X 28

(a) Let $x_1 = \log a_1$, and for $n > 1$,

let $x_n = \log a_n - \log a_{n-1} = \log \frac{a_n}{a_{n-1}}$.

Then $s_n = \frac{x_1 + \cdots + x_n}{n} = \log a_n^{1/n}$.

By X 23, we know that

$$\liminf x_n \leq \liminf s_n$$

$$\liminf \log \frac{a_n}{a_{n-1}} \leq \liminf \log a_n^{1/n}$$

$$\liminf \log \frac{a_{n+1}}{a_n} \leq \liminf \log a_n^{1/n}.$$

Since log is continuous and increasing,

we have $\liminf \frac{a_{n+1}}{a_n} \leq \liminf a_n^{1/n}$.

Similarly, $\limsup \frac{a_{n+1}}{a_n} \geq \limsup a_n^{1/n}$.