GOALS, CONSTRAINTS, AND PUBLIC ASSIGNMENT
A FIELD STUDY OF THE UEFA CHAMPIONS LEAGUE

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Abstract. We analyze a dynamic matching mechanism developed for the UEFA Champions League, the largest and most-watched football club competition worldwide. First, we theoretically characterize the assignment rule developed by UEFA by solving a complex constrained assignment problem with a publicly verifiable draw. Then, using a structural model of the assignments and data from the UEFA Champions League 2004 and 2019 seasons we show that the constraints cause quantitatively large spillovers to unconstrained teams. Nevertheless, we conclude that the UEFA draw is close to a constrained-best in terms of fairness. Moreover, we find that it is feasible to substantially reduce the distortions by only marginally slacking the constraints.

1. Introduction

When designing an institution economists typically prioritize theoretical properties, sometimes to the extent that the resulting mechanism can become too complex for participants to understand and trust. In contrast, in some high-stake public settings the focus is placed on transparency and simplicity, potentially (but not necessary) at a cost to the mechanisms' theoretic goals. In the present paper, we examine a randomized matching procedure designed to solve a complex assignment problem in a way that is transparent and comprehensible to the general public. The designed draw assembles football-team pairs in a sport tournament with huge public interest: the Union of European Football Association's (UEFA) Champions League (UCL). While the randomization's transparent procedure helps maintain the integrity of the draw under a series of non-trivial combinatoric constraints, we show—via a

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recent market-design tool—that to all practical extents the UEFA randomization procedure is a constrained-best in terms of ex ante fairness.

The UCL is one of the most successful pan-European ventures, and certainly the one with the most enthusiasm from the fans and the general public. The tournament brings together football clubs from across the continent (and beyond) that normally play within their own country-level associations. Selection into the competition is limited to the highest-performing clubs from each nation, where countries with a deeper history of success are allotted more places. A series of initial qualifying rounds whittle the number of participating teams down to 32 group-stage participants. From there, half of the clubs advance to a knockout stage that begins with the Round of 16 (R16), followed by four quarterfinals, two semifinals, and a final that determines a European champion. Outside of the World Cup, the UCL final game is one of the most-watched global sporting events, eclipsing even the viewership of the Superbowl in the United States.

Because the UCL is under a magnifying glass, from both the teams, fans and the media, UEFA has a clear interest in creating impartial, meritocratic, and publicly verifiable rules for the tournament. While a fair assignment rule with a public randomization would be simple to design in many situations—for instance, matching teams through urn draws without replacement—the tournament's problem is complicated due to three constraints imposed on match pairs: (i) Each pairing must be between a group winner and a group runner-up (*the bipartite constraint*); (ii) Teams that played one another in the prior group stage cannot be matched (*the group constraint*); (iii) Teams from the same national association cannot be matched (*the association constraint*).

In order to address the need for a transparent public randomization over a constrained assignment UEFA designed a novel matching procedure. The chosen randomization assembles each R16 team pairing through a dynamic public draw of balls from an urn, where the urns' composition adapts dynamically in order to respect the assignment constraints. Our paper sets out to analyze the properties of this chosen assignment mechanism, using the main tools of market design: theory, estimation, and computer simulation (Roth, 2002).

To begin our study we first theoretically formalize the natural to follow but combinatorically complex UEFA draw procedure. Next, we quantitatively analyze the constraint effects, showing that exclusions of same-nation match-ups (the association constraint) generate large distortions in teams' progression probabilities within the tournament. For example, we find that the constraints affect the chances of advancing to the tournament's semifinals by up to 20 percent, with an effect on expected tournament winnings of up to one million euros.

Of particular analytical interest is the indirect effect of the matching constraints, where spillovers from other team-pair exclusions disproportionately alternate the likelihoods of other teams' matches. A natural question is whether an alternative constrained-matching procedure exists that would result in a fairer randomization. Using an objective function focused entirely on the spillovers between otherwise equally treated teams, we use a recent theorem from the market-design literature (Budish et al., 2013) to operationalize this normative design question. Specifically, the theorem allows us to relax (without loss of generality) the combinatorically complex problem of looking for randomizations over constrained assignments through a switch in focus to the computationally tractable problem of finding *expected* assignments satisfying the constraints. The conclusion from our analysis of the UEFA assignment rule across the past 16 years shows that even though marginal improvements *are* possible, the tournament's transparent assignment procedure comes very close to achieving the same outcomes as the spillover-minimizing designs.

While our search for a superior constrained assignment procedure suggests only minimal scope for improvement—and with large potential costs from forgoing UEFA's simple-to-follow procedure—a related question is the extent to which better outcomes are possible from slacking the imposed constraints. As a constructive exercise, we conclude the paper by showing that a relatively small relaxation of the association constraint can go a long way towards reducing the distortions. In particular, we constructively show that slacking the constraints can be obtained with only minimal adjustment to the current matching procedure, retaining all the beneficial features of the transparent dynamic draw.

In summary, our paper documents a solution to a constrained matching procedure with three desirable properties: (i) a simple-to-follow sequential draw procedure; (ii) randomizations carried out through public draws from an urn, helping to dispel doubts about the designer cherry-picking realizations; and (iii) expected assignments that are close to optimal in terms of minimizing the constraint spillovers. Hence, in contrast to many other assignment mechanisms, our application's design achieves its theoretic objectives while remaining transparent and verifiable to all parties. This suggests that generalizations of this procedure might be useful in market-design applications, where openness and fairness in the randomization are paramount.

In terms of organization, Section 2 provides a brief review of the related literature. In Section 3, we describe the background of the UCL and discuss the constraints imposed on the R16 matching. In Section 4, we formalize the UEFA mechanism and some theoretic properties.

In Section 5, we provide a description of the data used in estimating our goal-outcome model, and then present our quantitative results. Finally, Section 6 concludes.[1]

2. Literature review

Our paper contributes to two main strands of economic literature: market design and tournaments. While there is large theoretic literature on the incentive effects of tournaments (see Prendergast, 1999) our paper is more closely related to a growing body of applied work exploiting sports-tournament outcomes as naturally occurring experiments. In recent years, using sports data from football through cricket to golf, the applied literature has provided evidence both for consistency with theory (Walker and Wooders, 2001; Chiappori et al., 2002; Palacios-Huerta, 2003) and behavioral biases (Bhaskar, 2008; Apesteguia and Palacios-Huerta, 2010; Pope and Schweitzer, 2011; Foellmi et al., 2016).

Where the literature on sports tournaments has been centered around various positive aspects of players' behavior, our paper instead emphasizes normative features of the tournament. In this sense, our work is more closely related to the market design literature, and a handful of applied papers examining well-structured environments. Key examples here are: Fréchette et al. (2007), demonstrating the problem of inefficient unraveling in a decentralized market through US college football bowls; Anbarci et al. (2015), designing a fairer mechanism for penalty shootouts in football tournaments; Baccara et al. (2012), investigating spillovers across participants in a faculty office assignment and the subsequent inefficiency of the procedure; Budish and Cantillon (2012), studying the superiority of a manipulable mechanism to the strategy-proof mechanism using data from a course assignment procedure in a business school. In each of these cases, market design conclusions are driven by the combination of theory and a structural analysis of the application data. Similarly, our paper analyzes the optimal procedure through a combination of theory tools, estimation techniques, and simulation within the application (see Roth, 2002).

The main insights for our application are made possible through the core theorem in Budish et al. (2013)—showing sufficiency for an analysis via expected assignment matrices as long as the constraints satisfy a biheirarchy separability condition.[2] This result allows us to show near-optimality of the existing UEFA assignment rule. To our knowledge our paper is one of the first ones to apply this market-design tool in a normative assessment of a procedure in

[1] An Appendix presents proofs of propositions together with additional (theoretical and empirical) results for interested readers. Full data, programs, and the Online Appendix are available at https://sites.google.com/site/martaboczon.

[2] Also see Hylland and Zeckhauser (1979) and Bogomolnaia and Moulin (2001), each of which are focused on assignments with individual choice, where our paper is focused on the alternative randomization procedures.

the field. While our tournament setting is of standalone interest,[3] the focus on constraints in matching is also related to topics in school choice such as the implementation of affirmative-action constraints (for example, Dur et al., 2016). Last but not least, our paper introduces a novel design consideration to a literature that is primarily focused on fairness, efficiency and strategy proofness (see Abdulkadiroğlu and Sönmez, 2003, and references thereof). In our setting instead of manipulation by participants, the main design consideration is that of minimizing the potential for deceptive behavior on the part of principals by removing any opaque or manipulable features in the randomization. This leads to an additional and non-trivial challenge: designing a matching procedure that incorporates substantial combinatoric complexity through the constraints, where the randomization is transparent to the general public, and as fair as possible to the participants in expectation.[4]

3. Application Background

The UCL is the most prestigious worldwide club competition in football. Its importance within Europe is similar to that of the Superbowl in the United States, though with stronger global viewership figures (details below). The tournament is played annually between late June and the end of May by the top-division teams from 57 national associations across Europe, featuring most of the sports' star players.

In the 2017 tournament, 6.8 million spectators attended UCL matches and more than 65,000 attended the final game alone.[5] In addition to the in-stadium audience the tournament has vast media exposure through television and the Internet. The UCL final game is globally the most-watched annual sporting event, where the 2015 final had an estimated 400 million viewers across 200 countries, with a live audience of 180 million. For comparison, the average Super Bowl viewership in the United States was equal to 103 million in 2018, 111 million in 2017, and 112 million in 2016.[6]

In financial terms, UEFA's revenues mainly arrive from the sale of broadcast and commercial rights to its three major club competitions: the UCL, the UEFA Europa League, and the UEFA Super Cup. In the 2017 season, UEFA generated a total revenue of 2.84 billion euro,

[3]In addition, our paper contributes to a literature on optimal tournament design in sports, see Dagaev and Sonin (2018), Guyon (2018, 2015), Ribeiro (2013), Scarf and Yusof (2011), Scarf et al. (2009), and Vong (2017).

[4]See also Bó and Chen (2019), which examines the importance of simplicity and transparency in a historical random assignment mechanism designed for civil-servant matching in Imperial China.

[5]Since each UCL season spans across two calendar years, for clarity and concision we refer to a particular season by the year of its final game; so 2019 would indicate the 2018–19 season.

[6]Furthermore, the UCL proves a massive success on social media. In the 2017 season, the UCL official Facebook page became the worlds' most followed for a sporting competition with 63 million fans, 300 million video views, and 98 million interactions over the final game alone.

where 75 percent of that amount was paid out to clubs and associations participating in UEFA competitions.[7] Clubs that made it to the UCL group stage were awarded 12.7 million euro each, while those that reached the R16 garnered a further 6 million. Beyond the R16, quarter-finalists won an additional 6.5 million euro, semi-finalists 7.5 million, and finalists 11 million (with the winner receiving a bonus of 4.5 million).[8]

Introduced in 1955 as a European Champion Club's Cup (and consisting only of the national champion from each association) the tournament has evolved over the years to admit multiple entrants from each national association (at most five).[9] The last major change to the tournament's design took place in the 2004 season. As such, in our empirical analysis we focus our attention on the seasons between 2004 and 2019 (the latest completed season at the time of writing).

Since the 2004 season, the UCL consists of a number of pre-tournament qualifying rounds followed by a group and then a knockout stage, similar in format to the World Cup, but played concurrently with the association's club seasons.[10] In the group stage, 32 teams are divided into eight groups of four.[11] Beginning in September each team plays the other three group members twice (once at home, once away). At the end of the group stage in December, the two lowest-performing teams in each group are eliminated, while the group winner and runner-up advance to the knockout stage. The knockout stage (except for the final game) follows a two-legged format, in which each team plays one leg at home, one away. Teams that score more goals over the two legs advance to the next round, where the remaining teams are eliminated.[12]

[7]The first-order beneficiaries are clubs participating in the group stage and onward. The remaining UEFA revenue is distributed among second- and third-order beneficiaries, which are clubs participating in the qualifying rounds and non-participating clubs, respectively. The solidarity payments made to the latter teams are distributed via national associations and allocated for the most part to youth training programs.

[8]"*UEFA Champions League Financial Report 2016/17,*" 42$^{\text{nd}}$ Ordinary UEFA Congress, Bratislava, 28 February 2018, available at `www.uefa.com`.

[9]For more details regarding the format changes, see Table 3 in the Appendix.

[10]"*Regulations of the UEFA Champions League 2015-18 Cycle*" and "*Regulations of the UEFA Champions League 2012-15 Cycle,*" available at `www.uefa.com`.

[11]Prior to the 2015 season, seeding was entirely determined by the UEFA club coefficients calculated based on clubs' historical performance, with the titleholder being automatically placed in Pot 1. Starting from the 2016 season, the titleholder together with the champions of the top-seven associations based on UEFA country coefficients are placed in Pot 1. The remaining teams are seeded to Pots 2-4 based on UEFA club coefficients. The eight groups are then assembled by making sequential draws from the four pots with a restriction that teams from the same association cannot be drawn against each other, enforced in a similar way to the R16 match we detail in the paper.

[12]Technically, the scoring rule is lexicographic over total goals, and goals away from home. A draw on both results in extra time, followed by a penalty shootout if a winner is is still not determined.

Our focus in this paper is on the assignment problem of matching the 16 teams at the beginning of the knockout phase into eight mutually disjoint pairs.[13,14] If the problem consisted simply of matching two equal-sized sets of teams under the bipartite constraint, the assignment could be conducted with two urns (one for group winners, one for runners-up) and sequential draws of team pairs without replacement. However, the presence of the group and association constraints prohibits such a simple mechanism for two reasons. First, after drawing a team to be partnered, the urn containing eligible partner draws must not contain any directly excluded teams. Second, a match with a non-excluded partner must not force an excluded match at a later point in the draw. While the first concern is easy to address, the second one requires a more-complicated combinatoric inference.

For illustration, consider a dynamic draw from two pots, one with teams A, B and C; one with teams d, e and f. Suppose that the matches Ad and Be are excluded. An initial draw from the first pot selects team A. The subsequent draw must directly exclude d. Assume f is selected from the two remaining valid partners. In the second round, C is chosen from the first pot. Although C has no *directly* excluded partners, the constraints *indirectly* imply that C cannot match with d. The reasoning is that if Cd is formed, B will have no valid partners in the third and final round. As such, given the initial Af draw, (Af, Ce, Bd) is the unique matching satisfying the constraints.

Although this logic is easy to follow when matching three teams to three teams, with eight teams on each side and many more constraints, the combinatorics become quite involved. While matchings could be easily formed via fully computerized draws, UEFA opts to make much of the randomization transparent through urn draws. The dynamic draw procedure UEFA developed randomizes the R16 tournament matching as follows: (i) eight blue balls representing eight runners-up are placed in the first urn and one ball is drawn without replacement; (ii) a runner-up is drawn and a computer algorithm determines the exhaustive set of group winners that can possibly match with the drawn runner-up given all constraints and any previous draws; (iii) white balls representing the possible match partners determined in the previous step are placed into the second urn and one ball is drawn without replacement; (iv) a pairing of the two drawn teams (one winner and one runner-up) is added to the aggregate R16 matching. This procedure repeats until all eight matches are formed. In what follows, we refer to the above algorithm as the constrained dynamic $\mathcal{R}$-to-$\mathcal{W}$ draw, where $\mathcal{W}$ and $\mathcal{R}$ stand for the sets of group winners and runners-up, respectively.

[13]Note that the quarter- and semifinal draws are free from seeding and association protection, and as such, are conducted in a standard fashion by drawing balls from an urn without replacement.

[14]A political constraint also excludes Russian teams from being drawn against Ukrainian teams. In what follows, we re-interpret this restriction as an extended association constraint.

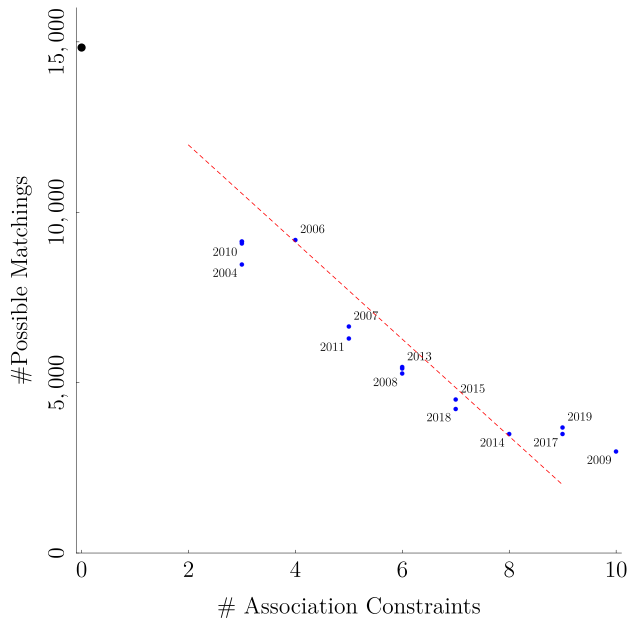


Figure 1. Possible matchings against the number of same-nation exclsions (2004–19)

Key features of the UEFA procedure are its simplicity and transparency with respect to the interim draw, and each specific urn draw. While representatives from each R16 team attend the draw ceremony, the event also draws substantial attention from the media and general public. The 20-minute draw ceremony is streamed live by UEFA over the Internet and broadcast by many national media companies. Examining the viewership figures for the Internet stream, the most recent UCL R16 draw attracted 935,000 viewers on UEFA.tv alone.[15] While this figure is small relative to game-day TV audiences, it is a *huge* viewership when considering a centralized assignment.

In the absence of the association constraint, the tournament has 14,833 possible matchings for the R16 teams where each same-nation exclusion significantly reduces the number of valid assignments. Across the 16 most-recent seasons, the number of valid assignments ranged from 2,988 in the 2009 season through 6,304 in 2011 to 9,200 in 2006. We graph the relationship between the number of possible matchings and the number of exclusions implied by the association constraint in Figure 1. While the number of valid assignments is not purely a function of the number of exclusions (it depends on their arrangement too) the relationship in question can be approximated by a linear function that decreases by 1,400 matchings for each same-nation exclusion.[16]

[15]Online audiences have increased for the draw ceremony over time. In the 2015 season the R16 draw was watched by 262,000 viewers, in 2016 by 415,000, in 2017 by 618,000, and in 2018 by 935,000.

[16]See Table 6 in the Appendix for the constraints in seasons 2004–19.

4. Theory for the Current Matching Procedure

We now turn to the theory for the current procedure, where we describe a generalized version of the dynamic mechanism used by UEFA to draw assignments satisfying the constraints. After characterizing the effective lottery over matchings induced by the dynamic draw, we show that the randomization in question is distinct from simpler static implementations, as well as from a number of dynamic variants. Finally, we demonstrate that the constraints enforced by UEFA satisfy the Budish et al. (2013) separability condition, which allows us to simplify our search for better assignment mechanisms on the more-computationally tractable space of expected assignments.

4.1. **Constrained Dynamic Draw.** Let $\mathcal{W} = \{w_1, w_2, ..., w_K\}$ and $\mathcal{R} = \{r_1, r_2, ..., r_K\}$ denote the sets of group winners and runners-up, respectively. Let $\mathcal{V}$ be the set of all *possible* perfect (exhaustive one-to-one) matchings between $\mathcal{W}$ and $\mathcal{R}$. We examine a random assignment mechanism $\psi : 2^{\mathcal{V}} \to \Delta\mathcal{V}$ that takes as input any subset Γ of $\mathcal{V}$ (a set of *admissible* matchings) and as output a probability distribution over the elements of Γ. The mechanism used by UEFA assembles a matching in the following manner:

Algorithm (Constrained $\mathcal{R}$-to-$\mathcal{W}$ dynamic draw)**.** *Given an input set of admissible matchings $\Gamma \subseteq \mathcal{V}$, the algorithm selects a matching $\psi(\Gamma)$ in $K = |\mathcal{W}|$ steps.*

Initialization: *Set $\mathcal{R}_0 = \mathcal{R}$, and $\Gamma_0 = \Gamma$.*

***Step-**k: (for $k = 1$ to K)*

- (i) *Randomly choose $v_k^{\mathcal{R}} \in \mathcal{R}$ through a fair draw over $\mathcal{R}_{k-1}$;*
- (ii) *Randomly choose $v_k^{\mathcal{W}} \in \mathcal{W}$ through a fair draw over the set of admissible partners for $v_k^{\mathcal{R}}$, $\mathcal{W}_k := \left\{ w \in \mathcal{W} \,\middle|\, \exists V \in \Gamma_{k-1} \text{ s.t. } v_k^{\mathcal{R}} w \in V \right\}$;*
- (iii) *Construct the sets of currently unmatched runners-up $\mathcal{R}_k = \mathcal{R}_{k-1} \setminus \{v_k^{\mathcal{R}}\}$ and valid assignments given current draw $\Gamma_k = \left\{ V \in \Gamma_{k-1} \,\middle|\, v_k^{\mathcal{R}} v_k^{W} \in V \right\}$.*

***Finalization**: After K steps the algorithm returns a vector of K match pairs*

$$\mathbf{v} = (v_1^{\mathcal{R}} v_1^{\mathcal{W}}, \dots, v_K^{\mathcal{R}} v_K^{\mathcal{W}}) \,.$$

The unordered matching $\left\{ v_1^{\mathcal{R}} v_1^{\mathcal{W}}, \dots, v_K^{\mathcal{R}} v_K^{\mathcal{W}} \right\}$ is set as the realization of $\psi(\Gamma)$.

This dynamic procedure as an interim output produces an ordered sequence of K matches, $\mathbf{v}$. In order to characterize the probability of the unordered matching V we define: (i) $\mathcal{P}(V)$, the set of all possible order-permutations for V; and (ii) $\mathcal{W}_k(\mathbf{v})$, the set of admissible match partners for runner-up $v_k^{\mathcal{R}}$ selected at Step-$k(i)$ in the permutation $\mathbf{v}$.[17]

Proposition 1. *Under the constrained $\mathcal{R}$-to-$\mathcal{W}$ draw the probability of any matching $V \in \Gamma$ is given by*

$$\Pr\{\psi(\Gamma) = V\} = \frac{1}{K!} \sum_{\mathbf{v}\in\mathcal{P}(V)} \prod_{k=1}^{K} \frac{1}{\mathcal{W}_k(\mathbf{v})}.$$

Proof. For any matching $V \in \Gamma$, each of the $K!$ possible permutations of V has a strictly positive probability, so $\Pr\{V\} = \sum_{\mathbf{v}\in\mathcal{P}(V)} \Pr\{\mathbf{v}\}$, which can be rewritten using the chain rule as

$$\Pr\{V\} = \sum_{\mathbf{v}\in\mathcal{P}(V)} \prod_{k=1}^{K} \left(\Pr\{v_k^{\mathcal{R}} \,|\mathbf{v}_{k-1}\} \cdot \Pr\{v_k^{\mathcal{W}} \,|v_k^{\mathcal{R}}, \mathbf{v}_{k-1}\}\right),$$

where $\mathbf{v}_{k-1}$ denotes the matches selected in steps 1 through $k-1$. Since the randomization is fair at each step, $\Pr\{v_k^{\mathcal{R}} \,|\mathbf{v}_{k-1}\}$ simplifies to $\frac{1}{K-k+1}$ and $\Pr\{v_k^{\mathcal{W}} \,|v_k^{\mathcal{R}}, \mathbf{v}_{k-1}\} = \frac{1}{\mathcal{W}_k(\mathbf{v})}$. $\square$

Proposition 1 shows that the mechanism's probability distribution over Γ requires $K! \times |\Gamma|$ calculations. Even though the cardinality of Γ can be substantially lower than $K!$, the exact computation of $\Pr\{V\}$ involves between $K!$ and $(K!)^2$ steps, and can be taxing even for our application with $K = 8$.

Though the above assignment rule is combinatorically involved, the UEFA draw procedure has three useful features. First, all draws are conducted using an urn, and thus each individual randomization is fair/uniform. Second, the number of possible realizations in each draw is always less than eight, implying that each individual draw is easy to comprehend, with a clear urn composition. Finally, all nontrivial elements of the draw conducted opaquely by the computer (the calculation of the set of valid partners at each step) can be checked ex post, and thus the draw procedure is fully verifiable by a viewer. Consequently, as long as the urn draws are conducted fairly and publicly,[18] it is not possible for the designer to cherry pick realizations, inoculating the mechanism against corruption on the part of the principal.

Given the characterization in Proposition 1, one question is the extent to which the above calculation can be simplified.

[17] That is: $\mathcal{W}_k(\mathbf{v}) := \left|\left\{w \in \mathcal{W} \,\middle|\, \exists V \in \Gamma \text{ s.t. } v_k^{\mathcal{R}} w \in V \text{ and } \wedge_{j=1}^{k-1} \left(v_j^{\mathcal{R}} v_j^{\mathcal{W}} \in V\right), \right\}\right|$.

[18] Unlike many state lotteries which use mechanical randomization devices to draw urn outcomes, the UEFA draw is conducted by human third-parties (typically famous footballers). Pointing to football fans' distrust in the process, the human draw has led to plausible allegations of UEFA rigging draws with hot/cold balls (here made by former FIFA president Sepp Blatter in an interview with Argentine newspaper *La Nacion*, June 13th, 2016).

We define two randomization mechanisms as being *equivalent* if they induce the same probability distribution over matchings $\mathcal{V}$, and *distinct* if they differ.

Proposition 2. *The constrained dynamic $\mathcal{R}$-to-$\mathcal{W}$ draw is distinct from:*

(i) *The constrained uniform draw over Γ.*

(ii) *The constrained dynamic procedure that fairly draws admissible pairs.*

(iii) *The constrained $\mathcal{W}$-to-$\mathcal{R}$ dynamic draw.*

Proof. See Appendix for counter-examples. □

The first two parts of Proposition 2 are essentially negative results, indicating that our environment is not equivalent to algorithmically simpler fair draws over complete matchings or individual matches, where the third part shows the procedure is asymmetric. While the main takeaway from Proposition 2 is negative, it does demonstrate three potentially constructive design channels, analogous to the reversal of the proposing sides in the National Resident Matching Program (NRMP) algorithm detailed in Roth and Peranson (1997).[19]

4.2. **Relaxing the UEFA Constrained Assignment Problem.** Above we characterize a generalized version of the dynamic draw employed by UEFA to assemble the R16 matching. Within this family of randomization procedures, the actual UEFA draw defines the admissible matchings Γ via a set of match exclusions $H \subset \mathcal{R} \times \mathcal{W}$, where the overall exclusion set $H = H_A \cup H_G$ is the union of the association-level exclusions H_A and group-level exclusions H_G. The precise set H varies across seasons depending on the composition of teams reaching the R16 stage and the group-level assignment. The admissible matching set for the UEFA implementation of the constrained dynamic draw is defined by

$$\Gamma_H := \{V \in \mathcal{V} \,|V \cap H = \emptyset\},$$

where the draw induces the random matching $\psi(\Gamma_H)$.

While our paper first analyzes the effects of the specific constraints in H on expected draw outcomes, we also examine the extent to which fairer random assignments might exist. To aid us in this endeavor we employ the core result in Budish et al. (2013) that guarantees the *existence* of an equivalent randomization over assignments in Γ_H for *every* feasible *expected*

[19]However, while distinct, we later show that in our particular setting the three draw procedures lead to only marginally different outcomes, where the fair draw over Γ has a computational advantage for approximating assignment probabilities. The standings of the UCL groups are only fixed two days before the draw is carried out. While analytic calculation of the exact matching probabilities for the draw would take approximately two months on a powerful desktop, probabilities for the fair draw over feasible assignments can be calculated in fractions of second.

assignment. This allows us to relax the constrained assignment problem over discrete final matches to the one of finding *expected* assignments allowing for fractional (and continuous) assignments of objects.

First, notice that any assignment V can be rewritten as a matrix $\mathbf{X}(V) \in \{0,1\}^{K\times K}$ with a generic entry $x_{ij}(V) = \mathbf{1}\{r_i w_j \in V\}$ indicating whether or not runner-up r_i is matched to winner w_j. Since V represents a perfect matching between $\mathcal{R}$ and $\mathcal{W}$, $\mathbf{X}(V)$ is a rook-matrix where each row and column have exactly one unit-valued entry with all other entries equal to zero. Second, for any random draw over Γ_H, the expected assignment matrix is defined as $\mathbf{A} := \mathbb{E}\mathbf{X}(V) = \sum_{V\in\Gamma_H} \Pr\{V\}\cdot\mathbf{X}(V)$, with the generic entry a_{ij} representing the probability of the $(r_i w_j)$ match.

An expected assignment matrix $\mathbf{A}$ is defined as satisfying the exclusion constraints H if: $\forall ij \in H : a_{ij} = 0$; $\forall i \in \mathcal{R}, j \in \mathcal{W} : 0 \le a_{ij} \le 1$, $\sum_{k=1}^{K} a_{kj} = \sum_{k=1}^{K} a_{ik} = 1$. That is, an expected assignment matrix $\mathbf{A}$ is said to satisfy the exclusion constraint set H if each of its entries is a non-negative real number representing a probability, all rows and columns sum to exactly one, and the excluded matches take value zero. While satisfying the exclusion constraint set H is clearly a necessary condition for any expected assignment resulting from a randomization over assignments in Γ_H, it is also sufficient:

Proposition 3 (Implementability)**.** *For any expected assignment matrix* $\mathbf{A}$ *satisfying the exclusion constraints* H *there exists an equivalent randomization over the constrained assignment set* Γ_H*.*

Proof. The matching exclusions can be grouped into two distinct sets: (i) the singleton exclusions in H and the K row constraints; and (ii) the K column constraints.[20] As such the matching constraints satisfy the bihierarchy condition in Budish et al. (2013) Theorem 1. ☐

Proposition 3 implies that when we search for elements of $\Delta\Gamma_H$ that optimize some objective, instead of searching directly in a space with $O(K!)$ degrees of freedom, we can focus without loss of generality on the search for expected assignments with $O(K^2)$ degrees of freedom. For the specific UEFA application with $K = 8$, the number of degrees of freedom is thereby reduced from 2,000–10,000 to 30–40.

A trivial corollary to the above result is that:

[20]The quotas for each element a_{ij} are therefore a min and a max of zero for the excluded singletons; a min of zero and a max of one for the non-excluded singletons; a min and a max of one for the row sum; and a min and a max of one for the column sum.

Corollary 1. *Any expected assignment matrix satisfying the exclusion constrains H is implementable by randomizing over a finite collection of J constrained dynamic $\mathcal{R}$-to-$\mathcal{W}$ draw mechanisms $\{\psi(\Gamma_j)\}_{j=1}^{J}$, with each feasible assignment set $\Gamma_j \subseteq \Gamma_H$.*

Proof. Set $J = |\Gamma_H| < K!$ and for each entry $V_j \in \Gamma_H$ set $\Gamma_j = \{V_j\}$. By Proposition 3, there exists a probability p_j of selecting each admissible matching V_j that is equivalent to for any implementable expected assignment matrix satisfying the constraint exclusions H. The result follows from setting $\Pr\{\Gamma_j\} = p_j$. □

While Proposition 3 facilitates the examination of whether better randomizations might exist, Corollary 1 provides us with a direct tool for implementing a randomization by a pre-draw over the input set of admissible assignments. After drawing a subset of the feasible assignments, the previously outlined dynamic matching procedure can be used to realize a specific matching.

5. Quantifying the Constraint Effects

In this section, we first quantify how the current UEFA mechanism affects expected assignments in the UCL R16, and how the spillovers from others' exclusions impact otherwise equally treated teams. Second, we examine the normative question of whether a fairer randomization mechanism is possible. Finally, after showing that substantially better mechanisms do not exist given the maintained constraints, we turn to the extent to which substantially fairer outcomes can be achieved through a slight relaxation of the matching constraints.

5.1. **Distortions in the Current UEFA Mechanism.** We start our empirical analysis with an illustrative example from the UCL R16 draw in 2018. The expected assignment matrix for the UEFA dynamic draw procedure is given in Table 1. Each row represents a group winner, and each column a runner-up, so the row-i–column-j cell indicates the probability the (ij)-pair is selected within the realized R16 matching.[21]

The constraints in the 2018 draw are as follows: First, along the diagonal, the probabilities of each match are zero, reflecting the eight group constraints.[22] Second, seven same-nation matches are excluded reflecting the 2018-specific association constraint. Finally, all rows

[21]We calculate all probabilities with a Monte Carlo simulation of size $N = 10^6$. At this size, 95 percent confidence intervals for each probability are within ± 0.001 of the given coefficient (see Proposition 4 in the Appendix).

[22]The bipartite and group constraints on their own impose symmetric restrictions, leading to an equal probability of matching with every non-excluded partner. Consequently, without the association constraint, the expected assignment would have a one-in-seven chance for each off-diagonal entry.

Table 1. Expected assignment matrix for the 2018 R16 draw

	Basel	*Bayern Munchen*	*Chelsea*	*Juventus*	*Sevilla*	*Shakhtar Donetsk*	*Porto*	*Real Madrid*
Manchester United	0.	0.148	0.	0.183	0.183	0.155	0.148	0.182
Paris Saint-Germain	0.109	0.	0.294	0.128	0.128	0.108	0.105	0.128
Roma	0.159	0.151	0.	0.	0.189	0.160	0.152	0.189
Barcelona	0.149	0.144	0.413	0.	0.	0.150	0.144	0.
Liverpool	0.159	0.151	0.	0.189	0.	0.160	0.152	0.189
Manchester City	0.156	0.148	0.	0.183	0.184	0.	0.148	0.183
Besiktas	0.109	0.105	0.293	0.128	0.128	0.108	0.	0.129
Tottenham Hotspur	0.160	0.152	0.	0.189	0.189	0.159	0.151	0.

Note: Probabilities derived from a simulation ($N = 10^6$) of the UEFA draw procedure.

and columns sum to exactly one, as each represents the marginal match distribution for the respective team through the bipartite constraint. [23]

Despite having fair urn draw at each point in time, the likelihoods of two teams playing each other are not uniform due to asymmetry in the constraints. For illustration, consider Paris Saint-Germain (PSG) in 2018, the second row of Table 1. As PSG is the only French team in the R16 in this season it has no same-nation exclusions and thus, seven feasible match partners. However, the likelihoods of each of the possible match-ups varies substantially, with the probability of PSG playing Chelsea almost three times larger than that of PSG playing either Basel, Shaktar, or Porto.

In what follows, we quantify the effects of distortions induced by the constraints in the R16 matching, using a structural model of game outcomes estimated using historical (and out-of-sample) data from the 2004 to 2019 UCL seasons. By simulating the R16 draws and all subsequent games within the tournament, we show that tournament matching constraints have major effects on teams' expected earnings and progression probabilities within the tournament. Finally, we outline a fairness objective function designed to isolate the indirect spillovers from enforcing the constraints, and demonstrate that even though the spillovers are sometimes substantial, the UEFA draw procedure comes very close to a constrained-best.

5.2. **Data and Estimation of Game-Outcome Model.** In order to account for variation in teams' ability while examining potential effects driven by the tournament's constraints, we estimate one of the most commonly used structural models for soccer outcomes in the sports economics literature: the bivariate Poisson (Maher, 1982; Dixon and Coles, 1997).

Model. Let S_i and S_j be the random variables for the number of goals scored by home-team i and guest-team j in a given game. In a bivariate Poisson model with parameters $(\lambda_1, \lambda_2, \lambda_3)$

[23]Note that the constraints are not mutually exclusive and consequently, even though the expected assignment is an 8×8 matrix, it has 34 degrees of freedom. The expected assignment matrices for the R16 draw in the other 15 seasons can be found in the paper's Online Appendix.

the realized scoreline (s_i, s_j) has a joint probability distribution given by

$$P_{(S_i,S_j)}(s_i, s_j) = \exp\{-(\lambda_1 + \lambda_2 + \lambda_3)\} \frac{\lambda_1^{s_i}}{s_i!} \frac{\lambda_2^{s_j}}{s_j!} \sum_{k=0}^{\min(s_i,s_j)} \binom{s_i}{k} \binom{s_j}{k} k! \left(\frac{\lambda_3}{\lambda_1 \lambda_2}\right)^k,$$

where $\mathbb{E}(S_i) = \lambda_1 + \lambda_3$, $\mathbb{E}(S_j) = \lambda_2 + \lambda_3$ and $\mathbf{Cov}(S_i, S_j) = \lambda_3$.

In our specification we follow Karlis and Ntzoufras (2003) and assume that $\ln \lambda_1 = \mu^t + \eta^t + \alpha_i^t - \delta_j^t$, $\ln \lambda_2 = \mu^t + \alpha_i^t - \delta_j^t$, and $\lambda_3 = \rho^t$, where α_k^t and δ_k^t measure the idiosyncratic offensive and defensive abilities for team k in season t (positive direction indicating greater ability), μ^t denotes a season-specific constant term, and η^t the season-specific home-advantage parameter.

We estimate the above bivariate Poisson model via constrained maximum likelihood separately for each season t between 2004 and 2019. For scale identification we impose two sum-to-zero constraints in each season, assuming that $\sum_k \alpha_k^t = \sum_k \delta_k^t = 0$. The model estimation uses scoreline data from the current tournament's group stage (season t) together with game-level data from the group and knockout stages (except for the final game which is played on a neutral soil) in seasons $t-1$ and $t-2$.[24]

In Figure 2, we graph the estimated defense parameters on the horizontal axis and the corresponding estimated attack parameters on the vertical axis (for all R16 teams across all seasons between 2004 and 2019). The strongest teams have large positive values for both the attack and defense parameters; see for example FC Barcelona or FC Shalke in 2011 in the first quadrant. Conversely, low-performing teams have either a negative value of the attack parameter (low offense) or for the defense parameter (low defense): see FC Lokomotiv Moskva in 2004 in the second quadrant or AC Sparta Praha in 2004 in the fourth quadrant. Teams of low-to-medium-strength with small but positive values of both attack and defense parameters are centered at zero in the first quadrant.[25]

Finally, in Table 2, we present summary statistics for the estimated attack and defense parameters broken out by the stage of the competition. We find that the eight teams that progress to the quarterfinals are stronger both offensively and defensively than the teams eliminated in the R16. Similarly, the four teams advancing to the semifinals have better offensive and defensive performance relative to those knocked out in the quarterfinals.

[24]This results in a total of 408 game-level observations used in the 2004 estimation, 376 observations for the 2005 season, and 348 observations for each season between 2006 and 2019. The differences in the number of observations result from a change to the tournament design in the 2004 season, where a second group stage feeding into the quarterfinals was replaced by the R16. Table 7 in the Appendix provides summary statistics for the UCL game outcomes in all seasons between 2002 and 2019.

[25]Table 8 in the Appendix provides estimates for the constant term, the home-advantage parameter, and the correlation coefficient between the number of goals scored by opposing teams in seasons 2004–19.

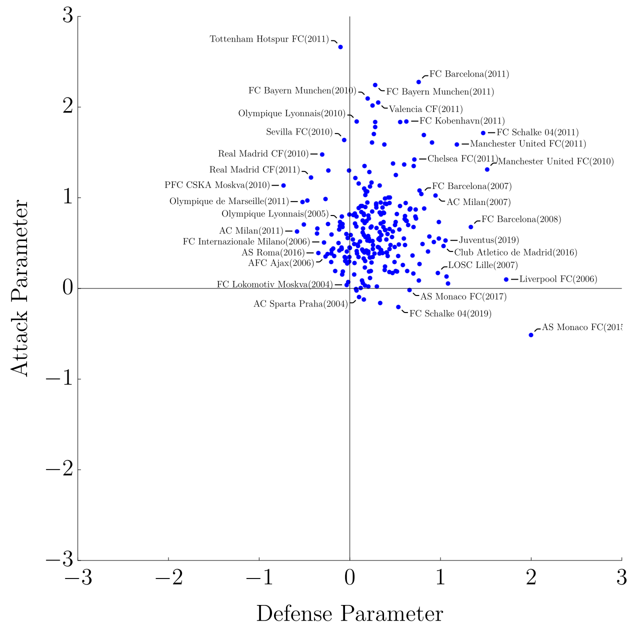


FIGURE 2. Estimated attack and defense parameters for the UCL teams (2004–19)

TABLE 2. Summary statistics for estimated parameters

Stage	**Attack parameter**				**Defense parameter**			
	Mean	Med.	Min	Max	Mean	Med.	Min	Max
Eliminated in R16	0.62	0.51	-0.20	2.25	0.27	0.17	-0.58	10.99
Eliminated in QF	0.69	0.64	-0.51	2.67	0.31	0.26	-0.73	2.00
Reach SF	0.78	0.74	-0.43	2.28	0.66	0.44	-0.43	11.52

Moreover, the two-sample Kolmogorov-Smirnov test suggests that the empirical distributions of attack and defense parameters in the R16 and the semifinals are statistically different (at 1 percent significance level). Interestingly, the same holds for the empirical distribution of defense parameters in the semi- and quarterfinals, what suggests that team's defensive (rather than offensive) skills are critical to its overall tournament success.

5.3. **Constraint effects in the UCL draw.** In what follows, we separately quantify the *total* and *spillover* effects from the constraints on two main metrics for teams' outcomes: their expected prize money and the probability of reaching the semi-final stages of the competition (and beyond). In summary, we find that:

Result 1. *The association constraint in the R16 generates substantial effects: (i) altering expected tournament prizes by millions of euro; (ii) significantly affecting the chances of*

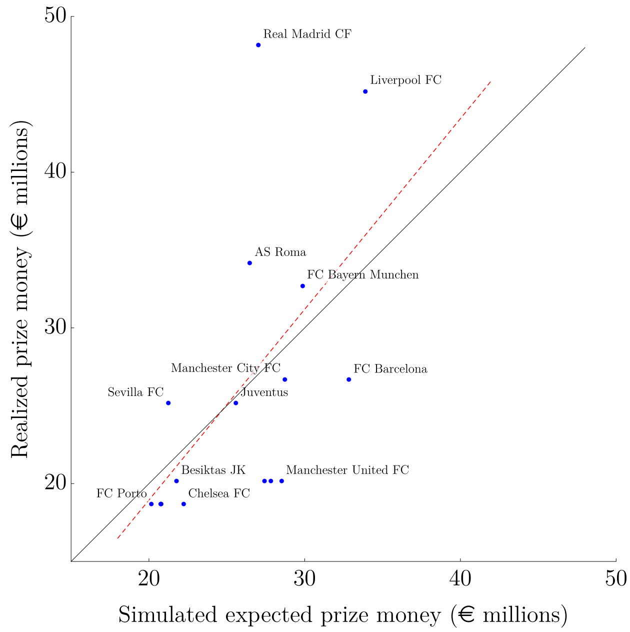


FIGURE 3. Expected prize money versus realized prizes under actual constraints (2018)

reaching later stages of the tournament; and (iii) creating spillovers to the matching chances of otherwise equally treated teams.

Evidence: We combine our model of the exact draw procedure and the estimated bivariate Poisson model to calculate expected tournament outcomes for each R16 team i in each season t. We conduct our calculations twice, once under the actual set of UEFA constraints (the bipartite, group, and association constraints), and once under a counterfactual set of constraints that drops the association constraint (so enforcing just the bipartite and group constraints).[26,27]

First, looking across all 16 seasons, we find that expected prize money under the model is strongly correlated with realized prizes ($\rho = 0.639$). A linear regression on the 256 team-year observations (16 teams across 16 seasons) finds expected prizes to be highly predictive of out-of-sample realized prizes ($p < 0.000$). Figure 3 provides an illustration of a relationship

[26]In detail, for both the actual and counterfactual calculations we proceed by first drawing $J = 1{,}000$ R16 matchings, $\left(\left\{V_j^t\right\}_{j=1}^J\right)$, using the exact $\mathcal{R}$-to-$\mathcal{W}$ dynamic draw mechanism and fixing the realized R16 teams. For each R16 draw V_j^t, we then simulate the remaining tournament outcomes $S = 1{,}000$ times (the R16 home/away games, quarter- and semifinal home/away games, and the final game on neutral soil). Consequently, each season is simulated one million times.

[27]Given the form of the simulation, we also calculate accurate metrics for the expected prize money conditional on each simulated R16 draw V_j^t as $\hat{\mathbb{E}}\left(\pi_{it} \,\middle|\, V_j^t\right) := \frac{1}{S}\sum_{s=1}^{S}\pi_{it,js}$. The unconditional expected prize is calculated as $\hat{\mathbb{E}}\pi_{it} := \frac{1}{J}\sum_{j=1}^{J}\hat{\mathbb{E}}\left(\pi_{it} \,\middle|\, V_j^t\right)$.

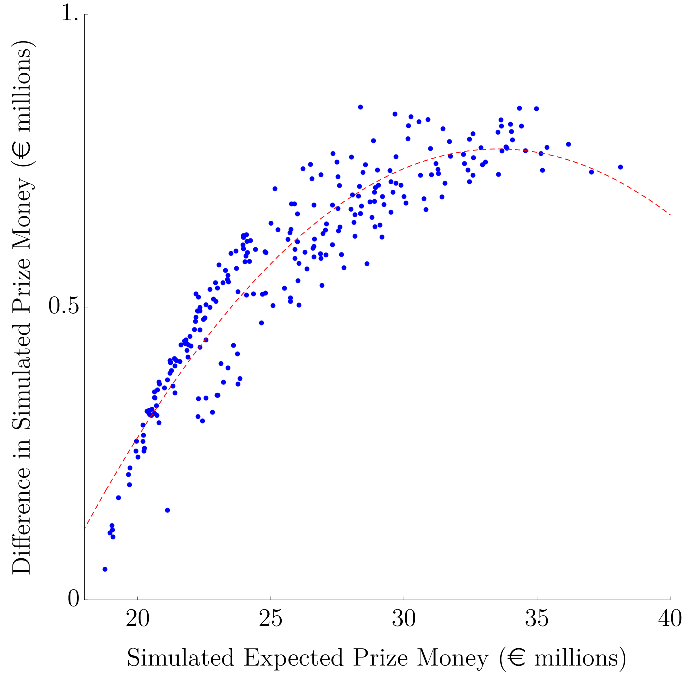
between the simulated and realized tournament prize money in the 2018 season (with the red line indicating a regression fit).[28]

Second, in terms of the importance of the R16 draw, we quantify the effects from a very good or very bad R16 draw on each team's expected performance. The bivariate Poisson model allows us to calculate the expected prize money conditional on any realized R16 draw V, $\Pi_{it}(V) = \hat{\mathbb{E}}\left(\pi_{it}^{A} \,|V\right)$. Through simulation of the dynamic draw procedure we approximate the distribution of $\Pi_{it}(V)$, and then measure the range in expected outcomes between the 90^{th} and 10^{th} percentiles of the $\Pi_{it}(V)$ distribution. In Figure 4(A) we graph the unconditional expected tournament prize money $\hat{\mathbb{E}}\pi_{it}^{A}$ for each team i in each season t under the current UEFA draw against the difference between the 90th and 10th percentile draws for the conditional expected prize money over the the draw realization. The figure indicates a strong link between a team's strength, and how affected they are by a good versus bad R16 draw. For clubs most likely to be eliminated at the very start of the knockout stage (and so expected to earn less than 20 million euro) the difference between a good (90th percentile) and bad (10th percentile) draw does not exceed a quarter million euro. Conversely, for the teams most likely to win the tournament (and so earning more than 30 million euro in expectation), the difference between a good and a bad R16 realization is three times larger. Hence, the realization of the R16 draw is of greater importance to the best-performing teams in the competition.

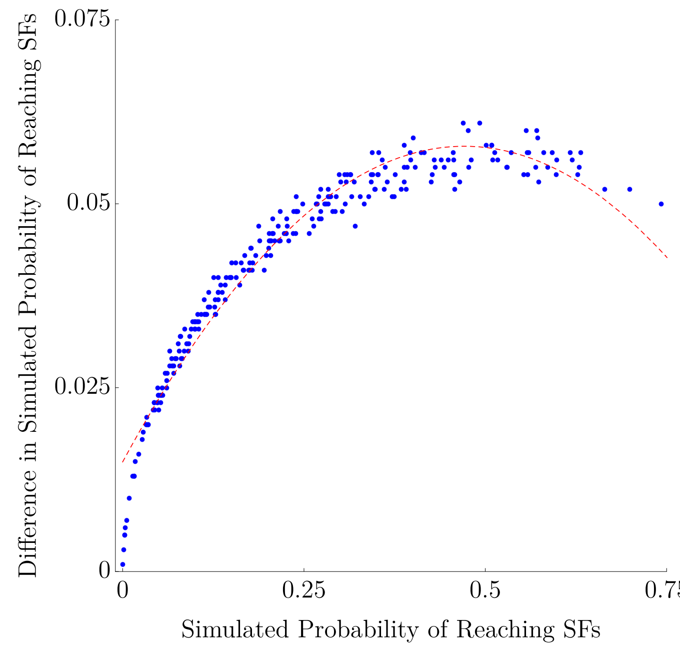
While Figure 4(A) illustrates the variability in prize money outcomes induced by the R16 draw, Figure 4(B) shows a complementary picture of the effects of the draw on another metric: the probability of reaching the competition's semifinals. For an upper-quartile team, a favorable R16 draw increases the chances of advancing to the semifinals by about five percentage points. On the extreme, even a best-case draw does not increase the chances for a lower-quartile-team by more than a percentage point. While similar results follow from alternative metrics, henceforth we focus on the prize money effects. While a monetary unit has obvious benefits of being practical and economically comprehensible, it has the added benefit of also being an objective measure for how the tournament organizers weigh reaching each stage of the competition.

Finally, having demonstrated that the model is predictive of realized prize money outcomes, and that the R16 draw matters to the teams (particularly those with higher-ability), we now turn to quantifying how the association constraint distorts the tournament outcomes. For each team i in season t we calculate the *association constraint effect* as the difference in

[28]Overall the expected prizes calculated under the model explain 41 percent of the total variation in realized prizes. Moreover, the remaining variability matches the intrinsic variability we would expect under the estimated Poisson model.



(A)



(B)

FIGURE 4. Unconditional expected outcomes against variability in draw outcome

Note: Both figures show simulated expected outcomes under the structural model on the horizontal axes against simulated differences between the 10^{th} and 90^{th} percentiles of the variable over the R16 draw V.

expected prizes from enforcing the association constraint, $\Delta\pi_i^t := \hat{\mathbb{E}}\bar{\pi}_{it}^A - \hat{\mathbb{E}}\bar{\pi}_{it}^B$, where $\hat{\mathbb{E}}\bar{\pi}_{it}^A$ and $\hat{\mathbb{E}}\bar{\pi}_{it}^B$ are expected earnings under the current UEFA procedure and the counterfactual mechanism that drops the same-nation exclusions, respectively. Teams with a positive value of $\Delta\pi_i^t$ are those benefiting from the association constraint, whereas those with a negative value are being disadvantaged. Across all 16 UCL seasons, the total constraint effect has a standard deviation of 0.3 million euro (it is mean-zero by construction within each season) and a range of 1.8 million euro (a cost of 0.8 million euro to Barcelona in the 2007 season and a subsidy of 1.0 million euro to Real Madrid in the 2017 season). The measured effects from enforcing the association constraint are therefore substantial.

In order to validate the measured association constraint effect, we further demonstrate that it is predictive of realized outcomes even after controlling for team ability. For each season t we create a zero-to-one ability index for the teams in the R16 using the estimated bivariate Poisson model.[29] By way of example, for the 2018 season, our ability index runs from Shaktar Donetsk at 0, FC Basel at 0.134 and Besiktas at 0.233, up to Real Madrid at 0.919, Liverpool at 0.999, and Barcelona at 1. Regressing the realized tournament prizes for each team-year observation on the ability index, we extract the fitted residuals as a measure of the prize-money outcome that is orthogonal to the teams' abilities.[30] Figure 5 illustrates the relationship between the association constraint effect $\Delta\pi_i^t$ as measured by our model on the horizontal axis, and the residuals from the regression of realized prizes on teams' ability on the vertical axis. Even though the measured constraint effect only explains 4.7 percent of the total variation in realized prizes after controlling for ability, our model-generated measure is statistically significant at any conventional significance level ($p < 0.001$).

A similar exercise conducted using a probit model, suggests that both the ability index and the association constraint effect are significant predictors of teams' success in the final stages of the competition. While a unit shift in the ability index—moving from the worst to the best team—increases the likelihood of a semifinal appearance by 67.3 percent ($p < 0.000$),

[29]Specifically, we calculate the average probability that team i wins a game across each of the 15 possible partners using the bivariate Poisson model as:

$$\omega_i^t = \frac{1}{15}\sum_{j\neq i}^{15} \hat{Pr}\left(S_i > S_j\right).$$

We then re-scale the probability to be an index that runs from zero to one at the season level as $\tilde{\omega}_i^t = (\omega_i^t - \underline{\omega}_i^t)/(\overline{\omega}_i^t - \underline{\omega}_i^t)$ where $\overline{\omega}$ and $\underline{\omega}$ are the best and worst values for ω_i^t in season t.

[30]The ability index coefficient indicates that a difference of 17.8 million euros across the expected prize can be explained by a unit change in the ability index.

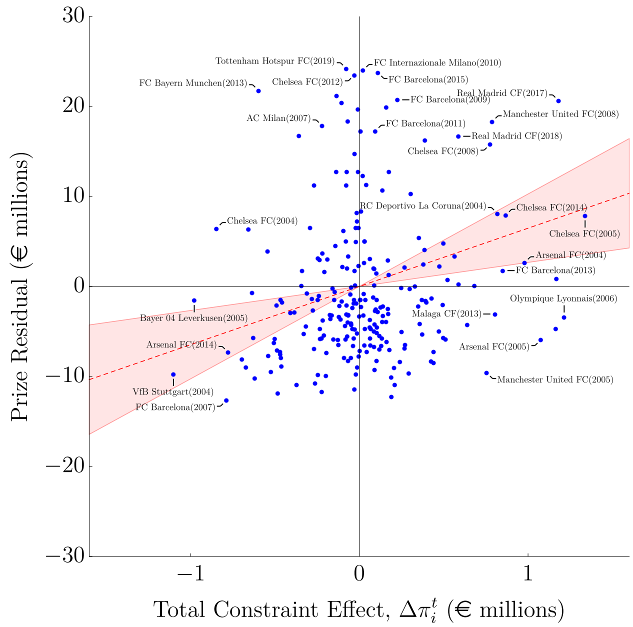


FIGURE 5. Constraint effect versus realized tournament prize residual

Note: Dashed line shows fitted relationship between the two variable; shaded band shows 95 percent confidence interval on slope effect.

a measured one million euro subsidy from the association constraint increases the likelihood by 19.7 percent ($p = 0.004$).[31]

Above, we quantify the *total* effects of the association constraint, consisting of both the *direct* effect on team i from ij matching exclusions as well as the *indirect* spillover effect from others' constraints (how a jk exclusion affects $i \neq j, k$). For illustration, consider the match probabilities for Real Madrid and Barcelona in the 2018 season (see Table 1, column 8 and row 4, respectively). While the Read-Madrid and Barcelona exclusion directly benefits the two Spanish teams (these are two of the three strongest teams that season according to our ability index), Barcelona additionally profits from substantial spillovers generated by six other same-nation exclusions. In particular, consider Barcelona and two other unconstrained group winners, PSG and Besiktas (each with seven potential match partners, rows 2 and 7, respectively). Even though none of the three aforementioned group winners are constrained from matching with the two lowest-performing teams (Basel and Shaktar Donetsk, columns 1 and 6) Barcelona is 1.35 times more likely to match to either of them than are either PSG or Besiktas, the less-constrained group winners.

[31]The association-constraint effects for reaching later stages of the tournament are diminishing in magnitude. Whereas a one million euro association-constraint subsidy increases the chances of reaching the quarterfinals by 31.2 percent ($p = 0.001$), the chances of reaching the final are raised by only 9 percent ($p = 0.086$).

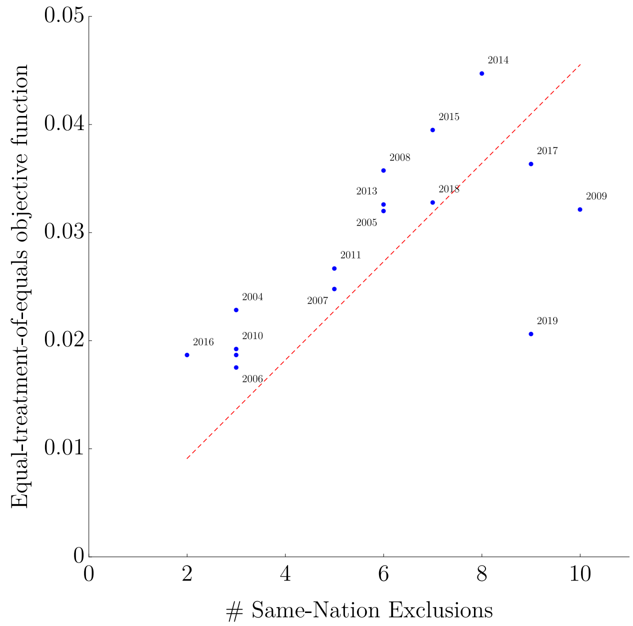


FIGURE 6. Spillovers by number of same-nation exclusions (2004–19)

Holding constant the matching exclusions, distortions caused by the direct effects are an unavoidable consequence. However, the indirect effects have the potential to be ameliorated through better randomization mechanisms. In order to quantify the indirect effects of the association constraint we construct a spillover measure, motivated by the idea of *equal treatment of equals* (ETE) and designed to compare the matching chances of otherwise equally treated team pairings.

We regard two teams i and j as *otherwise equally treated* with respect to third team k if both the ik and jk matches are not directly excluded, and as being *more equally treated* the closer are the likelihoods of the ik and jk matches. For any expected assignment $\mathbf{A}$ our ETE measure captures the mean absolute difference between all otherwise equally treated team pairings as:

$$Q\left(\mathbf{A}\right) = \frac{1}{|\Upsilon|} \sum_{(ik,jk)\in\Upsilon} |a_{ik} - a_{jk}|\,,$$

where

$$\Upsilon = \left\{(ik, jk)\,|i,j \in \mathcal{W}, k \in \mathcal{R}, ik, jk \notin H\right\} \bigcup \left\{(ki, kj)\,|k \in \mathcal{W}, i,j \in \mathcal{R}, ki, kj \notin H\right\}.$$

In Figure 6, we graph the ETE objective $Q\left(\mathbf{A}^t\right)$ for the expected assignment matrix $\mathbf{A}^t$ for each season t under the current UEFA draw procedures. Since the relationship in question is strongly positive, we conclude that the association constraint substantially distorts match chances of otherwise equally treated teams. Specifically, the estimated slope coefficient from the regression of $Q(\mathbf{A})$ on the number of same-nation exclusions suggests an average wedge in match-likelihoods of 0.05 for every ten exclusions.[32] This represents a relative swing of up to a third for unconstrained teams. Although this result points to quantitatively large spillovers even after accepting the constraints' direct effect, we now show that there is not much scope to ameliorate the spillovers through better randomizations.

5.4. **Near-Optimality of the Current Procedure.** A natural question raised by Result 1 is whether there exists a randomization procedure that generates less distortions than the one currently employed. In this section we provide the following largely negative answer:

Result 2. *While the UEFA mechanism is not optimal given the constraints, it comes very close to the optimal mechanism when considering the equal treatment of equals spillover measure.*

Evidence: We demonstrate Result 2 by making use of a tool provided by Proposition 3 in Section 4.2. The proposition shoes that a feasible assignment-producing mechanism exists for every feasible expected assignment matrix satisfying the constraints. This substantially simplifies the problem by reducing the effective degrees of freedom, allowing us a computationally tractable search for the optimal expected assignment $\mathbf{A}_t^\star$. The main result here is that the matrix $\mathbf{A}_t^\star$ that minimizes the spillover measure $Q(\cdot)$ is not *substantially* better than the expected assignment under the current dynamic draw procedure $\hat{\mathbf{A}}_t^\star$ for any UCL season between 2004 and 2019. [33]

The optimal expected assignment solves the following optimization problem:

$$\mathbf{A}_t^\star := \operatorname*{argmin}_{\mathbf{A}} Q(\mathbf{A}),$$

subject to the constraints (1) $\forall ij \in H_t : a_{ij} = 0$; (2) $\forall ij : 0 \leq a_{ij} \leq 1$; (3) $\forall i : \sum_k a_{ik} = \sum_k a_{ki} = 1$, where H_t denotes the set of match exclusions in season t and Q is the ETE spillover measure.

In Figure 7 we graph the spillover measure for the optimal expected assignments $Q(\mathbf{A}_t^\star)$ against the spillover measure for the actual UEFA mechanism, across the 16 past UCL

[32]When there are no same-nation exclusions our measure is zero by construction, so we estimate the relationship without a constant.

[33]One potential objection here is that we are not using the right objective function $Q(\cdot)$. While we are amenable to suggestions for other metrics, we have additionally tried minimizing the square differences, the differences between the maximal and minimal positive-probability matches for each team, as well as a measure based on the Kullbeck-Leibler divergence. None of these showed economically meaningful gains from optimization, where the interpretation of the quantitative measures became harder than the "average difference" interpretation of our chosen metric.

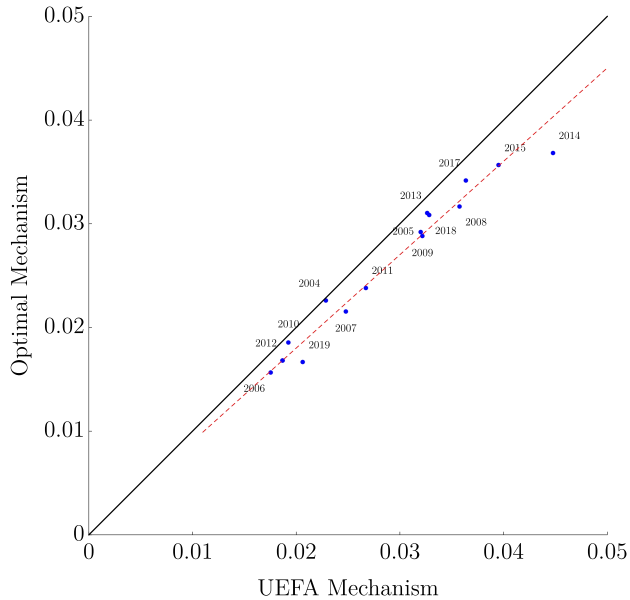


FIGURE 7. Spillovers under the constraints: Optimal mechanism versus UEFA mechanism.

seasons. While some improvement is possible across the tournament years, the gains are marginal (on average a less than 10 percent relative reduction in the size of the spillovers).[34]

Against the small potential benefits, there are large prospective implementation costs associated with modifying the existing procedure, as constructive mechanisms that produce the optimal expected assignment are potentially quite complex in comparison to the current procedure. While Corollary 1 provides a channel through which the optimal mechanism might be implemented—a pre-draw where the organizers randomize the feasible matching set—even this would be cumbersome and might engender suspicion from observers. Put against this transparency cost, the ability to reduce the average match distortion from a 5 percent average difference in matching chances under the current UEFA mechanism to a 4.5 percent difference does not seem alluring.[35,36]

[34]The size of the reduction is given by the estimated slope coefficient from a regression of the spillover under the optimal randomization against the actual UEFA mechanism for all years $t = 2004, ..., 2019$.

[35]Note that it is possible that there exist simple modifications of the current matching mechanism that would shift the expected assignments towards the optimum. In the Appendix, we show that the three distinct mechanisms in Proposition 2 are all near-identical in their expected assignments to the actual mechanism (see Figures 11-13 in the Appendix).

[36]We should note too that the inability to improve upon the expected assignments generated under the UEFA mechanism is not driven by a limited scope in moving expected assignments under the constraints. Taking 2018 as a (fairly representative) example, we can obtain any value for the spillover measure from the optimum value of just over 0.03 all the way up to an upper bound of 0.32.

5.5. **Weakening the Constraints.** One response to Result 2 is to accept the current mechanism and the distortions it generates. In this final section, however, we take a different approach, and examine the extent to which gains can be made by weakening the association constraint. Specifically, we investigate the extent to which the association constraint can be partially relaxed while still protecting the tournament from excessive R16 same-nation match-ups. There are several practicable ways in which this could be accomplished, but in what follows, we focus on a procedure that relaxes the association constraint while only marginally modifying the current draw procedure. Specifically, we study an alternative association constraint where we allow *at most one* same-nation match in the R16. We can therefore continue to use the constrained $\mathcal{R}$-to-$\mathcal{W}$ dynamic mechanism detailed in Section 4.1, with a sole modification to the (now expanded) admissible set given by

$$\Gamma_H := \{V \in \mathcal{V} \mid |V \cap H| \leq 1\}.$$

As such, the relaxation retains the desirable features of the current mechanism: the randomization is transparent (fair draws from a small-sample urn) and the more-opaque combinatoric check continues to be fully verifiable at all points during the draw.

Under this relaxation we find that:

Result 3. *Weakening the association constraint to allow for at most one same-association match in the R16 substantially reduces the distortions, while protecting associations from excessive same-nation match-ups. Moreover, as a secondary effect, weakening the association constraint reduces the number of same-nation games in the later stages of the tournament.*

Evidence: We start by constructing analog results to those presented in Section 5.1. In Figure 8, the blue points indicate the spillover measure for the UEFA draw procedure with the at-most-one same-association constraint set on the vertical axis against the values under the default constraint set. Allowing for a single same-nation match in the R16 decreases the total distortions by more than 70 percent. This is a sizable reduction, especially when compared to the 10 percent reduction obtained under the optimal mechanism maintaining the current constraint structure derived in Section 5.4 (the red stars in the figure).

Circling back to a measure of the total association effect when the constraint is relaxed we define the expected prize difference as $\Delta\tilde{\pi}_t^i = \hat{\mathbb{E}}\bar{\pi}_{it}^C - \hat{\mathbb{E}}\bar{\pi}_{it}^B$, where $\hat{\mathbb{E}}\bar{\pi}_{it}^C$ is the expected prize for team i in season t under the at-most-one association draw, and $\hat{\mathbb{E}}\bar{\pi}_{it}^B$ is the expected prize in the absence of any association constraints as before.[37] In Figure 9, we illustrate the relationship between $\Delta\tilde{\pi}_t^i$, which we refer to as the *counterfactual association constraint effect*, and the association constraint effect $\Delta\pi_t^i$ defined in Section 5.1. We conclude that

[37]We again numerically calculate $\hat{\mathbb{E}}\bar{\pi}_{it}^C$ via the same Monte Carlo simulation method used in Section 5.1.

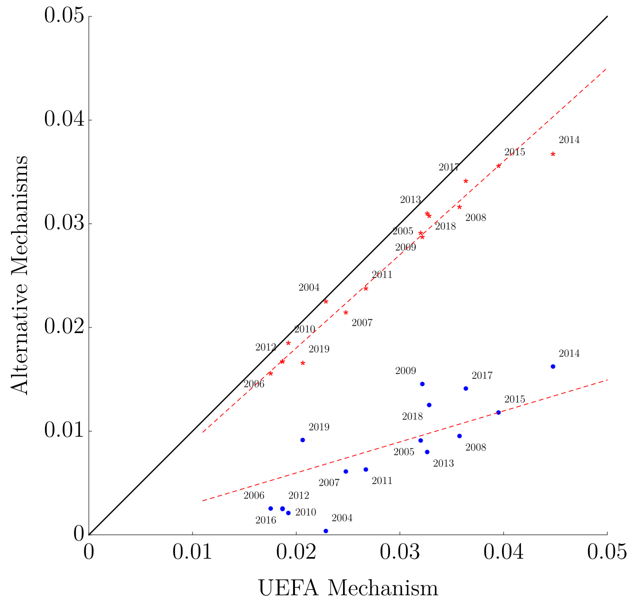


FIGURE 8. Spillovers under the relaxed constraints

Note: Horizontal axis shows the ETE spillover measure for the current UEFA mechanism. Vertical axis shows spillovers under the following two alternative mechanisms: (blue dots) at-most-one association match mechanism with dynamic draw; (red stars) the optimal mechanism under the constraints. Dashed red lines show regression fits.

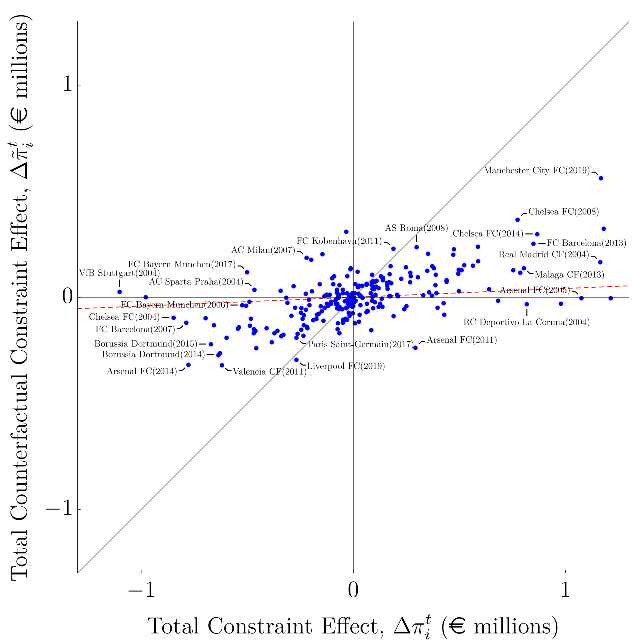


FIGURE 9. Counterfactual and current effects of the association constraint

allowing for a single same-association match in the R16 reduces the total prize distortions by 71 percent.[38]

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[38]Estimated slope coefficient of 0.29 from the regression of the direct counterfactual association effect on the current association effect, across all 16 teams in all seasons.

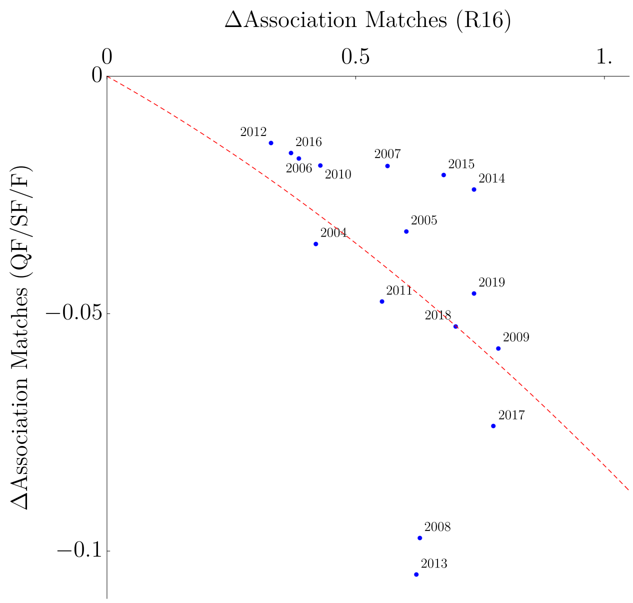

FIGURE 10. Within tournament association-game tradeoffs: later stages versus R16

The above demonstrates a substantial reduction in the matching distortions with only a slight relaxation of the association constraint. However, there are presumably nontrivial costs associated with allowing for same-nation R16 matches. Relaxing the association constraint as we have done leads to a single same-nation match-up in the R16 in approximately six out of every ten tournaments. In seasons with at least six same-nation exclusions, this ratio increases to seven-in-ten. Since the association constraint is imposed intentionally, UEFA likely has a clear underlying preference for the tournament to be primarily an international competition.[39]

As a final point in favor of our relaxation approach, we show that, perversely, imposing same-nation exclusions at earlier stages of the tournament has the effect of increasing the likelihood of same-nation match-ups in the subsequent rounds. Using our estimated model of goal outcomes and the at-most-one same-association match mechanism, we assess the predicted change in the number of same-nation matches in later stages of the tournament. We find that for every same-association pairing generated in the R16, there is a 0.10 reduction in the same-nation games in later stages of the tournament.[40] In Figure 10 we illustrate these two compensating effects in all seasons between 2004 and 2019.

[39]This, however, cannot be determined, since "the identification of design constraints is usually more difficult because they are rarely communicated by the organizers" (Csató, 2018).

[40]The effect would be larger if we additionally accounted for the same-nation exclusions in the group stage that precedes the R16.

6. Conclusion

We document a constrained-assignment problem—with huge public interest and millions of euro in prize money at stake in the draw outcomes—where the randomization mechanism needs to be transparent to both participants and the general public. This is in contrast to many market design solutions that are formed entirely over efficiency, fairness and strategic compatibility objectives—for example, in the assignment of students to schools (Abdulkadiroğlu et al., 2005; Pathak and Sönmez, 2013) or donors in a kidney exchange (Roth et al., 2004). While many matching procedures with desirable theoretical properties have been developed, the resulting algorithms can seem opaque and discriminatory, to the extent that participants can misunderstood even dominant strategies (Rees-Jones and Skowronek, 2018).

The alternative path pursued by UEFA in their assignment mechanism prioritizes transparency, potentially at a cost in terms of fairness. In the main body of the paper we evaluate the UEFA field mechanism, both theoretically and empirically. Through combination of a structural model of the matching procedure with a commonly-used econometric model for football outcomes, we quantify the distortions caused by the competition's matching constraints. While the enforced constraints on same-nation matches significantly distorts outcomes, we show that the chosen mechanism comes close to a constrained-best in terms of ex ante fairness. The randomization mechanism chosen by UEFA is therefore remarkable in the sense that it solves a complex constrained matching problem in a way that is transparent and comprehensible to the general public.

While our main result shows that the developed mechanism is close-to-optimal, the final section of the paper demonstrates that relatively large improvements are possible with only a slight relaxation of the constraints. In particular, we show that allowing for at most one out of eight R16 pairings to be between teams from the same nation, as opposed to none, reduces distortions by approximately 70 percent.

References

Abdulkadiroğlu, Atila and Tayfun Sönmez, "School choice: A mechanism design approach," *American Economic Review*, 2003, *93* (3), 729–747.

_______, **Parag A Pathak, and Alvin E Roth**, "The New York City High School Match," *American Economic Review*, 2005, *95* (2), 364–367.

Anbarci, Nejat, Ching-Jen Sun, and M Utku Ünver, "Designing fair tiebreak mechanisms: The case of FIFA penalty shootouts," January 2015. Boston College working paper.

Apesteguia, Jose and Ignacio Palacios-Huerta, "Psychological pressure in competitive environments: Evidence from a randomized natural experiment," *American Economic Review*, 2010, *100* (5), 2548–64.

Baccara, Mariagiovanna, Ayşe İmrohoroğlu, Alistair J Wilson, and Leeat Yariv, "A field study on matching with network externalities," *American Economic Review*, 2012, *102* (5), 1773–1804.

Bhaskar, V, "Rational adversaries? evidence from randomised trials in one day cricket," *Economic Journal*, 2008, *119* (534), 1–23.

Bó, Inácio and Li Chen, "Designing heaven's will: Lessons in market design from the Chinese Imperial Civil Servants Match," *Working Paper*, 2019.

Bogomolnaia, Anna and Hervé Moulin, "A new solution to the random assignment problem," *Journal of Economic theory*, 2001, *100* (2), 295–328.

Budish, Eric and Estelle Cantillon, "The multi-unit assignment problem: Theory and evidence from course allocation at Harvard," *American Economic Review*, 2012, *102* (5), 2237–71.

______ **, Yeon-Koo Che, Fuhito Kojima, and Paul Milgrom**, "Designing Random Allocation Mechanisms: Theory and Applications," *American Economic Review*, April 2013, *103* (2), 585–623.

Chiappori, Pierre-André, Steven Levitt, and Timothy Groseclose, "Testing mixed-strategy equilibria when players are heterogeneous: The case of penalty kicks in soccer," *American Economic Review*, 2002, *92* (4), 1138–1151.

Csató, László, "A simulation comparison of tournament designs for world men's handball championships," March 2018. arXiv:1803.10975v3.

Dagaev, Dmitry and Konstantin Sonin, "Winning by losing: Incentive incompatibility in multiple qualifiers," *Journal of Sports Economics*, 2018, *19* (8), 1122–1146.

Dixon, Mark J and Stuart G Coles, "Modelling association football scores and inefficiencies in the football betting market," *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, 1997, *46* (2), 265–280.

Dur, Umut, Parag A Pathak, and Tayfun Sönmez, "Explicit vs. Statistical Preferential Treatment in Affirmative Action: Theory and Evidence from Chicago's Exam Schools," Working Paper 22109, National Bureau of Economic Research March 2016.

Foellmi, Reto, Stefan Legge, and Lukas Schmid, "Do Professionals Get It Right? Limited Attention and Risktaking Behaviour," *Economic Journal*, 2016, *126* (592), 724–755.

Fréchette, Guillaume R, Alvin E Roth, and M Utku Ünver, "Unraveling yields inefficient matchings: evidence from post-season college football bowls," *RAND Journal of Economics*, 2007, *38* (4), 967–982.

Guyon, Julien, "Rethinking the FIFA World Cup Final Draw," *Journal of Quantitative Analysis in Sports*, 2015, *11* (3), 169–182.

______ , "What a fairer 24 team UEFA Euro could look like," *Journal of Sports Analytics*, 2018. in press.

Hylland, Aanund and Richard Zeckhauser, "The efficient allocation of individuals to positions," *Journal of Political economy*, 1979, *87* (2), 293–314.

Karlis, Dimitris and Ioannis Ntzoufras, "Analysis of sports data by using bivariate Poisson models," *The Statistician*, 2003, *52* (3), 381393.

Maher, Michael J, "Modelling association football scores," *Statistica Neerlandica*, 1982, *36* (3), 109–118.

Palacios-Huerta, Ignacio, "Professionals play minimax," *Review of Economic Studies*, 2003, *70* (2), 395–415.

Pathak, Parag A and Tayfun Sönmez, "School admissions reform in Chicago and England: Comparing mechanisms by their vulnerability to manipulation," *American Economic Review*, 2013, *103* (1), 80–106.

Pope, Devin G and Maurice E Schweitzer, "Is Tiger Woods loss averse? Persistent bias in the face of experience, competition, and high stakes," *American Economic Review*, 2011, *101* (1), 129–57.

Prendergast, Candice, "The Provision of Incentives in Firms," *Journal of Economic Literature*, 1999, *37* (1), 7–63.

Rees-Jones, Alex and Samuel Skowronek, "An experimental investigation of preference misrepresentation in the residency match," *Proceedings of the National Academy of Sciences*, 2018, *115* (45), 11471–11476.

Ribeiro, Celso C., "Sports scheduling: Problems and applications," *International Transactions in Operational Research*, 2013, *19* (1-2), 201–226.

Roth, Alvin E, "The economist as engineer: Game theory, experimentation, and computation as tools for design economics," *Econometrica*, 2002, *70* (4), 1341–1378.

Roth, Alvin E. and Elliott Peranson, "The Effects of the Change in the NRMP Matching Algorithm," *Journal of the American Medical Association*, 1997, *278* (9), 729–732.

Roth, Alvin E, Tayfun Sönmez, and M Utku Ünver, "Kidney Exchange," *Quarterly Journal of Economics*, 2004, *119* (2), 457–488.

Scarf, Philip and Muhammad M. Yusof, "A numerical study of tournament structure and seeding policy for the soccer World Cup Finals," *Statistica Neerlandica*, 2011, *65* (1), 43–57.

———, ———, and Mark Bilbao, "A numerical study of designs for sporting contests," *European Journal of Operational Research*, 2009, *198* (1), 190–198.

Vong, Allen, "Strategic manipulation in tournament games," *Games and Economic Behavior*, 2017, *102*, 562567.

Walker, Mark and John Wooders, "Minimax play at Wimbledon," *American Economic Review*, 2001, *91* (5), 1521–1538.

Appendix A. Technical Appendix: For Online Publication

A.1. **Proof of Proposition 2.** The constrained dynamic $\mathcal{R}$-to-$\mathcal{W}$ draw is distinct from: (i) The mechanism that fairly draws over Γ; (ii) The dynamic mechanism that fairly draws admissible pairs; and (iii) The constrained $\mathcal{W}$-to-$\mathcal{R}$ dynamic draw.

A.1.1. *Part (i).* Consider a problem of matching $\mathcal{R} = \{a, b, c, d\}$ to $\mathcal{W} = \{e, f, g, h\}$ under the set of constraints $H = \{ae, bf, cg, dh, ah, bg, dg\}$.[41] The three resulting feasible matchings are given by $V_1 = \{ag, be, ch, df\}$, $V_2 = \{ag, bh, ce, df\}$, and $V_3 = \{ag, bh, cf, de\}$. Under the fair draw from $\Gamma = \{V_1, V_2, V_3\}$, the probability of V_1 is equal to $\frac{1}{3}$. However, under the dynamic $\mathcal{R}$-to-$\mathcal{W}$ mechanism, (and knowing that a is degenerate) the probability of V_1 is given by:

$$\Pr\{V_1\} = \Pr\{be \in V^\star\} = \sum_{x \in \mathcal{W}} \Pr\{w_1 = x\} \cdot \Pr\{be \in V^\star \,|w_1 = x\} = \tfrac{13}{36}.$$

Hence, the two mechanisms are distinct.[42]

A.1.2. *Part (ii).* Consider a problem of matching $\mathcal{R} = \{a, b, c, d\}$ to $\mathcal{W} = \{e, f, g, h\}$ under the set of constraints $H = \{bf, cg, ch, bg, dh\}$. Among the five resulting feasible matchings, only $V_1 = \{af, bh, ce, dg\}$ contains the match af.[43] Under the dynamic $\mathcal{W} \to \mathcal{R}$ mechanism $\Pr\{V_1\} = \Pr\{af \in V^\star\} = \frac{161}{864}$; However, under the dynamic $\mathcal{R} \to \mathcal{W}$ mechanism $\Pr\{V_1\} = \Pr\{af \in V^\star\} = \frac{55}{288}$.[44] Hence, the two mechanisms are distinct.

A.1.3. *Part (iii).* Consider a problem of matching $\mathcal{W} = \{a, b, c, d\}$ to $\mathcal{R} = \{e, f, g, h\}$ under the set of constraints $H = \{bf, cg, ch, bg, dh\}$. Among the five resulting feasible matchings, only $V_1 = \{af, bh, ce, dg\}$ contains the match af. Under the dynamic draw of pairs $\Pr\{V_1\} = \Pr\{af \in V^\star\} = \frac{59}{308}$; However, under the dynamic $\mathcal{R} \to \mathcal{W}$ mechanism $\Pr\{V_1\} = \Pr\{af \in V^\star\} = \frac{55}{288}$. Hence, the two mechanisms are distinct.

A.2. **Additional Results.**

[41] The proof requires at least a 4×4 market, as a 3×3 is degenerate with the standard symmetric group constraints and a single asymmetric same-nation exclusion. The proof can obviously be extended to any $n \times n$ market by making the remaining $(n-3)$ match partners unique through exclusions.

[42] The proof becomes more cumbersome but still goes through if we removed the dg exclusion that forces ag to be degenerate.

[43] A similar counterexample can be constructed with the standard group restriction enforced, but would require a 5×5 market; We omit it for tractability and instead, focus on a 4×4 sub-market.

[44] The four other matchings are: $V_2 = \{ae, bh, cf, dg\}$; $V_3 = \{ag, bh, ce, df\}$; $V_4 = \{ag, bh, cf, de\}$; $V_5 = \{ah, be, cf, dg\}$.

A.2.1. *Simulation Errors.*

Proposition 4. *Simulating the mechanism* 10^6 *times leads to 95 percent confidence intervals smaller than* ± 0.001*.*

Proof. Assignments are independent draws from a fixed distribution with a probability of selecting assignment V given by $f(V)$. The probability that the particular match ab is selected is given by $p_{ab} = \sum_{V \in M(ab)} f(V)$ where $M_{ab} := \{V \in \Gamma \,|ab \in \mu\}$ is the set of matchings which include ab. We simulate the vector 8^2-vector $\hat{\boldsymbol{p}}$ where each element in $\hat{p}_{ab}$ is calculated from the N independent simulation assignments $\left(\hat{V}_i\right)_{i=1}^{N}$

$$\hat{p}_{ab} := \frac{1}{N} \sum_{i=1}^{N} \mathbf{1} \left\{ ab \in \hat{V}_i \right\}.$$

The vector $\hat{\boldsymbol{p}}$ has the obvious property that $\mathbb{E}(\hat{\boldsymbol{p}}) = \boldsymbol{p}$. We can use the central-limit theorem to show that $\sqrt{n}(\hat{\boldsymbol{p}} - \boldsymbol{p}) \xrightarrow{D} \mathcal{N}_{64}(\mathbf{0}, \boldsymbol{\Omega})$ for the variance-covariance matrix $\boldsymbol{\Omega}$ has a generic element given by:

$$\omega_{ab,cd} = \Pr \{ab \wedge cd\} - \Pr \{ab\} \Pr \{cd\},$$

which can be estimated by

$$\hat{\omega}_{ab,cd} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1} \left\{ ab, cd \in \hat{V}_i \right\} - \hat{p}_{ab} \hat{p}_{cd}.$$

However, given a simulation-size of $N = 10^6$, a conservative estimates (as $\omega_{ab,ab} \leq \frac{1}{4}$) for the 95 percent confidence interval for each probability p_{ab} is given by$\hat{p}_{ab} \pm \frac{1.96}{2000} \approx \hat{p}_{ab} \pm 0.001$. $\square$

Table 3. Format of the post-qualifying stages of the UCL between 1956 and 2019

Season(s)	**1st knockout phase**		**Group stage**		**2nd knockout phase**			
	K1	K2	G1	G2	R16	QF	SF	F
1956-1966	16					8	4	2
1967	30	16				8	4	2
1968–1991	32	16				8	4	2
1992–1993	32	16	8					2
1994	32	16	8				4	2
1995–1997			16			8	4	2
1998–1999			24			8	4	2
2000–2003			32	16		8	4	2
2004–2019			32		16	8	4	2

Note: K1 and K2 denote the number of teams competing in the 1st and 2nd knock-out round; G1 and G2 in the 1st and the 2nd round of the group stage; R16 in the R16, QF in the quarterfinal, SF in the semifinal, and F in the final game.

Table 4. Number of teams from each association participating in the UCL R16 by season

Season	**TOP5**						**BEL**	**CYP**	**CZE**	**DEN**	**GRE**	**NED**	**POR**	**RUS**	**SCO**	**SUI**	**TUR**	**UKR**	**Total**
	ENG	ESP	FRA	GER	ITA	Total													
2004	3	4	2	2	2	13			1				1	1					16
2005	4	2	2	3	3	14						1	1						16
2006	3	3	1	1	3	11					1	2	1		1				16
2007	4	3	2	1	3	13						1	1		1				16
2008	4	3	1	1	3	12					1		1		1		1		16
2009	4	4	1	1	3	13					1		2						16
2010	3	3	2	2	3	13					1		1	1					16
2011	4	3	2	2	3	14				1								1	16
2012	2	2	2	2	3	11		1					1	2		1			16
2013	2	4	1	3	2	12							1		1		1	1	16
2014	4	3	1	4	1	13					1			1			1		16
2015	3	3	2	4	1	13							1			1		1	16
2016	3	3	1	2	2	11	1					1	1	1				1	16
2017	3	4	2	3	2	14							2						16
2018	5	3	1	1	2	12							1			1	1	1	16
2019	4	3	2	3	2	14						1	1						16
Mean	3.4	3.1	1.6	2.2	2	13													

Note: BEL indicates Belgium, CYP Cyprus, CZE Czech Republic, DEN Denmark, ENG England, ESP Spain, FRA France, GER Germany, NED the Netherlands, ITA Italy, POR Portugal, RUS Russia, SCO Scotland, SUI Switzerland, TUR Turkey, UKR Ukraine. TOP5 denotes English, Spanish, French, German, and Italian associations together.

Table 5. Number of same-nation exclusions generated by each national association in the UCL R16 by season

Season	**TOP5**						**POR**	**RUS**	**Total**
	ENG	ESP	FRA	GER	ITA	Total			
2004		3				3			3
2005	4			2		6			6
2006	1	2				3			3
2007		2	1		2	5			5
2008	4				2	6			6
2009	4	3			2	9	1		10
2010			1		2	3			3
2011	3	2				5			5
2012				1	2	3			3
2013	1	4			1	6			6
2014	4			4		8			8
2015	2		1	4		7			7
2016	2					2		(1)	3
2017	2	4	1	2		9			9
2018	4	2			1	7			7
2019	3	2	1	2	1	9			9
Mean	2.8	2.8	1.0	2.5	1.7	5.5			5.6

Note: (1) indicates a constraint generated by FC Zenit (RUS) and FC Dynamo Kyiv (UKR).

Table 6. Same-nation exclusions in the UCL R16 by season

Season	**TOP5**					**POR**	**RUS**
	ENG	ESP	FRA	GER	ITA		
2004		1|3					
2005	2|2			1|2			
2006	1|1	2|1					
2007		1|2	1|1		1|2		
2008	2|2				2|1		
2009	2|2	1|3			2|1	1|1	
2010			1|1		2|1		
2011	3|1	2|1					
2012				1|1	1|2		
2013	1|1	2|2			1|1		
2014	2|2			2|2			
2015	1|2		1|1	2|2			
2016	2|1						1|1
2017	2|1	2|2	1|1	1|2			
2018	4|1	1|2			1|1		
2019	3|1	1|2	1|1	1|2	1|1		

Note: In a $(m|n)$-pair m indicates the number of seeded (group stage winners) teams and n the number of unseeded (group stage runners-up) teams. $m \times n$ is the total number of exclusions generated by a given association.

TABLE 7. Summary statistics for the number of goals scored in the UCL

Season	**# Games**	**Average**		**Std. Dev.**	
		Home	Away	Home	Away
2002	156	1.69	0.95	1.18	0.90
2003	156	1.58	1.19	1.36	1.07
2004	124	1.52	0.94	1.37	0.99
2005	124	1.69	0.97	1.46	1.07
2006	124	1.39	0.86	1.31	0.97
2007	124	1.47	1.00	1.29	1.05
2008	124	1.57	1.07	1.42	1.02
2009	124	1.45	1.19	1.34	1.28
2010	124	1.42	1.15	1.23	1.13
2011	124	1.64	1.19	1.44	1.27
2012	124	1.68	1.09	1.54	1.13
2013	124	1.63	1.31	1.30	1.10
2014	124	1.62	1.26	1.35	1.33
2015	124	1.70	1.19	1.60	1.33
2016	124	1.68	1.12	1.42	1.13
2017	124	1.84	1.19	1.68	1.23
2018	124	1.77	1.43	1.53	1.40
2019	124	1.71	1.23	1.43	1.20

Note: From the group stage onward except for the final game played on a neutral ground.

TABLE 8. Estimated bivariate Poisson model coefficients by season

Season	μ	η	ρ
2004	-0.25	0.42	-2.06
2005	-0.47	0.46	-2.00
2006	-0.71	0.57	-1.88
2007	-0.86	0.55	-1.99
2008	-0.49	0.46	-2.02
2009	-0.85	0.37	-1.71
2010	-1.34	0.29	-1.37
2011	-1.51	0.31	-1.33
2012	-0.69	0.27	-10.34
2013	-0.25	0.34	-2.23
2014	-0.05	0.31	-13.76
2015	-0.04	0.27	-16.38
2016	-0.09	0.34	-15.78
2017	-0.07	0.39	-14.96
2018	-0.05	0.37	-16.59
2019	-0.22	0.32	-16.06
Mean	-0.50	0.38	-7.53

Note: μ denotes the constant term, η the home-effect parameter, and ρ the correlation coefficient between the number of goals scored by the two opposing teams.

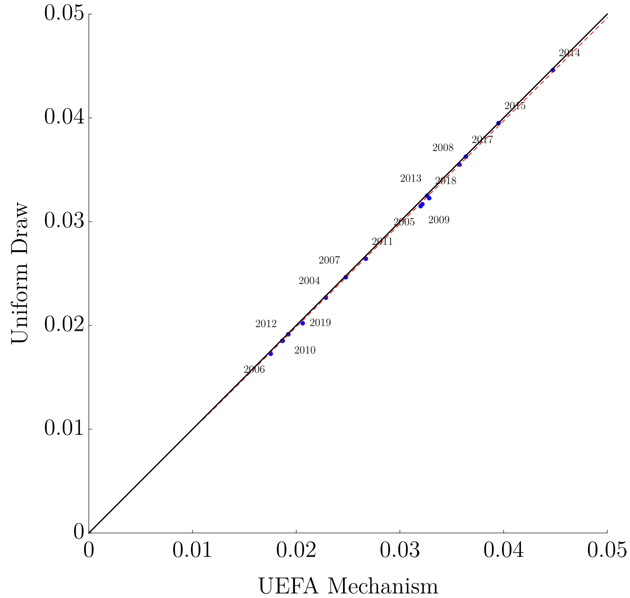

FIGURE 11. Spillovers under the constraints: Fair draw over feasible assignments versus UEFA mechanism (2004–19)

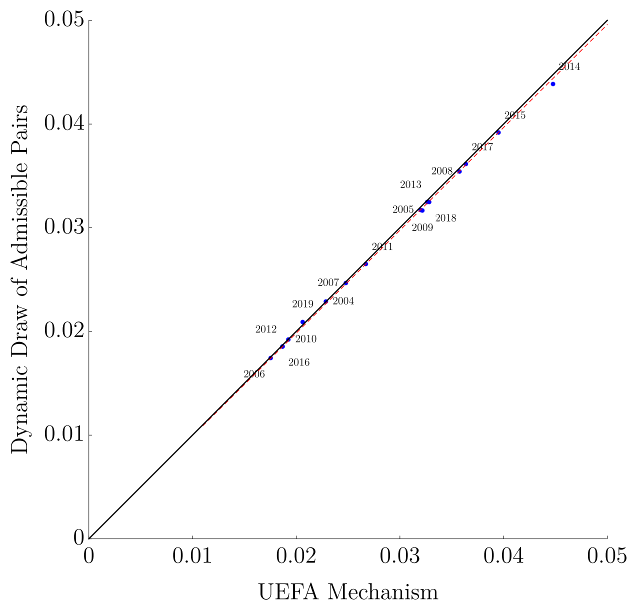

FIGURE 12. Spillovers under the constraints: Fair dynamic draw over admissible pairs versus UEFA mechanism (2004–19)

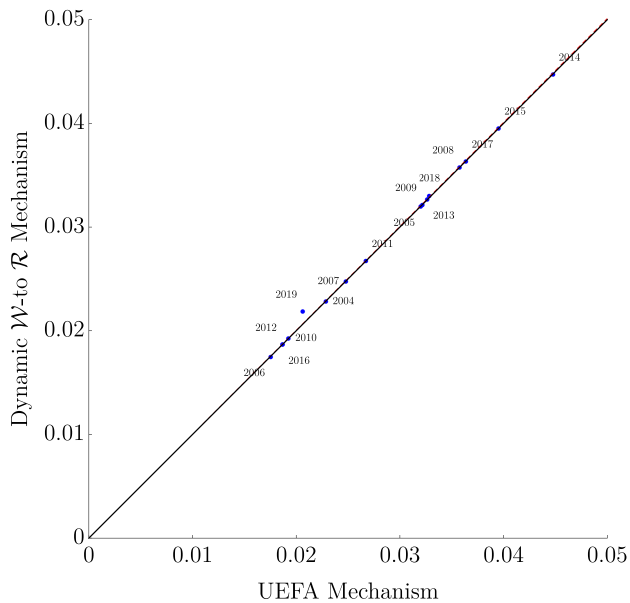

FIGURE 13. Spillovers under the constraints: Dynamic $\mathcal{W}$-to-$\mathcal{R}$ mechanism versus UEFA mechanism (2004–19)