**Quiz 4** MATH 5201, AUTUMN 2016 NAME:
(circle one): Auditing Registered

**Problem 1.**

3 (a) Given a sequence $\{a_n\}$, state the definition of the lim inf $a_n$.

$$\lim_{n\to\infty}\left\{\inf_{m\geq n}\{X_m\}\right\}$$

example:
$\{a_n\} = \{1,-1,1,-1,\dots\}$
$\{b_n\} = \{-1,1,-1,1,\dots\}$
$\{a_n+b_n\} = \{0,0,0,\dots\}$

2 (b) Circle the correct inequality:

lim inf $an$ + lim inf $bn \leq$ lim inf $(a_n + b_n)$

lim inf $an$ + lim inf $bn \geq$ lim inf $(a_n + b_n)$

**Problem 2.** (hint: be careful with the start and end points)

3 (a) Compute

$$\sum_{n=1}^{\infty}\frac{1}{5^n} = \frac{1}{1-\left(\frac{1}{5}\right)} - 1$$

$$= \frac{5}{4} - 1 = \boxed{\frac{1}{4}}$$

2 (b) Compute

$$\sum_{k=0}^{N}\frac{1}{5^k}$$

$$\sum_{K=0}^{N} X^K = 1 + X + X^2 + \dots + X^N$$

$$\sum_{K=0}^{N} X^{K+1} = X + X^2 + \dots + X^{N+1} = X[1 + X + X^2 + \dots + X^N] = \left(\sum_{K=0}^{N} X^K\right)X$$

$$\therefore \sum_{K=0}^{N} X^K = \sum_{K=0}^{N} X^{K+1} + 1 - X^{N+1} = \left(\sum_{K=0}^{N} X^K\right)X + 1 - X^{N+1}$$

$$\sum_{K=0}^{N} X^K = \frac{(1-X^{N+1})}{(1-X)} \overset{X=1/5}{=} \boxed{\frac{1-\left(\frac{1}{5}\right)^{(N+1)}}{1-\frac{1}{5}}}$$

$$\frac{5\left(1-\left(\frac{1}{5}\right)^{N+1}\right)}{4}$$