Midterm Topics
Math 5201, Autumn 2016

(*= know the proof)

- Prove that $\sqrt{2}$ is irrational*.
- State the least-upper-bound property. What is an ordered set?
- Prove that in an ordered field, $x + y = x + z$ implies that $y = z$.*
- Prove the Archimedean property of $\mathbb{R}$ (Theorem 1.20)*
- State the meaning of: $\mathbb{Q}$ is dense in $\mathbb{R}$(Theorem 1.20)
- Review basic facts about Complex numbers. Prove that $|z + w| \leq |z| + |w|$*
- Prove the Cauchy-Schwarz inequality for sequences $\{a_n\}$ and $\{b_n\}$ of reals. *

- A set A is countable if $\exists$ a bijection from $\mathbb{N}$ to A. Is the set of finite sequences with entries from a finite set countable? Is the set of all infinite sequences with entries from a finite set countable? Explain why or why not. *
- Prove that if p is a limit point of a set E, then every neighborhood of p contains infinitely many points of E.*
- Prove that the arbitrary union of open sets is open.*
- Prove that the complement of an open set is closed.*
- Prove that an infinite subset of a compact set K contains a limit point in K. *
- Review basic facts about Compact sets. Know the statement and meaning of the Heine-Borel Theorem (Theorem 2.41).

- Prove that if $\{p_n\}$ is a convergent sequence, then $\{p_n\}$ bounded.*
- Prove that convergent sequences are Cauchy, and provide an example to show that the converse need not be true.*
- What is a complete space?
- Let $\{s_n\}$ monotonic and bounded. Prove that $\{s_n\}$ converges to its supremum.*
- Let $\{x_n\}$ be a sequence, and set $a_n = \sup_{m \geq n} x_m$. Define

$$\limsup_{n\to\infty} x_n = \lim_{n\to\infty} a_n. \tag{1}$$

Prove that the expression in (1) is equal to the supremum over the set of subsequential limit points of $\{x_n\}$.**
- State the Cauchy criterion for the convergence of a series.
- Recall convergence tests and know the formula for a geometric series.
- Power series and radius of convergence: review Example 3.40 (Rudin).

- Prove that the inverse image of an open set under a continuous map is open.
- Prove that if f is a continuous mapping of a compact metric space X into a metric space Y. Then $f(X)$ is compact.

-review quizzes and homeworks (solutions online)