Quiz 1 MATH 5201, AUTUMN 2016 NAME: Key.

Problem 1.

(a) Suppose S is an ordered set, $E \subset S$, and E is bounded below. Let $m = \inf E$. Which two properties define m:

(i) "m is an ~~upper~~ lower bound on E" ($\forall e \in E$, $m \le e$)

(ii) "m is the ~~least~~ greatest lower bound on E" (if $\varepsilon > 0$, then $m+\varepsilon$ is <u>not</u> a lower bound on E)

(b) Let x be a real number. Let $\mathbb{Z}$ denote the integers. Set

$$m = \inf \{k \in \mathbb{Z} : x < k\}.$$

Prove that

$$\underset{(b)}{m - 1 \le} \underset{(a)}{x < m}.$$

Set $A = \{k \in \mathbb{Z} : x < k\}$

(a) m is the greatest lower bound, and so $m+1$ is <u>not</u> a lower bound of A. Therefore, $\exists\ k \in A$ so that $x < k < m+1$. If $x \ge m$, then this implies that $m \le x < k < m+1$, but no such integer k exists between m & $m+1$.
$\therefore \boxed{x < m}$

(b) We observed in class that m is an integer.
If $x < m-1 \longrightarrow m-1 \in A$, but m is a lower bound so $m \le m-1$ ⊗
$\therefore \boxed{m-1 \le x.}$

Problem 2. State the Cauchy-Schwarz inequality for real numbers.

Let $\{a_n\}_{n \in \mathbb{N}}$, $\{b_n\}_{n\in\mathbb{N}}$ be sequences of real numbers.
Then,
$$\left|\sum_{n\in\mathbb{N}} a_n \cdot b_n\right| \le \left(\sum_{n\in\mathbb{N}} a_n^2\right)^{1/2} \cdot \left(\sum_{n\in\mathbb{N}} b_n^2\right)^{1/2}.$$