Network topology of olivine–basalt partial melts

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Accepted 2017 April 21. Received 2017 April 19; in original form 2017 March 1

SUMMARY
The microstructural relationship between melt and solid grains in partially molten rocks influences many physical properties, including permeability, rheology, electrical conductivity and seismic wave speeds. In this study, the connectivity of melt networks in the olivine–basalt system is explored using a systematic survey of 3-D X-ray microtomographic data. Experimentally synthesized samples with 2 and 5 vol.% melt are analysed as a series of melt tubules intersecting at nodes. Each node is characterized by a coordination number (CN), which is the number of melt tubules that intersect at that location. Statistically representative volumes are described by coordination number distributions (CND). Polyhedral grains can be packed in many configurations yielding different CNDs, however widely accepted theory predicts that systems with small dihedral angles, such as olivine–basalt, should exhibit a predominant CN of four. In this study, melt objects are identified with CN = 2–8, however more than 50 per cent are CN = 4, providing experimental verification of this theoretical prediction. A conceptual model that considers the role of heterogeneity in local grain size and melt fraction is proposed to explain the formation of nodes with CN ≠ 4. Correctly identifying the melt network topology is essential to understanding the relationship between permeability and porosity, and hence the transport properties of partial molten mantle rocks.

Key words: Microstructure; Permeability and porosity; Mantle processes.

1 INTRODUCTION
Beneath mid-ocean ridges, adiabatic decompression produces partial melt in ascending mantle peridotite. The buoyant melt formed by this process rises through the mantle, is focused towards the ridge axis, and forms new oceanic crust. The chemical and physical properties of the upper mantle and lower crust are strongly influenced by the permeability and melt distribution of the partially molten rocks (McKenzie 1984; Spiegelman & McKenzie 1987; Kelemen et al. 1997).

There exist contradicting views on melt distribution beneath mid-ocean ridges. The uranium decay series disequilibria in mid-ocean ridge basalts implies a well-connected, permeable mantle for rapid melt segregation at melt fractions as low as 0.01 (e.g. Johnson et al. 1990; Lundstrom et al. 1995; Salters & Longhi 1999). In contrast, geophysical surveys detect a broad low-velocity region (e.g. The Melt Seismic Team 1998; Evans et al. 1999), which are interpreted as evidence for high melt fractions beneath the ridge. A recent magnetotelluric study suggests that local melt fractions may be as high as 0.10 (Key et al. 2013).

Permeability (Riley & Kohlstedt 1991; Wark & Watson 1998; Renner et al. 2003; Bernabé et al. 2010) and electrical conductivity (Waff 1974; Yoshino et al. 2010) are sensitive to the volume, composition and spatial distribution of melts. Understanding the 3-D melt microstructure in idealized systems, such as olivine plus basalt melt, is critical for interpreting a wide range of experimental and geological observations, and for creating models of geochemical and geophysical processes. To reconcile the discrepancies of current results, the first step is to obtain more accurate descriptions of the relationship between permeability (or electrical conductivity) and the melt distribution.

In partially molten systems, minimization of interfacial energy predicts various types of melt networks, which range from fully connected melt channels to isolated melt pockets (e.g. Smith 1964; Kingery et al. 1976). The connectivity of the melt depends on the dihedral or wetting angle of the liquid–solid interface, the volume of melt present and the grain shape. Crystalline materials can be packed in a variety of space-filling configurations and different grain shapes have been used to model partially molten rocks. Bulau et al. (1979) modeled grains as truncated octahedra, a space-filling tetrakaidecahedron with 14 faces. With this geometry, melt is predicted to reside along edges, where three grains meet, and in corners, where four grains meet. Many subsequent numerical studies have adopted this geometry to calculate permeability...
Table 1. Number of nodes and node density for samples investigated in this study.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Melt fraction</th>
<th>Number of nodes (this study)</th>
<th>Density of nodes [m$^{-3}$] (this study)</th>
<th>Number of nodes (Miller et al. 2014)</th>
<th>Density of nodes [m$^{-3}$] (Miller et al. 2014)</th>
</tr>
</thead>
<tbody>
<tr>
<td>scoba-09</td>
<td>0.02</td>
<td>206</td>
<td>$4.8 \times 10^{12}$</td>
<td>635</td>
<td>$1.5 \times 10^{13}$</td>
</tr>
<tr>
<td>scoba-12</td>
<td>0.05</td>
<td>175</td>
<td>$6.4 \times 10^{13}$</td>
<td>3407</td>
<td>$8.0 \times 10^{13}$</td>
</tr>
</tbody>
</table>

2 METHODS

Samples investigated in this study were synthesized from mixtures of San Carlos olivine and high-alumina basalt in a solid medium piston-cylinder apparatus. Quenched experimental run products were analysed using synchrotron-based X-ray microtomography at the Advanced Photon Source at Argonne National Laboratory (see Zhu et al. 2011 and Miller et al. 2014 for complete details of sample synthesis and microtomographic data collection). Two samples are examined in this study: scoba-9 (2 vol.% melt), which was annealed for 336 hr and scoba-12 (5 vol.% melt), which was annealed for 168 hr (Table 1). The average grain size of both samples is ~40 μm, and the samples are inferred to be in microstructural equilibrium (Miller et al. 2014).

Data were processed and reduced to binary form so that solid olivine is distinguishable from basalt glass. Subsets of these data were created, corresponding to cubes with an edge length of 500 pixels (350 μm) for scoba-9 (2 per cent melt) and 200 pixels (140 μm) for scoba-12 (5 per cent melt). Data were converted to isosurfaces, which delineate the boundary between solid and melt. These 3-D microstructural data sets are identical to those described in Zhu et al. (2011) and Miller et al. (2014); no additional smoothing or filtering was imposed. Data were imported into ArcScene software for 3-D visualization and a grid system was superimposed on the data to facilitate a spatially systematic survey of the melt geometry (Fig. 2). For some analyses, data were projected into the Foss3D visualization cave at Washington University in Saint Louis (Fig. 3).

Data are interpreted as a series of melt tubules, assumed to form at grain edges, connected at nodes, which are assumed to form within pores at grain corners. Nodes are characterized by the number of connecting tubules, which we define as the CN. The size of the nodes, or pores, is determined by measuring their length in three orthogonal directions and calculating the diameter of an equivalent sphere. Melt objects are identified visually using the following criteria: (1) nodes are identified as the junction where two or more distinct tubules meet. The terminus of a single tubule is not interpreted to be a unique node unless an obvious volume of melt is present, with a diameter significantly larger than the diameter of the associated tubule. (2) Tubules are classified as such if their...
Figure 2. Melt network for sample scoba-12 (5 per cent melt) with a super-imposed grid system used for systematic surveying. Grey objects represent melt and open spaces are the olivine matrix.

minimum length is greater than or equal to the diameter of the associated node. (3) Nodes with tubules that appear to be truncated by the edge of the data space are not included in the analysis unless their CN can be confidently inferred. Typically, this requires the node centre to be at least 10 \( \mu \text{m} \) from the edge of the data space.

(4) Clusters of melt or nodes with large CN (CN > 4) are split into their component nodes when clear distinctions can be made. For example, a cluster that appears to have six tubules could be classified as two discrete nodes with four tubules each if the two nodes are connected by a tubule that is long enough to be counted according to criterion (2).

3 RESULTS

A variety of melt objects were identified using this direct survey approach. The CN of these objects range from \( \text{CN} = 2 \) to 8. Examples of nodes with different CN are shown in Fig. 4.

The connectivity of the melt networks is quantified using CND, which show the normalized frequency of melt objects with a given number of associated melt tubules (Fig. 5). For sample, scoba-9 (2 vol.% melt), 206 melt objects were identified over a volume of \( 4.3 \times 10^7 \mu \text{m}^3 \) (a cube with edge length 350 \( \mu \text{m} \)), corresponding to a melt object density of \( 4.8 \times 10^{12} \text{m}^{-3} \) (Table 1). Within this volume, there are \( \sim 100 \) grains (Miller et al. 2014), and so an average of two melt objects per grain were identified. The CND has a mode of four, with 69.4 per cent of nodes populating this category (Fig. 5a). Nodes with \( \text{CN} = 3 \) are the next most abundant, making up 17.5 per cent of all observed nodes. 12.6 per cent of nodes have \( \text{CN} \) of 5 or 6. Only one node was identified with a \( \text{CN} = 2 \) (<1 per cent of all nodes). No nodes with a \( \text{CN} \geq 7 \) were observed.

For sample scoba-12 (5 per cent melt), data collection was performed three times by two different authors to test for reproducibility and analytical bias. In total, 175 \( \pm 9 \) melt objects were identified over a volume of \( 2.7 \times 10^6 \mu \text{m}^3 \) (a cube with edge length 140 \( \mu \text{m} \)), giving this sample an average melt object density of \( 6.4 \times 10^{13} \text{m}^{-3} \) (Table 1). Within this volume, there are \( \sim 30 \) grains (Miller et al. 2014), and thus an average of approximately six melt objects per

Figure 3. Data projected in the 3-D visualization cave in the Fossett Laboratory for Virtual Planetary Exploration at Washington University in Saint Louis. The visualization cave projects data onto three walls and the floor. Users, wearing 3-D glasses, are able to ‘fly’ into the sample and readily view objects from several orientations that might be obscured by other microstructural features.
4 DISCUSSION

The geometry of melt in isotropic materials can be predicted from theory if the dihedral angle of the system is known. von Bargen & Waff (1986) demonstrate that theoretical systems with a dihedral angle smaller than 60° will always exhibit a completely interconnected melt network at melt fractions of 0.005 or lower. For systems with dihedral angles greater than 60°, melt will not form an interconnected network unless the system is above a critical melt fraction. A number of experimental studies have characterized the distribution of dihedral angles in the olivine–basalt system. Waff & Bulau (1979), investigating samples equilibrated at high temperature and pressure, observe a median dihedral angle of 47° (1979), investigating samples equilibrated at high temperature and pressure, observe a median dihedral angle of 47°, below the critical threshold for interconnectivity at low melt fractions. Further microscopic study corroborated this observation, and also confirmed the absence of melt films along boundaries between two olivine grains (Vaughan & Kohlstedt 1982), although this latter conclusion has been debated (Garapić et al. 2013). Based on theory and experiments, it is generally assumed that the melt in the olivine–basalt system will form an interconnected network characterized by nodes with a predominant CN of four (Fig. 5a).

In an ideal system of isotropic surface energy and uniform grain size, melt topology at textural equilibrium is characterized by a constant mean curvature, and a correspondingly constant dihedral angle (Smith 1964; von Bargen & Waff 1986). However, in the olivine–basalt system, dihedral angle may not be constant, which could lead to more complex melt geometries. For example, anisotropic surface energy (Waff & Bulau 1979; Cooper & Kohlstedt 1982; Vaughan & Kohlstedt 1982; Laporte & Watson 1995), and the presence of faceted grains (Cmíral et al. 1998; Price et al. 2006; Yoshino et al. 2006), may introduce variation in the dihedral angle as a function of crystallographic orientation and texture. Grain-scale heterogeneities, such as non-uniform grain size and grain shapes that do not conform to the idealized foam-textured tetrakaidecahedra (e.g. Kingery et al. 1976) may modify melt geometry in a way that is not captured by isotropic theory (Bulau et al. 1979; Wark & Watson 2000). Finally, while not strictly relevant to this study, theory and experiments suggest that under non-static conditions melt will segregate through dynamic pressure gradients into bands of relatively high and low melt concentration, modifying the distribution of melts in partially molten rocks (Holtzman et al. 2003; Kohlstedt & Holtzman 2009).

Previous efforts by Zhu et al. (2011) to quantify melt connectivity in three dimensions used a skeletonization algorithm to reduce melt networks to a series of nodes connected by tubules, from which CNDs are readily calculated. This method is efficient, and capable of processing data from large sample volumes. However, there are limitations to this algorithm that impose artefacts on the resulting data. For example, any terminus in the melt network is identified as a node, with a CN equal to one. Therefore, the number of nodes with CN = 1 is likely overestimated. Furthermore, the algorithm was purposefully configured to exclude nodes with CN = 2, as these do not modify the sample’s permeability. Hence, the CNDs from Zhu et al. (2011) do not contain any data with CN = 2. Larger clusters of melt, with closely spaced nodes or unusually shaped tubules are also difficult for the algorithm to parse, which leads to a preponderance of nodes with CN > 4. The method used in this study, while relatively labor intensive, provides a more accurate view of the coordination of the melt networks. Objects that occur at the terminus of a melt tubule are readily identified as places where the melt pinches out, or becomes narrower than the spatial resolution of the microtomography technique (~0.7 µm). Nodes with large CNs can be evaluated in the context of the local microstructure to determine whether they are singular objects, or closely spaced clusters of smaller objects. In doing so, we gain a richer picture of the grain-scale organization of crystalline and molten material (Figs 2 and 4).

Considerable differences are observed between the CNDs for scoba-9 (2 per cent melt) determined by our survey and by Zhu et al. (2011, Fig. 5a). First, our microstructural analysis indicates that the large number of objects with CN = 1 (49.8 per cent) identified by the skeletonization algorithm are places where melt tubules

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Figure 4. Examples of melt objects with coordination numbers (CN) ranging from 3 to 8. The tubules associated with each node are numbered. Scale bars are 10 µm.
In this study, CNDs determined for olivine samples with 2–5 per cent basaltic melt exhibit a mode CN of four (Fig. 5), consistent with theoretical predictions of Bulau et al. (1979). Moreover, the distributions are narrow, and nodes with CN = 4 represent greater than 50 per cent of the objects identified. In both samples, the second most abundant nodes are those with CN = 3 (Fig. 3). We interpret these objects as nodes for which one of the four associated edges does not exhibit evidence of melting. The data do not allow us to distinguish between grain edges that contain no melt, and grain edges for which the local melt fraction was below detectable limit of the current microtomography technique.

As discussed earlier, the olivine–basalt system is not strictly isotropic and homogeneous, which could lead to grain-scale variation in melt fraction. If locally varying melt fraction is a feature of the olivine–basalt system, edges with lower than average melt fraction must be compensated for elsewhere by sites with higher local melt fractions. Indeed, locally elevated melt fractions seem to be a characteristic of melt objects with larger CNs. Figs 4(c)–(h) show several examples of melt objects with CN > 4. However, detailed examination of nodes with CN > 4 reveals that the majority of these objects are actually formed by joining closely spaced melt corners into a large pocket of melt. Objects with CN = 6 are particularly instructive (Figs 4c and f). Based on the shape and spacing of the melt tubules and nodes, we infer that CN = 6 nodes are formed by two adjoining CN = 4 nodes connected by a short and wide melt tubule. The short length of the connecting tubule is a consequence of the comparatively small size of the three adjacent grains. The width and volume of the connecting tubule may be related to locally elevated melt concentration. The three smaller grains that surround the connecting melt tubule, plus the two larger grains that form the opposite ends of the CN = 6 node, yield a melt object constructed of the interfaces between five grains. No true CN = 6 nodes, in which six melt tubules and six grains meet at a single grain corner, were observed in this survey. Based on these microstructural observations, we propose that most of CN > 4 nodes are formed by linking two or more closely spaced CN = 4 nodes (Fig. 4), and are not a result of true six-grain junctions (Fig. 1b). Therefore, we conclude that the olivine packing is best modeled by truncated octahedral grains; grains packed in a rhombic dodecahedral configuration are inconsistent with our results.

The coalescence of CN = 4 nodes into melt objects with CN > 4 may be due to (1) dissimilar grain size between adjacent grains, leading to short connecting tubules and/or (2) a larger local melt fraction that causes melt to ‘overflow’ into the grain boundaries, as is expected for larger melt fractions (von Bargen & Waff 1986). As illustrated in Figs 4(e) and (f), and 6(b), connecting two standard CN = 4 nodes forms an object with CN = 6. Connecting three standard nodes in a line or four nodes in a closed loop, produces an object with CN = 8 (Fig. 6c). Objects with odd CNs (CN = 5 or 7) may be generated in a similar way through combinations of nodes with CN = 3 and 4. With melt fractions between 2 and 5 per cent, we consider it unlikely that more than four corners should become connected in this manner, and indeed no objects were identified with CN > 8. Although not analysed in this study, Zhu et al. (2011) found that in a sample with 20 per cent melt, some grain boundaries are wet and objects with CN = 9 are found. Note that there are more objects with CN > 4 in higher melt fraction sample scoba-12 (5 per cent melt) than those in scoba-9 (2 per cent melt), and that objects with CN = 8 are observed in scoba-12 only (Fig. 5), which

Figure 5. Coordination number distributions (CND) for samples (a) scoba-9 (2 per cent melt) and (b) scoba-12 (5 per cent melt, bottom). Grey bars are the CNDs for this study, determined using a direct survey approach. White bars are the CNDs reported by Zhu et al. (2011), determined using an automatic computer algorithm. Short black lines on the scoba-12 CND (Fig. 5a) illustrate the CN detection limits, as suggested by Zhu et al. (2011). These observations also confirm that the abundance of nodes with CN = 3 detected by the algorithm is indeed an artefact of resolution limits, as suggested by Zhu et al. (2011). The density of melt objects detected by the algorithm is three times greater than the density of objects detected by the survey approach (Table 1), which is interpreted to be a result of misidentified nodes with CN = 1.

The CNDs for scoba-12 (5 per cent melt) determined by this study and by Zhu et al. (2011) are more comparable (Fig. 5b). Both methods find the mode of the CND to be equal to four, although this study identifies a greater abundance of nodes with CN = 4 and none with CN = 1, leading to a narrower distribution of CNs. For scoba-12, the density of objects identified by the two methods is similar, with the algorithm identifying approximately 25 per cent more objects than the survey approach (Table 1).
Olivine–basalt melt network topology

Figure 6. Schematic diagrams of melt networks for (a) CN = 4; (b) CN = 6 and (c) CN = 8. Inset figures show examples of nodes with these coordination numbers (see also Fig. 4). Nodes with CN = 4 are the most abundant melt objects in this study, and networks composed of CN = 4 are predicted to represent the equilibrium melt texture in the olivine–basalt system. Nodes with coordination numbers greater than four are interpreted to be formed by the coalescence of two or more standard nodes. The connection of these nodes may be due in part to local variation in the grain size, which shortens the lengths of some tubules, as well as locally elevated melt fractions.

is consistent with the ‘melt overflow’ conceptual model illustrated in Fig. 6.

In general, permeability of a porous rock depends on both pore-size distribution and pore connectivity, and the latter can be characterized by the average CN. Using network model simulations, Bernabé et al. (2010) derived ‘universal’ power-law relationships between permeability $k$ and the average CN $z$ of the network:

$$k \sim (z - z_c)^{\beta}$$  \hspace{1cm} (1)

where $z_c = 1.5$ for 3-D pore networks. Parameter $\beta$ is a non-dimensional constant that depends on the normalized standard deviation of the tubule diameter $\sigma$, (Bernabé et al. 2011):

$$\beta = 1.2343 + 0.93462\sigma + 1.4755\sigma^2$$  \hspace{1cm} (2)

$$\sigma = \sigma / (r)$$  \hspace{1cm} (3)

where $\sigma$ is the standard deviation of the tubule diameter distribution and $(r)$ is the average tubule diameter. For partially molten samples, the throat size of the melt tube is proportional to the grain size (i.e. von Bargen & Waff 1986). Therefore, parameter $\beta$ can be obtained from the grain size distribution of scoba-12, which is lognormal (Miller et al. 2014).

A histogram of grain size from sample scoba-12 is provided by Miller et al. (2014); the normalized standard deviation for a lognormal distribution is 0.125, which yields $\beta = 1.37$. In Fig. 7, we model permeability as a function of CN. From the 3-D microtomography data, Miller et al. (2014) calculated that the permeability of scoba-12 is $5.2 \times 10^{-15}$ m$^2$, which we use to anchor the calculation at $z = 4$. Assuming constant porosity and pore-size distribution, a network comprised of nodes with CN = 4 and 6 would yield an average CN $z = 5$, and would be $\sim 60$ per cent more permeable. This represents a conservative lower limit, as scoba-12 is synthesized experimentally and likely more homogeneous (therefore with permeability less sensitive to CN) than heterogeneous natural samples with comparable porosity. The sensitivity of permeability to CN is an important factor that must be considered when constructing numerical models of flow in porous media.

5 CONCLUSIONS

This study presents the first visual, 3-D survey of melt network coordination in the olivine–basalt system. Our survey approach represents a marked improvement over previous approaches by deciphering the CN and providing a more accurate picture of melt network microstructure at the level of individual melt pockets and tubules. Over a range of melt fractions (2–5 vol.%), melt forms an interconnected network, with tubules along grain edges meeting at melt nodes located at grain corners. A range of connectivity is observed, with CNs ranging from CN = 2 to 8. Conceptually, this variability in the melt network microstructure reflects the complexities introduced by anisotropic surface energy and variations in grain
size, leading to local variation in melt content. Our data indicate that most of CN > 4 nodes are formed by the coalescence of two or more closely spaced melt nodes. Although the melt microstructures are complex, the predominant CN is four. Therefore, it is concluded the grain packing in the olivine–basalt system is better approximated by truncated octahedra (Bulau et al. 1979) than rhombic dodecahedra (Park & Yoon 1985). This result confirms experimentally the theoretical prediction of melt connectivity proposed by Bulau et al. (1979).

ACKNOWLEDGEMENTS

The authors thank Bill Winston and Washington University Data and GIS Services for assistance with ArcScene and projection of the data in the Fossett Lab for Virtual Planetary Exploration. Ian Jackson and one anonymous reviewer are thanked for their constructive comments. This work is partially supported by the National Science Foundation EAR-1352306 (to PS) and EAR-1250338 (to WZ), and a grant from the Washington University Office of Undergraduate Research (to MMC). Micromosaic data for this paper are available upon request.

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