Capital Mobility and the Current Account

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This paper analyzes the effect of unanticipated changes in the terms of trade on consumption expenditures and the current account in a small open economy. Simple formulas are obtained for the covariances of changes in consumption expenditures, the current account and the expected current account with changes in the terms of trade. It is shown that, if investment is uncorrelated with the terms of trade, a necessary condition for unanticipated changes in the terms of trade to be correlated with changes in consumption expenditures and the expected current account is that the real expected return on foreign bonds differs from the real expected return on domestic bonds.

This paper uses an optimizing framework to study the effects on a small country's consumption expenditures and current account of an unanticipated change in the terms of trade. It is assumed that there are no restrictions to capital mobility, that an asset whose return is perfectly correlated with the terms of trade exists and that households can hold foreign equities in their optimally selected portfolio. Because of these assumptions, this paper provides results which differ substantially from the results presented in the earlier literature.[1] In particular, it shows that the effect on consumption and the current account of an unanticipated change in the terms of trade depends crucially on the reward households expect to receive for bearing terms of trade risks. In the absence of such a reward, households choose to hold a portfolio such that an unanticipated change in the terms of trade does not affect the current account through consumption, but only through investment. If one believes that the expected reward for bearing terms of trade risks is small, this paper shows that changes in consumption play at most a limited role in current account dynamics in the presence of well-developed international capital markets.[2]

The effect on consumption choices and the current account of an unanticipated change in the terms of trade has recently been re-examined within an optimizing framework. Obstfeld (1982) provides a model in which a fall in the home-good value of aggregate spending and a current account surplus result from an unanticipated, permanent worsening of the terms of trade. Obstfeld's results

provide a counter-example to the Harberger–Laursen–Metzler (HLM) effect,[3] which predicts that a fall in the terms of trade creates a current account deficit. The HLM effect is derived from the premise that domestic households adjust their consumption expenditures slowly. Consequently, as the terms of trade worsen, domestic households save less to smooth their consumption stream over time. The decline in savings brings about a current account deficit. In Obstfeld's analysis, the past level of consumption spending is irrelevant. As the terms of trade change, domestic households choose immediately a new, smooth path for consumption expenditures. To achieve the optimal level of steady-state consumption expenditures, domestic households have to acquire foreign assets. The required increase in domestic holdings of foreign assets can only be achieved by a current account surplus. Svensson and Razin (1983) show that the validity of the relation hypothesized by HLM depends crucially on the partial derivative of the subjective rate of time preference of home-country households with respect to the instantaneous flow of utility. In particular, they argue that Obstfeld's results hold because he assumes this derivative to be positive and that they do not hold when it is negative.

In this paper, we study optimal consumption choices in a small open economy in which the terms of trade change stochastically over time. The small economy assumption enables us to treat the terms of trade dynamics as exogeneous. In such an economy, the effect of an unanticipated change in the terms of trade on real consumption expenditures differs significantly from the effect which holds in existing models. These models provide useful answers only if households cannot hedge against unanticipated changes in the terms of trade. If households can protect themselves against the effect on their lifetime expected utility of unanticipated changes in the terms of trade, they allow these changes to affect their real consumption only if, by doing so, they can increase their expected real consumption. It is shown that households are rewarded for bearing terms of trade risks if the real rate of interest at home (assuming that a country uses the exported good as the numeraire) exceeds the real rate of interest abroad. Therefore, in such a setting, domestic households choose to have changes in their real consumption positively correlated with changes in the terms of trade only if the expected real rate of return on a domestic bond exceeds the expected real rate of return on a foreign bond.

The relation between the domestic and foreign real interest rates depends on asset demands. In an attempt to gain some insights into that relation, we relax slightly the small country assumption at the end of the paper and let domestic asset demands affect expected returns on risky assets. We show that, in this case, if foreign households do not demand domestic assets for hedging purposes, the HLM effect is likely to hold if foreign real wealth is uncorrelated with the terms of trade. This result arises because domestic households invest in foreign equities so that, when they have the same consumption preferences as foreign households, their real wealth is perfectly correlated with the real wealth of foreign households. As the expenditure share of domestic households on the domestic good increases, their real wealth becomes negatively correlated with the terms of trade if they hold the same portfolio as foreign households. While domestic households may want to reduce this negative correlation through hedging, they have to pay a risk premium to do so and consequently choose not to drive that correlation to zero. Interestingly, if domestic households are not allowed to invest in foreign equities

and if domestic equities are positively correlated with the terms of trade, the opposite result holds and consumption is positively correlated with the terms of trade.

The paper proceeds as follows. The model is described in Section I. In Section II, the main results of the paper for consumption choices are derived under the assumption that domestic households have a time-separable expected utility function of lifetime consumption. Section III presents the implications of the analysis for the current account. The case in which home-country households have a rate of time preference which is a function of their instantaneous flow of utility is discussed in Section IV. Section V relaxes the small country assumption and lets the domestic rate of interest be determined endogeneously. Section VI presents some concluding remarks.

I. The Model

We consider a small open economy in which all households are alike. This economy specializes completely in the production of one good which it uses as its numeraire. As domestic households also consume a good which is produced abroad only and serves as the numeraire in the foreign country, the domestic country must export the good it produces. The price of the imported good in terms of the exported good is e and is determined abroad because the foreign country is large and produces as well as consumes both goods. The terms of trade for the domestic country are therefore $1/e$. Markets for commodities and assets are assumed to be perfect. There are no taxes, transactions costs, or tariffs.

Households in the home country are assumed to be infinitely-lived and to maximize an expected utility function of lifetime consumption which is assumed to take the following form:

$$\langle 1 \rangle \qquad U = E_t \int_t^\infty \exp(-\rho\tau)\left(\frac{1}{\gamma}\right)[C_1(\tau)^{1-\alpha}C_2(\tau)^{\alpha}]^{\gamma}\, d\tau,$$

where $\gamma < 1$ and $0 \leqslant \alpha \leqslant 1$, and where $C_1(C_2)$ is the rate of consumption of the domestic (imported) good and $E_t(.)$ is the expectation operator. The expected utility function for domestic households implies that they have constant relative risk aversion and constant expenditure shares.[4] Finally, domestic households are assumed to have a constant subjective rate of time preference. This assumption is made here to simplify the analysis so that it is easier to understand the main point of the paper. In Section IV, we partially relax this assumption.

Contrary to the existing literature, we assume that domestic households can invest their real wealth in a variety of assets, including foreign equities.[5] The small country assumption implies that asset prices are determined abroad; consequently, there is no loss of generality if we assume that the investment opportunity set is constant.[6] One of the assets that domestic households can hold is a riskless bond which earns a domestic rate of return R per unit of time. Domestic households can invest in n risky assets which earn a domestic stochastic rate of return dI_i/I_i per unit of time, $i = 1, \ldots, n$. It is assumed that asset $i = 1$ is a default-free bond that earns R^* per unit of time in terms of the foreign good. Until Section VI, both R and R^* are determined abroad because the domestic country is a small country and capital mobility is perfect. However, if one assumes that the domestic country uses the

exported good as its numeraire while the foreign country uses the imported good as its numeraire, the bond which pays interest at the rate R corresponds to a risk-free bond in the domestic country while the bond which pays interest at the rate $R*$ is equivalent to a risk-free bond in the foreign country. Section V lets R be determined endogenously. Henceforth, we will therefore call the bond which earns interest at the rate R the domestic bond and the other bond the foreign bond. The domestic price of risky asset i, I_i, is assumed to satisfy a stochastic differential equation:

$$\langle 2 \rangle \qquad \frac{dI_i}{I_i} = \mu_i dt + \sigma_i dz_i, \qquad i = 1, \ldots, n,$$

where dz_i is the increment of a standard Wiener process.[7]

The price of the imported good is assumed to satisfy:

$$\langle 3 \rangle \qquad \frac{de}{e} = \mu_e dt + \sigma_e dz_e,$$

where dz_e is the increment of a Wiener process. While one might prefer to model the dynamics for the price of the imported good by some other stochastic process than the process given by equation $\langle 3 \rangle$, as long as that price follows a diffusion process and the investment opportunity set depends only on the price of the imported good and state variables whose changes are uncorrelated with changes in the price of the imported good, the main results of this paper remain the same. Notice that equation $\langle 3 \rangle$ implies that an unanticipated change in the terms of trade has no effect on the future dynamics of the exchange rate. Consequently, in this model, an unanticipated change in the terms of trade is equivalent to a permanent change in the terms of trade in a certainty model.[8]

With the expected utility function of domestic households in this model, there exists an exact price index P which satisfies:[9]

$$\langle 4 \rangle \qquad \frac{dP}{P} = \alpha \frac{de}{e} + (1 - \alpha)\alpha\sigma_e^2 dt.$$

Consumption expenditures using the domestic good as the numeraire are given by $C = C_1 + eC_2$, while real consumption expenditures c correspond to C/P.

II. Risk Tolerance and Real Consumption

In this model, changes in consumption expenditures are stochastic. In the absence of a change in the terms of trade, consumption expenditures can increase or decrease because the wealth of domestic households increased or decreased. To understand the effect of a change in the terms of trade on real consumption, one could take the total derivative of real consumption with respect to the terms of trade. However, such an approach would ignore the fact that changes in the value of some risky assets held by domestic households may be highly correlated with changes in the terms of trade, even though the value of these assets is not explicitly a function of changes in the terms of trade. As changes in real consumption are a function of many random variables, a more appropriate approach is to look at the covariance of changes in real consumption with changes in the terms of trade. This

covariance shows what one would expect to happen when an unexpected change in the terms of trade occurs.

We can now state the first major result of this paper:

Proposition I: The instantaneous covariance of the rate of change of real consumption expenditures with the rate of change of the terms of trade,

$$\text{Cov}\left(\frac{dc}{c}, \frac{d(1/e)}{1/e}\right),$$

is given by

$$\langle 5 \rangle \qquad \text{Cov}\left(\frac{dc}{c}, \frac{d(1/e)}{1/e}\right) = T^R(R - \mu_r - R^*)dt + T^R \alpha \sigma_r^2 dt,$$

where $T^R = 1/(1 - \gamma)$ is the relative risk-tolerance of domestic households.

Proof: See Appendix.

Proposition I is almost more interesting because of what does not appear in the equation for the covariance between consumption growth and the terms of trade than because of the various terms included in that equation. To wit, nothing related to the domestic home country's output appears in equation $\langle 5 \rangle$. While we made the standard assumption that the country is specialized in the production of the exported good, we could have made any arbitrary assumption about production in the domestic country and would have obtained the same result. That is because the distribution of asset returns — except for bond returns — does not enter equation $\langle 5 \rangle$ and because, furthermore, the domestic households' holdings of risky assets do not matter for that equation. To understand this surprising result, note that foreign and domestic bonds make it possible for domestic households to dynamically hedge terms of trade risks. Hence, if terms of trade risks can be costlessly hedged, risk-averse households bear no such risks. This means that the relevant variables in the households' decision about bearing terms of trade risks are simply the costs and benefits of hedging such risks.

In this model, a household can make its consumption deterministic by investing α in the foreign bond and $(1 - \alpha)$ in the domestic bond. With such an investment policy, the household bears no terms of trade risks. For a household to be willing to bear terms of trade risks, it must expect a reward in the form of a risk premium. A long investment in the domestic bond financed by a short position in the foreign bond has a return which is perfectly correlated with the terms of trade. To induce a household to bear terms of trade risks, it must therefore be the case that such an investment has a positive expected return. Otherwise, the household would bear risks without a compensation in the form of a higher expected return. However, such an investment has a positive expected real return only if the domestic real rate of interest exceeds the foreign real rate of interest. Denote by r_B and r_B^* the expected real rate of return of the domestic and the foreign bond for domestic households. It follows that real consumption is positively correlated with the terms of trade only if r_B is greater than r_B^*. Using Ito's Lemma, it follows that:

$$\langle 6 \rangle \qquad\qquad r_B = R^* - \alpha\mu_r + \tfrac{1}{2}(1 + \alpha)\alpha\sigma_r^2,$$

and

$$\langle 7 \rangle \qquad r_B^* = R^* + (1 - \alpha)\mu_r + \tfrac{1}{2}(1 - \alpha)^2\sigma_r^2 - \tfrac{1}{2}(1 - \alpha)\sigma_r^2.$$

Hence, r_B is greater than r_B^* if:

$$\langle 8 \rangle \qquad\qquad R - \mu_r - R^* + \varkappa \sigma_r^2 > 0.$$

This condition is exactly the same as the condition for real consumption to have a positive covariance with the terms of trade. Consequently, Proposition I can be restated as saying that domestic real consumption covaries positively with the terms of trade only if the domestic real rate of interest exceeds the foreign real rate of interest. Note that this result holds true even when one does not make the small country assumption. In the absence of this assumption, however, the relation between the expected real rate of return on domestic and foreign bonds cannot be taken to be exogeneous and hence has to be solved for explicitly. In particular, a change in the terms of trade would be associated with a change in R and R*.

There is no presumption that the real rate of interest at home differs from the real rate of interest abroad. However, in the absence of a presumption about the difference between domestic and foreign real interest rates, it immediately follows that one expects domestic consumption in units of the exported good to covary negatively with the terms of trade. To see this note that $C = ce^\gamma$. Using Ito's Lemma and Proposition I, it follows that:

$$\langle 9 \rangle \qquad\qquad \mathrm{Cov}\left(\frac{dc}{c}, \frac{d(1/e)}{(1/e)} \right) = T^R(r_B - r_B^*)dt - \varkappa \sigma_r^2 dt.$$

This result is not surprising, as consumption measured in units of the exported good must increase when the terms of trade fall unexpectedly to keep real consumption unaffected by the adverse change in the terms of trade.

III. Dynamics of the Current Account

The current account is the difference between national income and absorption. Let planned domestic real investment be i per unit of time and assume that i depends at most on the terms of trade. By definition, absorption is equal to $(c + i)dt$, while national income is equal to the change in national real wealth, *i.e.*, dw.[10] As, over the next instant, both c and i are fixed, it follows that if da is the real current account:

$$\langle 10 \rangle \qquad\qquad \mathrm{Cov}\left(da, \frac{d(1/e)}{(1/e)} \right) = T^R(r_B - r_B^*)wdt,$$

where w is real wealth. Hence, the sign of the covariance of the real current account with the terms of trade is the same as the sign of the covariance of real consumption with the terms of trade. This result is not surprising because, with the assumed utility function, real consumption is proportional to real wealth and investment is non-stochastic over a period of length dt. However, it follows from this result that the relation between the current account and the terms of trade does not depend on domestic production.

We can now consider how an unexpected change in the terms of trade affects the expected value of the real current account, *i.e.*, $E(dw - cdt - idt)$. Because the terms of trade follow a continuous-time stochastic process, over each interval of time the current account will be affected by the contemporaneous unanticipated change in the terms of trade. To evaluate the effect of a change in the terms of trade on subsequent current accounts, one wants therefore to focus on the expected or *ex*

ante rather than on realized or *ex post* current accounts, as *ex post* current accounts depend on unanticipated changes in the terms of trade which occur after the shock whose effect one wishes to analyze. It is proved in the Appendix that:

> *Proposition II.* The covariance of changes in the expected real current account μ_a and the rate of change of the terms of trade is:

$$\langle 11 \rangle \qquad \mathrm{Cov}\left(d\mu_a, \frac{d(1/e)}{(1/e)}\right) = (\mu_w w - c)T^R(r_B - r_B^*)dt - i'\sigma_r^2 dt,$$

where $\mu_w w$ is the expected real income and i' is $\partial i/\partial(1/e)$.

Notice that when the domestic real rate of interest is equal to the foreign real rate of interest, the sign of the covariance of the change in the expected real current account with the terms of trade depends only on the sign of i'.[11] In particular, if domestic investment increases with the terms of trade, the expected current account falls with an unexpected increase in the terms of trade.[12] When i' is positive, the Harberger–Laursen–Metzler effect holds, provided that income exceeds consumption expenditures, only if the domestic real rate of interest exceeds the foreign real rate of interest by a sufficient amount. If r_B is greater than r_B^*, domestic real wealth is positively correlated with the terms of trade. Hence, a positive i' and a high r_B make it possible for the increase in investment caused by an improvement in the terms of trade to be financed out of the increase in saving which results from the associated increase in real wealth. If i' is negative, the HLM effect holds unless the foreign real rate of interest is much higher than the domestic real rate of interest. However, if one believes that differences in real rates of interest tend to be small, then the main determinant of the covariance of the expected current account with the terms of trade is the covariance of changes in investment with changes in the terms of trade.

IV. An Extension

Recent research on the HLM effect shows that the validity of this effect may depend on the sign of the partial derivative of the domestic households' subjective rate of time preference with respect to the instantaneous flow of utility. In particular, it has been shown that in some models, if this derivative is positive, the HLM effect does not hold.[13] In this section, we extend our analysis to show how our results are affected if the domestic households' subjective rate of time preference is a function of the instantaneous flow of utility. As we assumed that domestic households spend a constant share α of consumption expenditures on the foreign good, we can use an instantaneous utility function of real consumption $u^*(c)$ for domestic households. Following Uzawa (1968), we assume that domestic households maximize the expected value of:

$$\langle 12 \rangle \qquad \int_0^\infty e^{-\Delta(s)s} u^*(c(s))ds,$$

where the discount rate $\Delta(s)$ is given by

$$\langle 13 \rangle \qquad \Delta(s) = \int_0^s \delta(u^*(c(v)))dv,$$

and $\delta(u^*(c(v)))$ is the subjective rate of time preference at time v. It is assumed that $\delta(.)$ is positive and that $\delta(.)$ and $u^*(.)$ are twice differentiable.[14] Let $J(w, t)$, where w is the real wealth of domestic households, be a function which satisfies:

$$\langle 14 \rangle \qquad J(w, t) = \mathrm{Max}\, E_t \int_t^T e^{-\Delta(s)s} u^*(c(s)) ds.$$

Using well-known techniques, we can then show that:

$$\langle 15 \rangle \qquad \underline{w} = \left(\frac{-J_w}{J_{ww}w}\right) = V_{aa}^{-1}\underline{\mu}^R ,$$

where μ^R is the vector of real expected excess returns, *i.e.*, $(1/dt)E(d(I_i/P)/(I_i/P)) - r$ is the ith element of the vector μ^R, and r is the real rate of return on an asset which promises a safe real return. By a proof similar to the proof used in obtaining Proposition I in Section III, we get:

$$\langle 16 \rangle \qquad \mathrm{Cov}\left(\frac{dc}{c}, \frac{d(1/e)}{1/e}\right) = \left(\frac{-c_w^* J_w}{J_{ww}w}\right)(r_B - r_B^*)dt.$$

Equation $\langle 16 \rangle$ is similar to equation $\langle 5 \rangle$ in Section III. In particular, the sign of $\mathrm{Cov}(dc/c, d(1/e)/(1/e))$ is the same as in Section III as long as domestic households behave like risk-averse households, *i.e.*, as long as the first term in parentheses in equation $\langle 16 \rangle$ is positive. This result is not surprising, as households still choose how changes in their real wealth are correlated with changes in the terms of trade. When the utility function of domestic households is given by $\langle 12 \rangle$ and $\langle 13 \rangle$, there is no reason for the indirect utility function of wealth to exhibit constant relative risk tolerance when the instantaneous utility function $u^*(c(t))$ exhibits constant relative risk tolerance. Let $T(w, t)$ be the inverse of the coefficient of relative risk aversion of the indirect utility function of wealth $J(w, t)$. If $T(w, t)$ increases with wealth, an increase in wealth increases expected income per unit of invested wealth as the expected income on an (efficient) portfolio of risky assets exceeds the return on a safe asset and as an increase in $T(w, t)$ means that households invest more in risky assets. It follows that, with the indirect utility function given by equation $\langle 14 \rangle$, the effect of an unanticipated change in the terms of trade on the expected current account can also involve a change in μ_w, *i.e.*, the expected real income per unit of wealth invested. In this case, the covariance of changes in the expected real current account and the rate of change of the terms of trade is:

$$\langle 17 \rangle \qquad \mathrm{Cov}\left(da, \frac{d(1/e)}{1/e}\right) = (T\mu_w w + T_w \mu_w w - Tc_w)(r_B - r_B^*)dt.$$

While no interesting results can be provided about T_w, it is still the case that if households are not rewarded for bearing terms of trade risks, changes in the current account are uncorrelated with changes in the terms of trade. With the approach used in this paper, households can choose to hold their wealth in such a way that its real value is not affected by an unanticipated change in the terms of trade. If households choose to invest their wealth in this way, the expected real current account is not affected by unanticipated changes in the terms of trade, whether the households' subjective discount rate varies with instantaneous utility or not.

V. Relaxing the Small Country Assumption

The most narrow interpretation of the small country assumption would be to assume that all households are the same, so that all households have the same asset demands and no household can hedge. In this section, we show how, when domestic households differ from foreign households, their desire to hedge against unanticipated changes in the terms of trade makes the domestic real rate of interest differ from the foreign real rate of interest. It turns out that the sign of the difference between the domestic and the foreign real rates of interest depends crucially on the investment opportunity set of domestic investors and on their degree of relative risk aversion. Consequently, so does the existence of the HLM effect. For our analysis to be meaningful, households in at least one country must have a utility function with a coefficient of relative risk aversion which differs from one. Otherwise, no household has asset demands which depend on expenditure shares. However, a two-country analysis with households who do not have a logarithmic utility function is not tractable in the framework used in this paper. Such an analysis would have to solve explicitly for asset demands in a setting in which the investment opportunity set changes stochastically. To avoid this difficulty, we compare economies in which the domestic country's asset demand functions differ from those of the foreign country while maintaining the assumption of a constant investment opportunity set.

Throughout this discussion, we assume that all foreign households have the same utility function which is time-additive, exhibits constant relative risk aversion and yields constant expenditure shares. Suppose now that a foreign household's consumption of the good produced in the small country is negligible, so that it consumes almost exclusively the good imported by the small country. In this case, the instantaneous real rate of return of a domestic bond for foreign households is $(R^* + \sigma_c^2)dt - de/e$. The first-order conditions of a foreign household's optimization problem imply that the expected real rate of return of the domestic bond for that household is proportional to the covariance of the real rate of return of the bond with the real rate of growth of foreign wealth. More precisely, it must be the case that:

$$\langle 19 \rangle \qquad T^R{}^*(R - \mu_e + \sigma_e^2 - R^*) = -\sigma_{e,w^*},$$

where $T^R{}^*$ is the coefficient of relative risk tolerance of foreign households and σ is the covariance of the rate of growth of the domestic price of the imported good with the rate of growth of foreign real wealth.[15] The small country assumption has been interpreted so far to mean that σ_{e,w^*} does not depend on the portfolio choices of households in the small country, so that $R - \mu_e + \sigma_e^2 - R^*$ can be treated as fixed. In this section, we let σ_{e,w^*} be determined endogeneously.

A natural starting point for the analysis of this section is to assume that domestic households have the same consumption preferences as households in the (large) foreign country and the same degree of relative risk aversion. In this case, all households hold the same portfolio of risky assets. The risk-free asset is the foreign bond and no bonds are held in equilibrium. Let σ_{e,w^*} be equal to zero in that case. Now we consider an otherwise identical economy in which a domestic household's consumption of the good produced in its country is not trivial, *i.e.*, α is substantially smaller than one. In this economy, when the expected returns are the same as in the economy with α close to one, domestic households want to hedge against

unanticipated changes in the price of their consumption basket if their coefficient of relative risk aversion exceeds one.[16] If, for these expected returns, domestic households hold the same portfolio as foreign households, their real wealth is positively correlated with the price of imports (as $\sigma_{r,y^*} = 0$ by assumption) and hence is negatively correlated with the terms of trade. To hedge, these households must therefore change their asset holdings to make their real wealth less negatively correlated with the terms of trade (for instance, by purchasing domestic bonds and assets whose return is positively correlated with the terms of trade). However, in equilibrium, hedging by domestic country households implies that the real wealth of foreign households must become negatively correlated with the terms of trade (for instance, because they have to sell the domestic bonds purchased by domestic investors). This means that σ_{r,y^*} must increase. From equation $\langle 18 \rangle$, an increase in σ_{r,y^*} implies that the expected real return on foreign bonds becomes greater than the expected real return on domestic bonds for foreign households. Proposition I implies that, in this case, domestic real consumption is negatively correlated with the terms of trade.

To understand why domestic real consumption is negatively correlated with the terms of trade when α is smaller than one, it is important to remember that domestic households can invest in foreign equities. Hence, when α is close to one, the income of domestic households is perfectly correlated with the income of foreign households because, as in Lucas (1982), households share all production risks. This means that when α falls, the real wealth of domestic households becomes negatively correlated with the terms of trade in the absence of hedging if foreign real wealth is uncorrelated with the terms of trade. Sufficiently risk averse domestic households may want to reduce the magnitude of this correlation through hedging, but they have to pay a risk premium to do so and hence will not choose to drive this correlation to zero.

The analysis of this section suggests that the HLM effect holds if, in the absence of hedging by domestic households, the real wealth of foreign households is uncorrelated with the terms of trade. This result depends however crucially on the ability of domestic households to invest in foreign equity. To see this, suppose that domestic households cannot invest in foreign equity and that, in the absence of hedging, their real wealth is positively correlated with the terms of trade. In this case, if domestic households try to hedge against unanticipated changes in the price of their consumption basket, they want to buy foreign bonds and sell domestic bonds. This means that, in equilibrium, σ_{r,y^*} must fall, so that the expected real return on domestic bonds exceeds the expected real return on foreign bonds for foreign households. Hence, if domestic households cannot invest in foreign equity, Proposition I suggests that their real consumption is positively correlated with the terms of trade (assuming that the resulting σ_{r,y^*} is low enough).

The analysis of this section can be extended in at least three directions. First, one can let σ_{r,y^*} differ from zero when α is close to one. σ_{r,y^*} is a measure of the risk for foreign households of an asset which is perfectly negatively correlated with the terms of trade. Suppose that σ_{r,y^*} is negative when α is close to one. In this case, the expected real rate of return on the domestic bond exceeds the one on the foreign bond in the absence of hedging. While hedging by domestic households increases σ_{r,y^*} unless it is so low that the real wealth of domestic households is positively correlated with the terms of trade, the increase in σ_{r,y^*} may not be sufficient to make that covariance positive. Hence, if foreign real wealth is positively correlated with

the terms of trade without hedging by domestic households, it is possible for domestic real consumption to be positively correlated with the terms of trade in equilibrium. Secondly, it is possible to relax the assumption that foreign households spend only a small fraction of their consumption expenditures on the good produced in the small country. If foreign households wish to hedge also, the more their consumption resembles the consumption of domestic households, the more expensive it is for domestic households to hedge. Finally, one could let the coefficient of relative risk aversion of foreign households fall. This decreases the cost of hedging for domestic households.

VI. Concluding Remarks

In this paper, we have constructed a simple model of the behavior of households in a small open economy. The model has been useful to show how optimizing households can be expected to behave when an unanticipated change in the terms of trade occurs. The major innovation of the model is that we assume that households know the distribution of the rate of change of the terms of trade and can hedge against unanticipated changes in the terms of trade. In this model, asset holdings are derived as solutions to an optimal control problem. It turns out that this approach leads to a simple formula for the covariance of terms of trade changes with changes in the real consumption rate and for the covariance of terms of trade changes with changes in the expected real current account. We show that an unanticipated change in the terms of trade affects real consumption expenditures and the expected real current account only if the expected real return on foreign bonds differs from the expected real return on domestic bonds.

It would be interesting to pursue the analysis of this paper in a number of ways. Introducing money in the model would allow us to look at the covariance of changes in real consumption with changes both in the nominal and the real exchange rate. The assumption of constant expenditure shares could be relaxed. Production could be modeled explicitly and labor could be introduced as a factor of production. While pursuing the analysis of this paper in these various directions would improve our understanding of optimal consumption choices in open economies, we believe that the central message of this paper is robust. This message is that changes in a country's real consumption expenditures are positively correlated with changes in the exchange rate only if the expected reward for holding a long position in the foreign currency is large enough. However, as evidenced by our discussion in Section V, extensions of our model can add important insights into the determinants of the sign and magnitude of the correlation of changes in consumption with changes in the terms of trade.

Appendix

By investing x in the foreign bond and $(1-x)$ in the domestic bond, households can form a safe real asset whose real return is r. We can therefore solve the household's portfolio problem by assuming that the investment opportunity set includes the n assets risky in the domestic numeraire plus the safe real asset. In this case, the flow budget constraint is:

$$\langle A1 \rangle \qquad dw = \sum_{i-1}^{n} n_i \left(\frac{d(I_i/P)}{I_i/P} - rdt \right) w + rwdt - cdt,$$

where c corresponds to the real consumption expenditures and n_i is the fraction of the household's wealth invested in risky asset i. Using well-known techniques, the optimal portfolio can be shown to be:

$$\langle A2 \rangle \qquad\qquad \underline{n} = T^R I'^{-1} \mu^R,$$

where $\underline{n}$ is the vector of investment proportions, T^R is the household's relative risk tolerance $1/(1 - \gamma)$, I' is the variance–covariance matrix of real asset returns and μ^R is the vector of real expected excess returns, *i.e.*, the ith element is

$$(1/dt)E\left(\frac{d(I_i/P)}{I_i/P}\right) - r.$$

Let n_1 be the fraction of the household's wealth invested in the foreign bond. Using Ito's Lemma, it follows that:

$$
\langle A3 \rangle \qquad\qquad
\begin{aligned}
\operatorname{Cov}\left(\frac{dc}{c}, \frac{de}{e}\right) &= \frac{1}{c}\operatorname{Cov}\left(c_w dw, \frac{de}{e}\right) \\
&= T^R (I'^{-1}\mu^R)' I'_e dt \\
&= T^R \mu^R I'^{-1} I'_e dt \\
&= T^R (r_B^* - r)(1/(1 - \alpha))dt,
\end{aligned}
$$

where I'_e is the vector of covariances of real asset returns with the rate of change of e. The second line of $\langle A3 \rangle$ comes from the fact that the household's portfolio of risky assets is the only risky component of the change in the household's wealth dw given in $\langle A1 \rangle$. The last line of $\langle A3 \rangle$ follows from the fact that I'_e is just the first line of $(1/(1 - \alpha)) I'$.

Proposition I follows after substituting r in the last line of $\langle A3 \rangle$, as $r_B^* - r = r_B^* - (\alpha r_B^* + (1 - \alpha)r_B) = (1 - \alpha)(r_B^* - r_B)$, and noting that $\operatorname{Cov}(dc/c, de/e) = -\operatorname{Cov}(dc/c, d(1/e)/(1/e))$.

Next, note that:

$$
\langle A4 \rangle \qquad\qquad
\begin{aligned}
\operatorname{Cov}\left(dw - cdt - idt, \frac{de}{e}\right) &= wT^R (I'^{-1}\mu_R)' I'_e dt \\
&= wT^R \mu'_R I'^{-1} I'_e dt \\
&= wT^R (r_B^* - r_B)dt,
\end{aligned}
$$

which proves equation $\langle 10 \rangle$.

Finally, letting $\mu_w w = E(dw)/dt$, note that:

$$
\langle A5 \rangle \qquad
\begin{aligned}
\operatorname{Cov}\left(dE(dw - cft - idt), \frac{de}{e}\right) &= \operatorname{Cov}\left(d(\mu_w wdt - cdt - idt), \frac{de}{e}\right) \\
&= \operatorname{Cov}\left(\mu_w dw - c_w dw - i'\frac{de}{e}, \frac{de}{e}\right) \\
&= \operatorname{Cov}\left((\mu_w w - c)\frac{dw}{w} - i'\frac{de}{e}, \frac{de}{e}\right) \\
&= [(\mu_w w - c)T^R(r_B^* - r_B) - i'\sigma_e^2]dt,
\end{aligned}
$$

which proves Proposition 2.

Notes

1. See, for instance, Sachs (1981), Obstfeld (1982), Dornbusch (1983), Svensson and Razin (1983), and Persson and Svensson (1985).
2. Sachs (1981) also emphasizes the role of investment.
3. See Harberger (1950) and Laursen and Metzler (1950).
4. These assumptions are convenient. Without them, the analysis would be substantially more complicated but our main results would not be significantly altered. See Stulz (1983) for a discussion of asset demands in a similar model when these assumptions are omitted.
5. Stockman and Svensson (1987) constitute an important exception. However, their analysis focuses on different issues than the HLM literature.
6. Foreign households must also be able to invest in domestic equities to make this assumption sensible.
7. A convenient description of the methodology used here and appropriate references are given in Fischer (1975). The continuous-time methodology is useful here because it makes it possible to obtain exact formulas for the covariance of changes in the terms of trade with changes in consumption and with changes in the current account which are linear in means and variances of observable variables. A discrete-time approach which would exploit the first-order conditions of the domestic households' optimization problem would yield distribution-free formulas which, however, would be nonlinear functions of observable variables in the absence of extremely restrictive assumptions on the utility function of domestic households.
8. One could model temporary shocks by letting $\mu = \lambda(\bar{e}/e) - \lambda$. This change in assumption would yield a mean-reverting process for the price of the imported good.
9. See Breeden (1979).
10. Note that dw does not correspond to income as measured by the national accounts because it includes changes in the value of domestic and foreign assets, while national accounts include only the dividends paid by these assets. For a detailed analysis of the implications of these measurement issues, see Stockman and Svensson (1987) or Obstfeld (1986).
11. Note that this result differs from the time-additive literature on the current account (see, for instance, Sachs, 1981), because here households can *choose*, through their portfolio holdings, how their wealth is correlated with the terms of trade.
12. Persson and Svensson (1985), for instance, have a model in which they derive an investment function which has the property $i' > 0$.
13. See Svensson and Razin (1983).
14. Obstfeld (1982) uses equations ⟨13⟩ and ⟨14⟩. He assumes that $\delta' \geqslant 0$. For discussion of the methodological issues associated with using equations ⟨13⟩ and ⟨14⟩ in a continuous-time stochastic optimization model, see Bergman (1985).
15. This result can be verified by using Stulz (1981) or by inverting equation ⟨A2⟩ in the Appendix and substituting T^{R*} for T^R.
16. See Stulz (1983) for a discussion.

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