TAKE HOME QUIZ KEY

① Use definitions

(2a) Use C.S. inequality w/ $\{a_n\}_{n=1}^{N}$ and $\{1\}_{n=1}^{N}$.

(2b) Use C.S. inequality w/ $\{a_n^{1/3}\}_{n=1}^{N}$ and $\{a_n^{2/3}\}_{n=1}^{N}$

③ Compare the x^3 coefficient. This will require understanding (writing out) the x^3 coefficient for $x \prod_{n=1}^{\infty}\left(1-\frac{x^2}{n^2\pi^2}\right)$.

④ Use $ab \overset{(*)}{\leq} \frac{a^2}{2}+\frac{b^2}{2}$ & repeat.

Prove: $(a_1 \cdots a_n)^{1/n} \leq \frac{a_1+a_2+\cdots+a_n}{n}$ when $n=8$.

Let's consider $n=4$:

$$(a_1a_2a_3a_4) \overset{\text{use }(*)}{\leq} \frac{(a_1a_2)^2}{2}+\frac{(a_3a_4)^2}{2}$$

$$= \frac{a_1^2a_2^2}{2}+\frac{a_3^2a_4^2}{2}$$

$$\overset{\text{use }(*)}{\leq} \frac{a_1^4}{4}+\frac{a_2^4}{4}+\frac{a_3^4}{4}+\frac{a_4^4}{4}.$$

Similar arg. for $n=8$.