of aggregate supply to depend positively on the rate of unemployment ($\theta_1 + \rho_1 > 0$). There must be a structural channel that allows supply to adapt to demand. Note that there is no similar necessary condition for ρ_2 , the parameter that connects

¹⁸ This result is somewhat similar to the implications of the model in Palley (1997) according to which different rates of demand growth can shift the economy among a possible multiplicity of supply growth rates.

labor productivity to capital replacement. The size of ρ_2 affects u^* and \hat{g} (see equations 7 and 8), but a positive ρ_2 is not necessary to reconcile steady-state supply and demand growth.¹⁹

This approach to reconciliation resembles, in part, the classic approach of Robinson (1956, chapter 5 and 1962). On the one hand, Robinson argued that economies usually operate with unemployment and therefore do not face constraints from labor supply. On the other hand, she recognizes that the equilibrium “desired” growth rate driven by the demand effects of investment and profits may be high relative to labor force growth but that situation would “call forth the innovations that it needs” (1962, page 52).²⁰ In our model, high demand growth “calls forth” supply growth, as long as demand growth is less than the maximum feasible level given by equation 8.

The model also shows that the steady-state demand effects of changes in saving discussed earlier will also impact steady-state supply. An increase in saving does not change steady-state growth rates or unemployment rates. But it would induce a transitory increase in unemployment that reduces supply. If the system is dynamically stable, the level of supply will converge to the lower demand path, as determined by equation 4. In general, persistent effects of demand on output and supply are basic predictions of this model.²¹

5. Empirical Calibration and Dynamic Stability

What conditions are necessary for the steady-state results described in the previous section to attract the actual dynamics of the system? Because of non-linearities and the rather large number of parameters in the model, detailed analytical results are difficult to obtain. Simulations show the

¹⁹ Dutt (2006) presents a model that reconciles demand-led growth with labor supply growth, allowing for endogenous technical change. The key adjustment mechanisms are similar to those in the neoclassical synthesis, but in the Kaleckian growth model these structures produce a continuum of possible growth equilibria. Also see Tavani and Zamparelli (2017a).

²⁰ Also see Harris (1978, pages 186-192).

²¹ In an innovative empirical study, Girardi et al. (2017) find convincing evidence that demand changes have highly persistent effects on output, capital, and labor productivity.

demand dynamics are cyclical out of steady state with cycles that may be stable or unstable, as in Allain (2015, 2017).²²

A linearized version of the model provides insights into the system dynamics. If the modulus of the maximum eigenvalue of the Jacobian matrix from the linearized model is less than one, the model will be dynamically stable after a shock in the neighborhood of the steady state. Simulations show the linear stability condition corresponds very closely with the stability of the nonlinear model for shocks of reasonable economic magnitude. The eigenvalues of the demand side of the model are a complex function of the parameters s , v^* , α , and λ . The effects of these parameters on dynamic stability are explored empirically below. The linearized model does lead to a strong analytical result about the relation between the supply side and demand side of the model, however. Under very general conditions, if the demand side of the model is dynamically stable, *the supply side will converge to the demand side*. This finding supports our claim that demand leads supply in our model.²³

Because the stability of the model is ultimately an empirical question, we simulate the model with key parameters that vary across a reasonable empirical range summarized in table 1.

²² As discussed in Fazzari, et al. (2013), however, the addition of any amount of autonomous demand will prevent the dynamics from collapsing to zero, but that outcome does not guarantee global dynamic stability. Even if the output path is bounded away from zero, endogenous cycles can be explosive upward for some parameter configurations.

²³ Derivation of the linearized model and a detailed discussion of the findings summarized in this paragraph are contained in an appendix available from the authors. A sufficient condition for convergence of the supply side to a stable demand path is that the sum of the parameters relating unemployment to labor force and productivity growth ($\theta_1 + \rho_1$) is less than two. Violating this assumption would imply an extreme sensitivity of supply growth to unemployment. The linearized model also shows that stability cannot be guaranteed in a stationary state with all supply parameters equal to zero.

Table 1: Summary of Empirical Calibration for Key Model Parameters

Parameter	Benchmark Value	Bottom of Range	Top of Range
Demand-Side Parameters			
Propensity to Consume ($1 - s$)	0.50	0.40	0.80
Target Capital-Output Ratio (v^*)	1.20	1.00	1.50
Adjustment Speed for v (λ)	0.09	0.05	0.20
Adjustment Speed for expected growth (α)	0.90	0.75	0.95
Capital Depreciation rate (δ)	0.084	--	--
Supply-Side Parameters			
Effect of unemployment rate on labor force growth (θ_1)	0.20	0.10	0.30
Effect of unemployment rate on labor productivity growth (ρ_1)	0.50	0.30	0.70
Effect of capital replacement rate (output growth + depreciation) on labor productivity growth (ρ_2)	0.50	0.30	0.70

The empirical appendix describes the data analysis used to justify the entries in table 1.

The benchmark value for the propensity to consume (0.50) may appear somewhat low, especially for a longer run horizon. In the empirical context of our model, however, income not consumed is not just saving but includes other leakages from demand. Federal, state and local taxes were 29 per cent of US GDP in 2017; and imports were 15 percent. These figures imply that the propensity to consume domestic output out of GDP is unlikely to exceed 0.60, although we consider the effects of higher values in the simulations based on empirical evidence discussed in the appendix. The table does not provide information on the share of autonomous demand in output. We can infer the relevant range in steady state (f^*), however, by calibrating it to reasonable values for steady-state growth according to the equation:

$$f^* = s - v^*(g^* + \delta).$$

With a steady-state growth rate of 2.5 percent and s , v^* , and δ set at their benchmark values, the steady-state autonomous demand share is 37 percent. This share varies between 4 and 49 percent across the range of values for s and v^* specified in table 1.²⁴

The model determines a dynamic path for the endogenous variables of consumption (C_t) and investment (I_t). Given the growth path of autonomous demand (F_t), aggregate demand determines total output ($Y_t = C_t + I_t + F_t$) and growth (g_t). The capital stock evolves according to investment spending and depreciation of existing capital. Expectations and the target capital-output ratio for future investment are all updated for period t with variables pre-determined in period $t-1$. For all parameter values in the empirical range described in table 1, the model dynamics are cyclical. Supply in period t evolves based on pre-determined $t-1$ variables, so supply in t is also pre-determined and the period t unemployment rate (u_t) is determined by demand. The supply side imposes a ceiling on possibly unstable demand cycles. We define the dynamics as stable if the actual path of output converges to steady state.

With the benchmark parameter values the model's dynamics are cyclical and stable. That said, the model can become unstable for some parameter combinations within the range given in table 1. An extensive iteration over the entire parameter range from table 1 shows that the cycles converge to steady state 67 percent of the cases.²⁵ These iterations also justify the perspective that demand is the proximate constraint on output almost always. The simulations begin at a 4 percent unemployment rate and autonomous demand growth is shocked from 2.5 to 3.0 percent. In less than 0.2 percent of the observations, including along paths that will ultimately become unstable asymptotically, does the

²⁴ In the US, government consumption and investment in 2016 was 18 percent of GDP; exports were 12 percent. Most spending financed by the "social safety net" (Social Security, Medicare, and Medicaid, primarily) is likely also autonomous and accounts for about 15 percent of GDP. Therefore, the autonomous demand share may well be higher than 40 percent, although this value is undoubtedly affected by different historical circumstances.

²⁵ The iterations were run over the four key demand-side parameters (s , v^* , α , and λ) by dividing the range in table 1 into 20 steps for each parameter. The prevalence of stable results helps address questions raised by Skott (2016) about whether models with autonomous demand can be dynamically stable for plausible parameter values.

unemployment rate fall below 2 percent in the first 10 years following the shock. The share of observations with unemployment below 2 percent is just 4 percent even if we extend the horizon to 30 years.

Table 2 shows the effect on stability of an increase in the value of each key parameter and the level of the parameter that shifts the dynamics from stable to unstable (holding all other parameters at their benchmark values from table 1).

Table 2: Effect of Individual Parameters on Model Stability

Parameter	Effect of Increase on Stability	Value That Induces Instability (other parameters at benchmark)
Propensity to Consume ($1 - s$)	Destabilizing	0.82
Target Capital-Output Ratio (v^*)	Destabilizing	3.11
Adjustment Speed for Capital-Output Ratio (λ)	Destabilizing	0.75
Adjustment Speed for Expected Growth (α)	Stabilizing	0.64

The effect of parameters on stability is intuitive. Changes in parameters that make induced demand less sensitive to the state of the economy make the model more stable (a decrease in v^* or δ and an increase in s). More persistence in expectation formation (higher value of α) and capital adjustment (lower value of λ) also stabilize the dynamics.²⁶

The supply parameters (θ, ρ) have little effect on dynamic stability with the structure of this model. Not surprisingly, supply lags demand since demand is the engine of output growth. Higher

²⁶ Cesaratto et al. (2003, footnote 19) also model expectations adaptively and identify the speed of adjustment of expectations as central to the dynamic stability of a similar model. An anonymous referee notes it may be unrealistic to allow current expected growth to be influenced by an arbitrarily long lag of past growth rates. We explored the effects of an alternative expectation model with weights on 10 years of lagged growth, declining geometrically and normalized to sum to one. Results were very similar. For example, the value of α that induces instability rises to just 0.65, from 0.64 in our benchmark model.

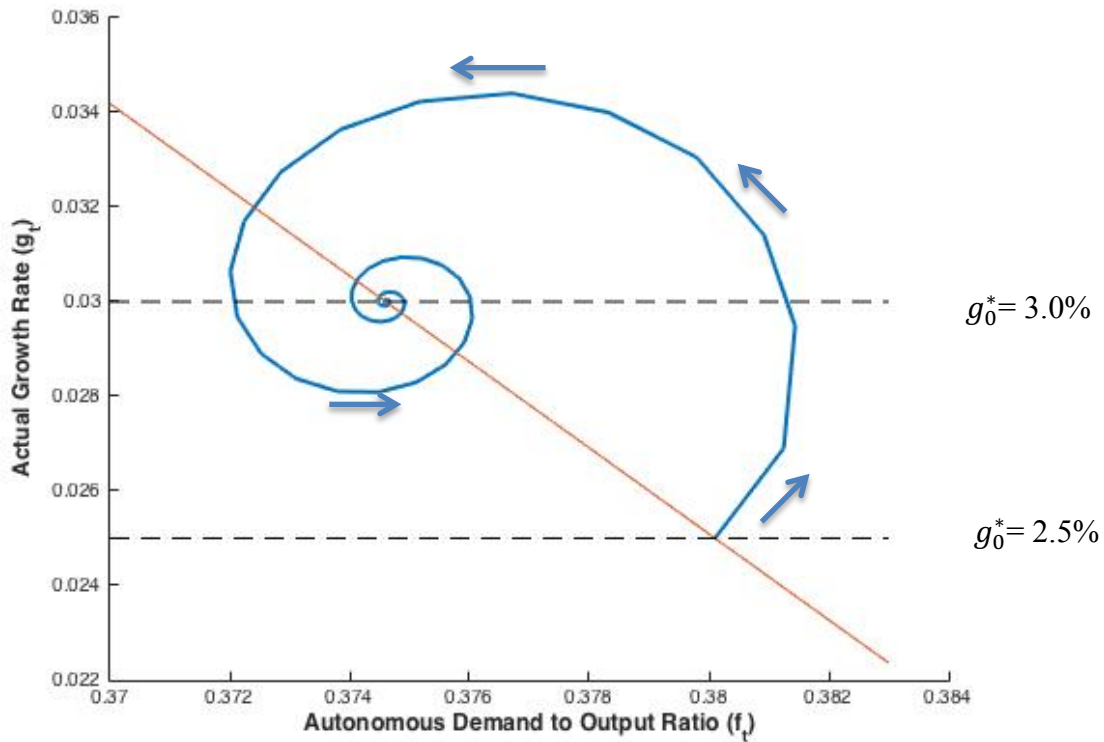
values of the supply parameters reduce the time it takes for supply to catch up with demand in either direction.

To build further intuition, consider figure 1 that graphs the rate of growth (g_t) against the ratio of autonomous demand to output (f_t). The downward sloping line plots the rate of growth such that expected growth is realized as a function of f_t (see equation 3):

$$g_t = E g_t \text{ if } E g_t = \frac{s - f_t}{v_t} - \delta.$$

Steady state occurs at the intersection of the downward sloping line and the prevailing g^* . Let the growth of autonomous demand rise from g_0^* to g_1^* with a parameter set that induces stable dynamics. Steady state shifts to the northwest in the diagram, but the actual trajectory of the model moves to the northeast initially because faster demand growth raises actual and therefore expected growth, while output growth below the new higher rate of growth of autonomous demand initially raises the f_t ratio. Growth continues to accelerate and f_t continues to rise until actual growth reaches the new level of autonomous demand growth g_1^* . When the model trajectory crosses g_1^* , f_t immediately begins to fall (because now output growth exceeds autonomous demand growth). Growth continues to have positive momentum due to adaptive expectations, but for a stable parameter configuration the effect of the falling value of f_t eventually offsets the unstable expectation dynamics (see equation 2) causing output growth to peak and then decline. When the falling growth rate crosses g_1^* , at the left extreme value of the cycle, the direction of f_t reverses, eventually causing actual growth to rise again. Subsequent cycles rotate in ever smaller orbits as the system converges to the new steady state.

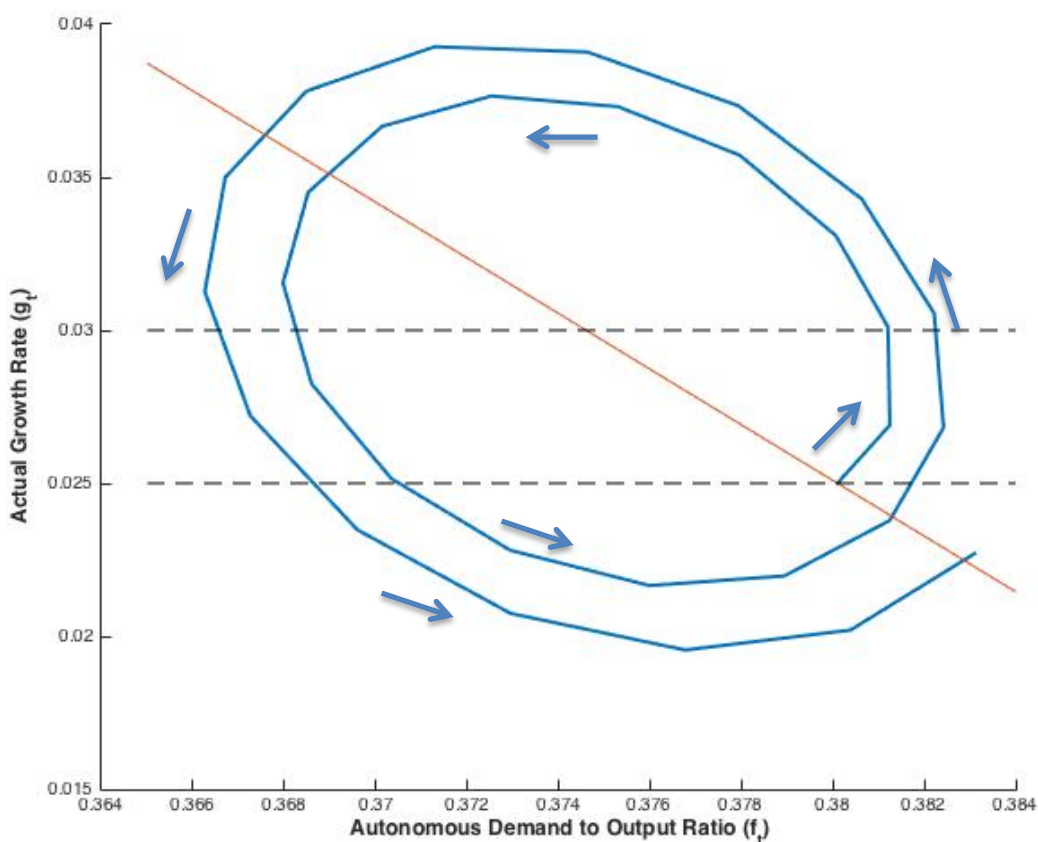
Figure 1
Stable Cyclical Dynamics of g_t and f_t



Now consider what happens if growth expectations are more sensitive to changes in output growth (a reduced value of α). Then growth rises more quickly after a positive shock to autonomous demand growth. The falling value of f_t when growth exceeds g_1^* , can still reverse the growth cycle, but perhaps not until the cycle trajectory moves so far to the left in the figure that the cycle becomes unstable (figure 2). A faster adjustment speed of the capital-output ratio (λ) has a similar effect. An increase in s shifts the negatively sloped line in the figure to the right meaning that the entire cycle trajectory has a higher value of f_t . This change is stabilizing because it raises the share of autonomous demand relative to the induced demand and makes it more likely that the stabilizing anchor of autonomous demand can overcome the destabilizing dynamics of expectations and the accelerator-

multiplier process. A reduction in v^* rotates the downward sloping line counter-clockwise, again raising f_t and stabilizing the system, other things equal.

Figure 2
Unstable Cyclical Dynamics of g_t and f_t



6. Implications for Demand-Led Growth

In our model, strong demand causes an increase in supply, unless the lower limit on unemployment imposed by equation 8 is binding. Weak demand growth always reduces supply. From equation 7, we have

$$\frac{du^*}{dg^*} = -\frac{1 - \rho_2}{\theta_1 + \rho_1}$$

which equals -0.7 with the three parameters at the calibrated benchmark values and ranges from -0.3 to -1.7 over the parameter range from table 1. Therefore, with benchmark parameter values, if

autonomous demand growth rises by one percentage point, the steady-state unemployment rate must fall by just 0.7 percentage points to equate supply growth with the higher demand path. Even with parameters set at the limits of the empirical range that make supply the least responsive to demand, a one percentage point increase in autonomous growth requires a drop in unemployment of 1.7 percentage points. Adjustments of these sizes seem entirely feasible in a realistic setting. For example, consider an economy with stagnant growth of 1.5 percent and unemployment of 6 percent. A one percentage point increase in autonomous demand growth raises growth 2.5 percent. To induce supply growth to adapt to the higher demand path, unemployment must decline to just 5.3 percent with the benchmark parameters. The accommodation of demand by supply, what might be called “reverse Say’s Law,” is not just theoretically possible but empirically relevant.

These results lead to a very different perspective on macroeconomics than what has become conventional wisdom in modern mainstream macro and policy analysis. Consider a positive shock to exogenous component labor productivity growth (an increase in ρ_0). What would be considered a positive growth shock from a mainstream supply-driven perspective on growth does not change growth in our model because there has been no change in the growth rate of autonomous demand. Furthermore, from equation 7, a positive shock to labor productivity will *raise unemployment*. The reason is straightforward: demand does not automatically adjust to supply in this model and a more productive economy requires less labor to satisfy given demand.

This result seems to support the common “populist” (perhaps even “Luddite”) claim that labor-saving innovation destroys jobs. But the result also suggests the solution to the problem: raise aggregate demand. In one sense, this perspective is similar to the mainstream New Keynesian view that monetary policy provides the key mechanism to reconcile aggregate demand to aggregate supply. Therefore a positive technology shock would justify expansionary monetary policy to boost demand. But our dynamic perspective with endogenous supply pushes the argument further. If labor

productivity growth rises above autonomous demand growth the resulting gap between actual and maximum feasible output will grow with time. It is far from clear that interest rate adjustment or even unconventional monetary policy (“quantitative easing” for example) will be adequate to fill the gap. A one-time adjustment is not sufficient; monetary policy would need to be continuously eased. In addition, since supply adapts to demand, a failure of monetary policy to raise demand growth after a positive productivity shock will compromise the supply side. The economy may appear to converge to something like “potential output,” but the level of potential itself is lower than it could have been with faster demand growth.

It is also important to consider whether there is a “natural” rate of unemployment or a “natural” rate of growth in this model. Autonomous demand growth can be affected by a variety of private behaviors and policy regimes. In the U.S., we interpret spending growth generated by the recent housing or technology bubbles as shifts in the dynamics of autonomous demand. Furthermore, the end of bubble-supported demand will reduce autonomous demand. In both Europe and the U.S., the “pivot to austerity” for fiscal policy in the aftermath of the financial crisis affects the path of autonomous demand. If something like the supply-side mechanisms from our model operate in real-world economies, supply growth will adapt to changes in the dynamics of autonomous demand. There is no “natural” rate of growth of the supply side independent of demand dynamics.

The results here also show that a potentially wide range of unemployment rates is consistent with aggregate demand leading aggregate supply. One might associate a target rate of unemployment with the minimum unemployment rate and maximum rate of autonomous demand growth defined earlier (\hat{u} and \hat{g}). But it will be difficult to know what these rates are in practice. Because supply adapts to demand, the economy might follow a lower growth path than feasible but without clear signals of excess supply. Furthermore, while the unemployment rate *in steady state* indicates the degree to which growth is lower than what could be feasible, the actual economy will operate on a

cyclical path outside of steady state and the unemployment rate could be a misleading indicator for years about the extent to which the economy falls short of its possible growth path.²⁷

This model seems particularly relevant to the recent incarnation of what Summers (2014) labeled “secular stagnation.”²⁸ The engine of growth is autonomous demand. A significant drop in autonomous demand growth will drag medium-run actual growth along with it. Consider the dynamics of U.S. household demand and its relation to housing finance. In the decades leading up to the financial crisis of 2008 and 2009, rising household debt ratios signaled that a substantial portion of U.S. household demand was financed not by rising incomes but by borrowing. Part of this kind of spending can be interpreted as autonomous in the context of our model, that is, demand driven by factors other than the growth of incomes (although some of the borrowing and spending may have been induced by income growth). When the crisis hit, a significant source of autonomous demand growth was lost and not replaced by faster autonomous growth of any other demand component. The result has been persistent stagnation of output growth and a convergence of the supply side to a weaker demand path.²⁹

This approach also has implications for fiscal policy. Government spending is a significant source of demand, and much government spending should be classified as autonomous.³⁰ Government transfer payments for retirement income and health care also likely support a large and growing amount of autonomous demand. With so much autonomous demand coming from fiscal policy, our model implies that acceptable aggregate growth simply may not be feasible under fiscal austerity. A

²⁷ Skott (2005) presents a model with endogenous wage-setting norms in which demand affects equilibrium unemployment and there is no “structurally determined natural rate of unemployment.”

²⁸ Summers took this term from Alvin Hansen although related ideas are widely discussed in heterodox research during the intervening period, for example Robinson (1956, 1962) and Steindl (1972).

²⁹ Consider the continuous reduction in the Congressional Budget Office forecasts for U.S. potential output following the Great Recession. The actual level of output in 2017 that was approximately equal to 2017 estimates of potential output would have generated an output gap of over 10 percent based on the decade-ahead potential output forecast from 2007.

³⁰ Government spending also includes “automatic stabilizers” that respond to the state of the economy. These structures could be an important stabilizer, which we intend to explore in further research.

reduction in the rate of autonomous demand growth induced by fiscal austerity will *always* lead to a lower growth path for both demand *and supply* in our model.

While our model generates demand-led growth, labor supply can impose constraints on the extent to which supply can accommodate demand.³¹ This outcome is obvious from equation 8 that derives the maximum steady-state demand growth rate consistent with the lower bound on the unemployment rate. But the constraints imposed by supply are more nuanced than the simple ceiling on production that we imposed in Fazzari, et al. (2013). The model developed here proposes that there is an empirically relevant range of growth rates of demand that induce supply growth to adapt to demand growth, reversing the direction of causation of mainstream macro growth theory.

Finally, because there is no explicit role for income distribution, our model sidesteps a key issue. But some changes in the specification of saving and consumption could extend the findings to connect distribution and growth. Suppose the saving rate depends on income distribution along standard lines, with a higher wage share reducing s . Holding autonomous demand growth constant this change will not affect steady-state growth. But as argued by Lavoie (2016) and Dutt (2019), a rise in the wage share will shift the *level* of the steady-state path upward (see equation 4) and therefore increase the *average* growth rate as the system converges toward a new steady state. This result is, in a broad sense, consistent with the short-run concept of wage-led growth pioneered by Bhaduri and Marglin (1990).³²

The structure of our investment function, which does not have an independent effect of the profit share or profit rate on investment, excludes by assumption the possibility of profit-led growth described by Bhaduri and Marglin (1990). That said, a different kind of extension could add some

³¹ This result differentiates our model from most research on demand-led growth that usually does not consider constraints imposed by supply.

³² Indeed, Dutt (2019, page 299) writes that wage-led growth in the Kaleckian growth models following from Bhaduri and Marglin (1990) is “analogous to wage-led growth in an average sense” in models such as the one presented here.

aspects of Bhaduri-Marglin profit-led growth. Suppose the source of autonomous demand is capitalist consumption. Dutt (2019) proposes such a specification with the growth of capitalist consumption given exogenously. Extend this approach, however, to posit a behavioral relationship between the *growth* of autonomous capitalist consumption and the *level* of the profit share. Then a shift in distribution towards profits would raise steady-state growth. Moreover, this capitalist consumption behavior would be validated as actual growth of both demand and supply would rise in steady state supporting a faster increase in profit income (assuming unemployment is above its minimum level \hat{u} , see equation 8).

7. Conclusion

The first message of this paper is that demand can be the engine of economic growth. Of course what is demanded must be feasible to produce, that is, adequate supply is necessary to realize a demand-led growth path. This paper's primary result is that endogenous linkages between the demand-determined state of the economy and both labor supply and labor productivity provide channels through which supply accommodates growth driven by demand. This approach reconciles Harrod's warranted and natural rates of growth. The model shows how the demand and supply sides interact with each playing a substantive role in macroeconomic growth determinants over the medium run (also see Mason, 2017). Our results differ from the mainstream perspective that relegates demand to a short run of a few quarters with supply alone determining growth over multi-year horizons.

Demand growth in our model cannot generate arbitrarily high rates of actual economic growth because of limits on labor supply. We believe that this feature is an important and empirically relevant implication of our model, not often recognized in Keynesian growth models. While one can debate whether developed economies can feasibly grow at 2, 3, or 4 percent for a sustained period of time, it seems unreasonable to propose that mature economies could grow at 10 percent or more for an

extended period without running hard into supply constraints.³³ That said, a simple empirical calibration of our model shows it is entirely reasonable that acceleration of demand growth from something like 2 to 3 percent could be accommodated by supply. Perhaps more important, misguided attempts at demand-side “austerity” will drag supply down with them. “Output gaps” may seem to disappear as supply adjusts to weak demand growth but nonetheless the economy stagnates relative to what would be feasible with more robust demand.

The results here are consistent with the idea of hysteresis, or “history matters,” in the sense that the dynamic path of demand fundamentally affects the economy’s productive potential (as in Setterfield, 1997; DeLong and Summers, 2012; Mason, 2017; among many others). Although analytical results often focus on steady states with constant growth rates of autonomous demand, the actual path of autonomous spending in real economies likely evolves over time. For example, there is little doubt that the institutional changes in US household credit access in the 1970s and 1980s affected the path of autonomous household demand for the decades leading up to the financial crisis of 2008 and 2009. This example illustrates the more general point that “autonomous” need not mean “exogenous” or “constant.” Rather autonomous demand refers to spending that is not induced by the state of the economy. To apply the message of our model empirically is to explore how autonomous demand evolves in particular historical periods.³⁴ In this sense, our approach relates to the rich historical analysis of demand generation and its effects on aggregate growth in earlier heterodox growth research such as Steindl (1976).

In addition to empirical analysis of the dynamics of autonomous demand, further work is needed on the key structural parameters that link the demand and supply sides of the system. The

³³ Skott, (2016, p. 15) writes the “supply side matters, and there is nothing particularly Keynesian about an exclusive focus on the demand side.” Dosi et al. (2010) explore the complementarities between Keynesian demand dynamics and Schumpeterian technical change on the supply side.

³⁴ This perspective is at least partially consistent with the views presented by Nikiforos (2018).

calibration presented earlier is empirically based, but research needs to better pin down the relationship between the state of the economy, labor supply, and labor productivity (see Storm, 2017). These parameters are the key determinants of the range of demand-led growth paths that can be accommodated by supply.

The results have critical implications for policy. Most obviously, policy objectives must focus on demand well beyond the mainstream “short run.” When private autonomous demand falters public demand can help avoid stagnation. Furthermore, there is the danger of what Palley (2017) describes as policy “lock in,” demand-side austerity policy can limit supply growth and cause the appearance of a small traditionally defined output gap even though more aggressive demand growth policies could pull supply upward to generate better medium-term outcomes. As Mason (2017, p. 10) writes “a belief that hysteresis just reflects the ‘new normal’ can be self confirming.”

Cornwall and Cornwall (1994, page 238) write that their paper “can be seen as outlining a research strategy for investigators who might wish to put some numbers on programmes designed to better utilize available resources and to reduce unemployment.” Our paper is also step in this direction. We show that demand-led reductions in unemployment and greater resource utilization can be accommodated by the economy’s supply side within limits. Original empirical work shows that expanding autonomous demand can be a stable anchor for an actual path of demand-led growth and empirically relevant changes in demand growth can be accommodated by supply, inverting the logical sense of the old Say’s Law argument.

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Empirical Appendix and Parameter Calibration for Simulations

Estimates of Sensitivity of Consumption Spending to Changes in Aggregate Output

(Parameter $1 - s$)

This parameter measures the magnitude of *induced* consumption, that is, consumption spending that varies with the state of the economy.

We estimate this parameter in several ways. First, we perform a simple regression of real personal consumption expenditures (*PCE*) on real GDP.

$$PCE_t = Constant + (1 - s) * GDP_t$$

This regression directly estimates the basic specification for the consumption in the model.³⁵

Because GDP and consumption are almost certainly cointegrated, an OLS regression should provide a consistent estimate of the key parameter. To examine further robustness, however, we also estimate “dynamic OLS” (DOLS) models that will be asymptotically consistent under a broader range of assumptions.³⁶

$$PCE_t = Constant + (1 - s) * GDP_t + \sum_{j=-p}^p \Delta GDP_{t+j}$$

The estimates of $1 - s$ are remarkably consistent:

Model	Point Estimate of $1 - s$	Standard Error
OLS	0.7266	0.0055
DOLS, 1 lead and lag	0.7288	0.0049
DOLS, 2 leads and lags	0.7271	0.0046

Because our theoretical model is recursive, consumption in t depends on expectations formed based on earlier information. Therefore, we also estimated a two-stage least squares model with current output instrumented by two lags of output. The results were almost identical to those reported above.

We also considered adding more structure to the empirical model by linking consumption to disposable personal income (*DPI*, that is, income net of taxes) and then regressing *DPI* on GDP. In this case, the parameter $1 - s$ in the theoretical model would be proxied by the product of the two regression coefficients. The OLS results are:

³⁵ Annual data are taken from the US national accounts for 1980 through 2016. The results are not too sensitive to the choice of years, although the results for $1 - s$ tend to become slightly higher when the sample is restricted to more recent observations.

³⁶ Dynamic OLS adds leads and lags of the difference in the independent variable (in this case differences in real GDP) to the regressors.

Model	Point Estimate of $1 - s$	Standard Error
PCE on DPI	0.9449	0.0065
DPI on GDP	0.7672	0.0092
Product of the Estimates	0.7249	

It is not surprising that consumption is more sensitive to disposable income than it is to total output. But when one considers the effect of total output on disposable income the estimate for $1 - s$ is virtually identical with the estimates from the the first table above.

While *PCE* is the standard measure of consumption, it is in some ways misleading for our purposes. As discussed in detail in Cynamon and Fazzari (2017), *PCE* includes significant parts of consumption that likely should be considered autonomous, most obviously government-financed health care. The standard *PCE* measure also includes significant imputed components that are not cash expenditures on final output by the household sector, most obviously imputed rent on owner-occupied housing. Using updated data through 2016 following the methods described in Cynamon and Fazzari (2017) provides time-series data on household demand (*HH_DEM*), a measure more consistent with the consumption concept used in the theoretical model.

Replacing *PCE* with *HH_DEM* in the OLS and DOLS regressions discussed above gives the following results:

Model	Point Estimate of $1 - s$	Standard Error
OLS	0.4913	0.0080
DOLS, 1 lead and lag	0.4908	0.0081
DOLS, 2 leads and lags	0.4892	0.0080

The adjustments to remove items from *PCE* that are not cash expenditures of the household sector also imply the need to adjust *DPI*. For example, if government-financed health expenditure is not considered part of household demand then these payments should not be included in household disposable income (as they are in the standard definition of *DPI*).

We revised the data on disposable income to estimate actual cash income of the US household sector (again based on Cynamon and Fazzari, 2017) and then re-estimated the *DPI* regressions:

Model	Point Estimate of $1 - s$	Standard Error
<i>HH_DEM</i> on <i>CASH DPI</i>	0.8919	0.0169
<i>CASH DPI</i> on <i>GDP</i>	0.5466	0.0103
Product of the Estimates	0.4875	

Again, there is virtually no difference between the two approaches.

What the substantially lower estimate of $1 - s$ with *HH_DEM* implies is that if one adjusts consumption and income measures to remove government-financed consumption (which is substantial) and other imputations, a greater share of GDP is autonomous. This makes sense. Government spending on health care (as well as smaller categories like unemployment insurance) will not likely vary with income (indeed they are likely counter-cyclical).

This analysis leads us to a benchmark estimate for $1 - s$ more closely tied to the adjusted household demand approach, that is, about 0.5. Nonetheless, in the parameter range considered in the paper we allow for values derived from standard measures of *PCE* and set the range for $1 - s$ between 0.40 to 0.80.

Estimates of the Capital-Output Ratio and Capital Dynamics

(Parameters v^* and λ)

The measurement of capital is far from straightforward owing to the problems of aggregating heterogeneous capital goods based on value measures. That said, for our purposes the issue is not about measuring the value of a “factor of production.” Rather, our model requires the long-run capital-output ratio parameter (v^*) to quantify how investment depends on expected output growth.

For this reason, it is most useful to calibrate v^* using investment data. In steady state, the investment share of output is given by the simple expression:

$$\frac{I}{Y} = v^*(g^* + \delta).$$

The aggregate investment share in the US (I/Y) had a positive secular trend in the postwar decades that ended with the early 1980s recession. Since the early 1980s the investment share has been highly cyclical but it has little long-term trend. There was quite a large peak in the late 1990s associated with the “technology bubble,” however. To avoid the influence of that peak, we use the average investment share from 2002 to 2016 of 12.5% to calibrate v^* . (The average value for a longer period of 1984-2016 is not much different at 12.8%.) Assuming a steady state growth rate g^* of 2.5% and an average depreciation rate of 8.4% (Bureau of Economic Analysis, BEA, fixed capital depreciation also averaged over 2002 to 2016) yields an estimate for v^* of 1.14.

We also set the benchmark depreciation rate (δ) at 8.4%. This parameter has been quite stable in the data and has little effect on dynamic results, so we do not consider a range for δ .

It is noteworthy that if one calculates v^* from BEA non-residential capital stock data (value aggregates), again averaged over 2002 to 2016, the estimate is just slightly higher: 1.22.

Furthermore, we can estimate the dynamics of the capital-output ratio from the BEA capital stock data. The actual capital-output ratio in any year is a good proxy for the “target” ratio (\hat{v}_t) in our model. Using the actual value of the capital-output ratio in place of the target gives the simple auto-regression equation

$$v_t = \lambda v^* + (1 - \lambda)v_{t-1}$$

We used data from 1985 through 2016 to estimate this equation, a period in which v_t is largely without trend. The implied value for λ is 0.087 with a standard error of 0.055. The value implies substantial persistence in the capital-output ratio, which stabilizes the dynamics of our model. The regression constant can also be used to solve for an independent estimate of v^* which is 1.18, again consistent with the results discussed above.

From these estimates we set the benchmark value for v^* at 1.2, with a range from 1.00 to 1.50 and the benchmark value for λ at 0.09 with a range from 0.05 to 0.20.

Dynamics of Growth Expectations

(Parameter α)

In the model, growth expectations drive both consumption and investment. Expectations adapt in the direction of actual growth according to the equation:

$$Eg_t = (1 - \alpha)g_{t-1} + \alpha Eg_{t-1}.$$

To calibrate α , we employ data from the survey of professional forecasters on one-year ahead real GDP growth (available from 1992 through 2017) to measure expected growth. Actual growth is annualized real GDP growth. A regression that constrains the coefficients on lagged actual growth and lagged expected growth to sum to one implies extreme persistence with an estimate of α equal to 0.989 (standard error of 0.017). With lagged actual growth and lagged expected growth unconstrained in the regression the α estimate is 0.759 (standard error of 0.065) with the coefficient on lagged actual growth of 0.018 (standard error of 0.016).

These results imply persistent expectations. We set the benchmark value of α to 0.90 and the range from 0.75 to 0.95.

Labor Force Growth and the Unemployment Rate

(Parameter θ_1)

One of the key parameters that causes the convergence of the growth of supply to the growth of demand is θ_1 , the effect of the unemployment rate on the growth rate of labor supply:

$$g_t^{LS} = \theta_0 - \theta_1 u_{t-1}$$

The estimates of θ_1 depend to some extent on the time period chosen. The following table shows estimates of θ_1 from a regression of US labor force growth on the headline unemployment rate for three different sample lengths:

Sample Period	Point Estimate of θ_1	Standard Error
1980 - 2016	0.140	0.062
1990 - 2016	0.243	0.050
2000 - 2016	0.299	0.055

Results are similar using the dynamic least squares approach described earlier, although standard errors are somewhat higher. The rise in the size of the coefficient with a more recent sample may be the result of removing years when substantial labor force growth was driven by the large increase in female labor force participation.

From these results, we choose a benchmark value for θ_1 of 0.20 with a range from 0.10 to 0.30.

Labor Productivity Growth

(Parameters ρ_1 and ρ_2)

To calibrate the productivity growth equation:

$$g_t^A = \rho_0 - \rho_1 u_{t-1} + \rho_2 (g_{t-1}^K + \delta)$$

we need to estimate the effect of the unemployment level on productivity growth and the effect of capital replacement ($g_{t-1}^K + \delta$). Because of the difficulty of measuring the capital stock, we use the growth of output to proxy the growth of capital; in the long-run, the two measures should converge. We estimated the parameters with dynamic least squares (with two leads and lags) as described above. The table below reports point estimates and standard errors for ρ_1 and ρ_2 estimated in separate regressions and also in a single regression.

Model	Estimate ρ_1	Standard Error	Estimate ρ_2	Standard Error
DOLS for u effect alone	0.84	(0.19)	--	--
DOLS for g effect alone	--	--	0.47	(0.22)
DOLS for u and g together	0.51	(0.42)	0.51	(0.51)

In the regressions with the two effects estimated separately the coefficients are significant (especially for ρ_1) and economically large. Combining the two effects in a single regression with rather limited data the estimates are imprecise, but the point estimates are largely consistent with the separate regression results. (Note that the leads and lags in the DOLS regressions add five independent variables for each effect estimated. Therefore lack of precision in the combined regression is not surprising.)³⁷ Furthermore, the results for the growth effect are consistent with other estimates in the literature. The preferred “demand-side” estimate of the Verdoorn coefficient (equivalent to ρ_2 here) relating productivity growth to output growth in Angeriz, et al. (2008) is 0.67. Values greater than 0.5 are regularly cited. Michl (1985, table 2) finds Verdoorn coefficients estimated from international panel data that vary from 0.49 to 0.68 for manufacturing industries. Hein (2014, table 8.2, pp. 327-8) surveys a wide range of empirical estimates. While the estimate can be as low as 0.11, the vast majority of estimates for many countries fall between 0.3 and 0.6.

Based on this information, we set the benchmark value of ρ_1 and ρ_2 to 0.5 with ranges from 0.3 to 0.7. The possibility that ρ_1 in particular may be somewhat higher than the upper end of this range would only magnify the results discussed in the text.

A summary of the empirically based benchmark parameter values and their relevant range appears in table 1 in the text.

Simulations use the benchmark parameter values except as noted in the text. All simulations are calibrated with initial values to be in steady state as follows:

³⁷ Degrees of freedom are limited by the fact that the labor productivity series we use starts in 1987. If we use real output per hour rather than labor productivity the data go back to 1948. We explored this alternative data set as well. Results are within the range of parameters obtained from more recent data on labor productivity with somewhat smaller standard errors.

- Initial growth rate of autonomous demand (g^*): 2.5%
- Initial unemployment rate (u): 4.0%
- The intercept of the labor force growth equation (θ_0) is set to generate initial labor force growth of 0.5%
- The intercept of the productivity growth equation (ρ_0) is set to generate initial productivity growth of 0.5%

Note that with initial labor force growth of 0.5% and productivity growth of 2.0%, initial aggregate supply growth matches the steady-state rate of demand growth.

Simulations to explore stability begin in steady-state and the model is then shocked with a permanent increase in the autonomous demand growth rate from 2.5% to 3.0%, except as described in the text.

Appendix: Dynamic Stability in the Linearized Model

The squared term in the investment function as well as the presence of endogenous variables in the denominators of ratios introduce nonlinearities in our model. General statements about dynamic stability are difficult to obtain in nonlinear discrete-time models. Results from a linearized version of the model can be informative, however. If the eigenvalues of the linearized model are less than one in modulus, a region exists around the model's steady state such that, if the steady state is disturbed within that region, the dynamics will converge back to the steady state. The size of the stability region for our model does not have an analytically tractable representation. That said, simulations of our model show that the eigenvalue condition for stability in the linearized model correspond very closely to results for the full nonlinear model, as discussed below.

We derive the linearized model in two steps. First we present the linearized equations for the demand side of the model. We then add the supply-side equations and prove that if the linearized demand model is stable, the full model will be stable with supply converging to the demand-led growth path.

Demand-Side Model

Specify the demand-side model with a vector of five endogenous variables: the actual growth rate (g_t), expected growth rate (Eg_t), target capital-output ratio (\hat{v}_t), actual capital-output ratio (v_t), and the ratio of autonomous demand to output (f_t).

To simplify notation let G_t denote $1 + g_t$ and EG_t denote $1 + Eg_t$. As described in the paper the model has a unique (non-degenerate) steady state. Steady-state conditions for the key variables are as follows (with steady-state values denoted by an asterisk):

Growth: $G_t = EG_t = G^*$

Capital-output ratio: $v_t = \hat{v}_t = v^*$

Ratio of autonomous demand to output: $f_t = f^*$

Unemployment rate: $u_t = u^*$

All parameters in the model are non-negative. Economic logic would likely restrict all parameters to be less than one with the exception of the various v parameters (capital-output ratio). In principle, v could take on any positive value, but the sensible empirical range is likely from 0.5 to 3.

Equation for actual growth G_t (equation 2 in the main text):

$$G_t = (1 - s)(EG_t) + \hat{v}_t(EG_t)^2 - \frac{K_t}{Y_{t-1}}(1 - \delta) + \frac{F_t}{Y_{t-1}}$$

Using the identity $G_t = Y_t/Y_{t-1}$:

$$G_t = (1 - s)(EG_t) + \hat{v}_t(EG_t)^2 - v_t(G_t)(1 - \delta) + f_t(G_t)$$

where $v_t = K_t/Y_t$ and $f_t = F_t/Y_t$

We obtain partial derivatives of G_t with respect to all endogenous variables with total differentiation as follows:

$$dG_t = [-v_t(1 - \delta) + f_t]dG_t + [(1 - s) + 2\hat{v}_t(EG_t)]d(EG_t) + (EG_t)^2 d\hat{v}_t - (G_t)(1 - \delta)dv_t + (G_t)df_t.$$

The relevant partial derivatives in the neighborhood of steady-state are:

$$\begin{aligned} \frac{\partial G_t}{\partial (EG_t)} &= \frac{(1 - s) + 2v^*G^*}{1 + v^*(1 - \delta) - f^*} \\ \frac{\partial G_t}{\partial \hat{v}_t} &= \frac{(G^*)^2}{1 + v^*(1 - \delta) - f^*} \\ \frac{\partial G_t}{\partial v_t} &= \frac{-(1 - \delta)G^*}{1 + v^*(1 - \delta) - f^*} \\ \frac{\partial G_t}{\partial f_t} &= \frac{G^*}{1 + v^*(1 - \delta) - f^*} \end{aligned}$$

The linearized growth equation in the neighborhood of steady state:

$$G_t^L = \left[\frac{(1 - s) + 2v^*G^*}{1 + v^*(1 - \delta) - f^*} \right] (EG_t) + \left[\frac{(G^*)^2}{1 + v^*(1 - \delta) - f^*} \right] \hat{v}_t - \left[\frac{(1 - \delta)G^*}{1 + v^*(1 - \delta) - f^*} \right] v_t + \left[\frac{G^*}{1 + v^*(1 - \delta) - f^*} \right] f_t$$

or, equivalently:

$$0 = G_t^L [1 + v^*(1 - \delta) - f^*] - EG_t^L [(1 - s) + 2v^*G^*] - \hat{v}_t^L (G^*)^2 + v_t^L [(1 - \delta)G^*] - f_t^L (G^*)$$

The “L” superscript denotes a linearized version of the variable.

Equation for the capital-output ratio v_t :

$$v_t = \frac{K_t}{Y_t} = \frac{I_{t-1} + (1 - \delta)K_{t-1}}{Y_t}$$

Using the investment equation from the text and lagging it one period gives:

$$v_t = \frac{1}{Y_t} [\hat{v}_{t-1} (EG_{t-1})^2 Y_{t-2} - K_{t-1} (1 - \delta) + (1 - \delta) K_{t-1}]$$

$$v_t = \frac{1}{Y_t} [\hat{v}_{t-1} (EG_{t-1})^2 Y_{t-2}] = \hat{v}_{t-1} \frac{(EG_{t-1})^2}{(G_t)(G_{t-1})}$$

Derivatives evaluated at steady-state values:

$$\frac{\partial v_t}{\partial \hat{v}_{t-1}} = \frac{(G^*)^2}{(G^*)(G^*)} = 1$$

$$\frac{\partial v_t}{\partial G_t} = \frac{\partial v_t}{\partial G_{t-1}} = \frac{-v^*(G^*)^2}{(G^*)^3} = \frac{-v^*}{G^*}$$

$$\frac{\partial v_t}{\partial EG_{t-1}} = \frac{2v^*G^*}{(G^*)^2} = \frac{2v^*}{G^*}$$

Linearized capital-output ratio equation in the neighborhood of steady state:

$$v_t^L = \hat{v}_{t-1} - G_t^L \left(\frac{v^*}{G^*} \right) - G_{t-1}^L \left(\frac{v^*}{G^*} \right) + EG_{t-1}^L \left(\frac{2v^*}{G^*} \right)$$

$$v_t^L + G_t^L \left(\frac{v^*}{G^*} \right) = \hat{v}_{t-1} - G_{t-1}^L \left(\frac{v^*}{G^*} \right) + EG_{t-1}^L \left(\frac{2v^*}{G^*} \right)$$

Equation for expected growth EG_t :

The specification for growth expectations is:

$$Eg_t = (1 - \alpha)g_{t-1} + \alpha Eg_{t-1}$$

In terms of the notation used in this appendix:

$$EG_t - 1 = (1 - \alpha)(G_{t-1} - 1) + \alpha(EG_{t-1} - 1) = (1 - \alpha)(G_{t-1}) + \alpha(EG_{t-1}) - 1$$

which simplifies to the linear form:

$$EG_t^L = (1 - \alpha)(G_{t-1}^L) + \alpha(EG_{t-1}^L)$$

Equation for target capital-output ratio \hat{v}_t :

The equation is already in linear form:

$$\hat{v}_t^L = (1 - \lambda)v_{t-1}^L + \lambda v^*$$

Equation for autonomous demand-output ratio f_t :

Note that F_t grows at G^* and Y_t growth at G_t . Therefore:

$$f_t = \frac{F_t}{Y_t} = \frac{(G^*)F_{t-1}}{(G_t)Y_{t-1}} = \frac{G^*}{G_t} f_{t-1}$$

Derivatives evaluated at steady-state values:

$$\frac{\partial f_t}{\partial f_{t-1}} = \frac{G^*}{G^*} = 1$$

$$\frac{\partial f_t}{\partial G_t} = \frac{-f^* G^*}{(G^*)^2} = \frac{-f^*}{G^*}$$

Linearized autonomous demand-output ratio equation in the neighborhood of steady state:

$$f_t^L = f_{t-1}^L - G_t^L \left(\frac{f^*}{G^*} \right)$$

$$f_t^L + G_t^L \left(\frac{f^*}{G^*} \right) = f_{t-1}^L$$

Demand-side system in recursive form:

The full system can be written in matrix form as follows:

$$\begin{bmatrix} 1 + v^*(1 - \delta) - f^* & (1 - \delta)G^* & -[(1 - s) + 2v^*G^*] & -(G^*)^2 & -G^* \\ v^*/G^* & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ f^*/G^* & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} G_t^L \\ v_t^L \\ EG_t^L \\ \hat{v}_t^L \\ f_t^L \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -v^*/G^* & 0 & 2v^*/G^* & 1 & 0 \\ 1 - \alpha & 0 & \alpha & 0 & 0 \\ 0 & 1 - \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} G_{t-1}^L \\ v_{t-1}^L \\ EG_{t-1}^L \\ \hat{v}_{t-1}^L \\ f_{t-1}^L \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \lambda v^* \\ 0 \end{bmatrix}$$

or $\mathbf{B}\vec{X}_t = \mathbf{C}\vec{X}_{t-1} + \vec{Z}$. The eigenvalues of the Jacobian matrix $\mathbf{B}^{-1}\mathbf{C}$ determine the local stability of the model's demand side. Analytical solutions for these eigenvalues are difficult to derive and largely impossible to interpret economically. As mentioned above, however, simulations confirm that if the modulus of the maximum eigenvalue of $\mathbf{B}^{-1}\mathbf{C}$ is less than one then the model is stable unless the modulus of the maximum eigenvalue is very close to unity or the shock is very large. More specific information appears below after the supply side is added.

Supply Side

The growth of supply is specified as a linear function (labor supply growth plus productivity growth). But nonlinearities are induced by including the growth of the capital stock (g_t^K) and the unemployment rate (u_t) in the specification of supply growth.

Growth of aggregate supply (sum of labor supply growth and productivity growth):

$$G_t^{SL} = 1 + [\theta_0 - \theta_1 u_{t-1}] + [\rho_0 - \rho_1 u_{t-1} + \rho_2 (g_t^{KL} + \delta)]$$

$$= 1 + \theta_0 + \rho_0 + \rho_2 \delta - (\theta_1 + \rho_1) u_{t-1} + \rho_2 g_t^{KL}$$

$$G_t^{SL} - \rho_2 g_t^{KL} = 1 + \theta_0 + \rho_0 + \rho_2 \delta - (\theta_1 + \rho_1) u_{t-1}$$

Capital growth at a rate determined by last period's investment less depreciation:

$$g_t^K = \frac{I_{t-1}}{K_{t-1}} - \delta = \frac{\hat{v}_{t-1}(EG_{t-1})^2 Y_{t-2} - K_{t-1}(1 - \delta)}{K_{t-1}} - \delta$$

$$g_t^K = \frac{\hat{v}_{t-1}(EG_{t-1})^2 Y_{t-1}}{G_{t-1} K_{t-1}} - 1 = \frac{\hat{v}_{t-1}(EG_{t-1})^2}{v_{t-1} G_{t-1}} - 1$$

Linearization of g_t^K evaluated at steady state:

$$g_t^{KL} = \left(\frac{G^*}{v^*}\right) \hat{v}_{t-1}^L + (2)EG_{t-1}^L - \left(\frac{G^*}{v^*}\right) v_{t-1}^L - G_{t-1}^L - 1$$

Unemployment rate dynamics:

$$u_t = 1 - \frac{L_t}{L_t^S} = 1 - \frac{Y_t}{Y_t^S} = 1 - \frac{G_t Y_{t-1}}{G_t^S Y_{t-1}^S} = 1 - \left(\frac{G_t}{G_t^S}\right) (1 - u_{t-1})$$

Linearization of u_t :

$$u_t^L = 1 - \left(\frac{1}{G^*}\right) (1 - u^*) G_t^L + \left(\frac{1}{G^*}\right) (1 - u^*) G_t^S + u_{t-1}^L$$

$$u_t^L + \left(\frac{1}{G^*}\right) (1 - u^*) G_t^L - \left(\frac{1}{G^*}\right) (1 - u^*) G_t^S = 1 + u_{t-1}^L$$

Incorporating the supply side adds three variables to the demand-side model (G_t^{SL} , g_t^{KL} , and u_t^L). The full linearized model in recursive form becomes:

$$\begin{bmatrix} 1 + v^*(1 - \delta) - f^* & (1 - \delta)G^* & -[(1 - s) + 2v^*G^*] & -(G^*)^2 & -G^* & 0 & 0 & 0 \\ v^*/G^* & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ f^*/G^* & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\rho_2 \\ \frac{1 - u^*}{G^*} & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} G_t^L \\ v_t^L \\ EG_t^L \\ \hat{v}_t^L \\ f_t^L \\ G_t^{SL} \\ g_t^{KL} \\ u_t^L \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -v^*/G^* & 0 & 2v^*/G^* & 1 & 0 & 0 & 0 & 0 \\ 1 - \alpha & 0 & \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 - \lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(\theta_1 + \rho_1) \\ -1 & -G^*/v^* & 2 & G^*/v^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} G_{t-1}^L \\ v_{t-1}^L \\ EG_{t-1}^L \\ \hat{v}_{t-1}^L \\ f_{t-1}^L \\ G_{t-1}^{SL} \\ g_{t-1}^{KL} \\ u_{t-1}^L \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \lambda v^* \\ 0 \\ 1 + \theta_0 + \rho_0 + \rho_2 \delta \\ -1 \\ 1 \end{bmatrix}$$

Again, this matrix equation has the form $\mathbf{B}\vec{X}_t = \mathbf{C}\vec{X}_{t-1} + \vec{Z}$ and the eigenvalues of the Jacobian matrix $\mathbf{B}^{-1}\mathbf{C}$ determine the stability of the system in a neighborhood of the steady state.

Note the 5x3 block of zeroes in the upper right corners of the \mathbf{B} and \mathbf{C} matrices. This structure reflects the fact that there is no feedback from the supply side to the demand side and it provides useful

information about the dynamics of supply and demand. Write the matrices \mathbf{B} and \mathbf{C} in block form as follows:

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{0} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} \mathbf{C}_{11} & \mathbf{0} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{bmatrix} \text{ with } \mathbf{C}_{22} = \begin{bmatrix} 0 & 0 & -(\theta_1 + \rho_1) \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix multiplication verifies that

$$\mathbf{B}^{-1} = \begin{bmatrix} \mathbf{B}_{11}^{-1} & \mathbf{0} \\ \mathbf{D}_{21} & \mathbf{D}_{22} \end{bmatrix} \text{ with } \mathbf{D}_{22} = \begin{bmatrix} 1 & \rho_2 & 0 \\ 0 & 1 & 0 \\ (1 - u^*)/G^* & -\rho_2(1 - u^*)/G^* & 1 \end{bmatrix}$$

$$\mathbf{B}^{-1}\mathbf{C} = \begin{bmatrix} \mathbf{B}_{11}^{-1}\mathbf{C}_{11} & \mathbf{0} \\ \mathbf{E}_{21} & \mathbf{E}_{22} \end{bmatrix} \text{ with } \mathbf{E}_{22} = \mathbf{D}_{22}\mathbf{C}_{22} = \begin{bmatrix} 0 & 0 & -(\theta_1 + \rho_1) \\ 0 & 0 & 0 \\ 0 & 0 & 1 - (\theta_1 + \rho_1)(1 - u^*)/G^* \end{bmatrix}.$$

Because $\mathbf{B}^{-1}\mathbf{C}$ is block triangular, its eigenvalues are the union of the eigenvalues of the two blocks $\mathbf{B}_{11}^{-1}\mathbf{C}_{11}$ and \mathbf{E}_{22} . The eigenvalues of $\mathbf{B}_{11}^{-1}\mathbf{C}_{11}$ are the eigenvalues of the demand side of the system which, as described above, are a complicated function of demand-side parameters. But the eigenvalues of the simple matrix \mathbf{E}_{22} are two zeroes and the diagonal element $1 - (\theta_1 + \rho_1)(1 - u^*)/G^*$. This additional eigenvalue introduced by the supply side is always less than one. It will exceed -1 except in the very unlikely case that $\theta_1 + \rho_1 > 2$, which would imply implausibly large effects of the unemployment rate on labor force and/or productivity growth.

These observations prove that if the modulus of the maximum eigenvalue of the linearized demand side is less than one, the modulus of the maximum eigenvalue of the full linearized system with the supply side will be less than one. Therefore, in a neighborhood around the steady state of the model, *if the demand side is stable, supply will converge to demand.*

Extensive simulations show that this neighborhood corresponds very closely with the behavior of the full nonlinear model. For example, with the benchmark parameter values the maximum eigenvalue of the linearized model is 0.943 and simulations of the nonlinear model are stable for shocks exceeding 20 percent of GDP. Reducing the expectation adjustment parameter (α) from the benchmark value of 0.90 to 0.70 raises the maximum eigenvalue to 0.953 and the amplitude of the cycles rises, but the model remains stable for shocks to the growth of autonomous demand exceeding 20 percent of GDP. Modest further reductions of α push the model toward instability. With $\alpha = 0.68$, the maximum eigenvalue becomes 0.970 and an autonomous demand shock greater than 18% of GDP induces unstable cycles. This outcome occurs even though the linearized model is stable because the shock is large enough to push the system outside the stable neighborhood of the nonlinear model. With $\alpha = 0.65$, the linearized model is just barely stable with a maximum eigenvalue of 0.996. In this case shocks as small as 3.1% of GDP lead to unstable dynamics.