PRICING CAPITAL ASSETS IN AN INTERNATIONAL SETTING: AN INTRODUCTION

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Abstract. This paper shows how differences across countries of 1) inflation rates, 2) consumption baskets of investors, and 3) investment opportunity sets of investors matter when one applies capital asset pricing models in an international setting. In particular, the fact that countries differ is shown to affect the portfolio held by investors, the equilibrium expected returns of risky assets, and the financial policies of firms.

SECTION 1: INTRODUCTION

■ Capital asset pricing models play a central role in finance theory.[1] For instance, they provide the basis for finding appropriate discount rates in capital budgeting problems. When one tries to extend such models to an international setting, however, one is confronted with several important problems.

First, whereas most basic capital asset pricing models use the simplifying assumption that returns are denominated in a numeraire good whose price is fixed, such an assumption makes little sense when one has to choose a portfolio in a world in which inflation rates are stochastic and differ across countries.

Second, in deriving capital asset pricing models, one generally assumes that investors have identical tastes with respect to commodities and consume identical baskets of goods. While this may be a reasonable approximation in a single country, it is not very useful in an international setting since investors based in different countries consume locally produced, nontraded goods, and, probably, differing proportions of internationally traded goods. Because the relative prices of traded and nontraded goods fluctuate significantly, real returns on any particular asset depend upon which investor's perspective is taken.

Third, capital asset pricing models generally assume that all investors have the same investment opportunity set—that is, that any distribution of returns feasible for one investor is feasible for all investors. In an international setting, taxes, transactions costs, and holding costs differ across securities and across investors. Furthermore, some investors are restricted from buying some securities.

This paper shows how differences across countries of 1) inflation rates, 2) consumption baskets of investors, and 3) investment opportunity sets of investors matter when one applies capital asset pricing models in an international setting. In particular, the paper shows how the fact that countries differ affects the portfolio held by investors, the equilibrium expected returns, and the financial policies of firms. Many results presented in this paper are already known. The main contribution of this paper is a systematic and unified examination of the implications of what the author believes to be the 3 most important differences among countries for application of capital asset pricing models in an international setting.[2] The purpose of this paper is to discuss problems that one encounters when applying capital asset pricing models, rather than to derive international asset pricing models under the most general assumptions. Throughout the paper, references are provided to more general derivations of the results presented, when such references are available, and otherwise directions are provided for

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more general derivations of the results. It is hoped that the reader will find that the added clarity in the discussion, which results from its not being burdened by a lot of technical baggage, makes up for the lack of generality and sophistication of the approach.

Although all 3 differences among the countries to be explored affect asset prices in a significant way, discussion will first introduce the difference among countries whose impact on capital asset pricing models is the easiest to understand. As discussion proceeds, some initial assumptions will be relaxed to obtain more realistic but also more complex models of capital asset pricing. The plan of the paper is as follows: Section 2 discusses differences in inflation rates. The effects of these differences on asset holdings and expected returns are further explored in Section 3. Section 4 introduces differences in consumption baskets. Differences in investment opportunity sets are discussed in Section 5. Section 6 offers some concluding remarks.

SECTION 2: DIFFERENCES IN PURCHASING POWER RISKS ACROSS CURRENCIES

This section shows how differences in purchasing power risks across currencies affect an investor's holdings of risky assets and the equilibrium expected returns of risky assets when there are no real differences among investors. "Purchasing power risks" refers to the differing, uncertain purchasing power of individual currencies under the assumption that goods markets are in equilibrium. In other words, in this section all investors face the same real investment opportunity set and consume the same basket of commodities. While this may appear to be an unrealistic set of assumptions, it is one that has been used in most of the international asset pricing models. Further, it provides an excellent case to understand just under which conditions internationalization of capital asset pricing models makes no difference. Section 4, below, deals with the more complex but realistic situation in which there are not only differing rates of inflation across currencies, but there also are relative price changes and differing consumption preferences among investors.

While earlier research has presented models that show how differences in purchasing power risks across currencies can be incorporated in a model of asset pricing, this section uses yet another model.[3] In the perspective of this paper, the model used here is attractive for at least 2 reasons. First, it is simpler than earlier models in that it constitutes a straightforward extension of the Black [1972] capital asset pricing model without risk-free lending and borrowing. Second, throughout this paper, equilibrium expected returns are derived by perturbating an investor's portfolio at the optimum. The model used in this section is helpful to introduce this approach.

Some Assumptions

To focus the discussion on inflation rates, it is assumed that all investors consume the same commodity and that the price of the commodity in any particular currency is the same in all countries. In other words, there are no barriers to international trade and the law of one price as well as purchasing power parity hold all the time.[4] Furthermore, a one-period world is assumed in which investors maximize a utility function that is increasing in the expected value of end-of-period consumption and, because of risk aversion, decreasing in the variance of end-of-period consumption. End-of-period consumption, in a one-period world, is equal to end-of-period real wealth—that is, nominal wealth divided by the price of the commodity consumed. With these assumptions it follows that investors care only about the mean and variance of real returns. Finally, it is assumed that financial markets are perfect—that is, all investors are price-takers, there are no transactions costs, no taxes, and no restrictions on short sales.

Without loss of generality, it is assumed that there are only 2 countries, the domestic country and the foreign country. Prices and returns denominated in foreign currency are denoted by stars. As the law of one price holds, real returns are the same for all investors. To understand this, let P_{ij} (P^*_{ij}) be the price of security i in domestic (foreign) currency and P_j (P^*_j) be the domestic (foreign) price of the commodity at date j, j = 0,1. Notice that the price at home of security i in terms of the commodity at date j is P_{ij}/P_j whereas abroad it is P^*_{ij}/P^*_j. Let e_j be the price of one unit of foreign currency at date j. The law of one price implies that $P_j = e_jP^*_j$. It immediately follows that $P^*_{ij}/P^*_j = e_jP^*_{ij}/e_jP^*_j = P_{ij}/P_j$. As the price of a security in terms of the commodity is the same everywhere, real returns must be the same everywhere.

Equilibrium Expected Real Returns

To show the determinants of equilibrium expected returns, it is assumed that markets are in equilibrium and the first-order conditions of the representative investor's portfolio optimization problem are used. The fact that real returns on securities are the same for all investors means that the country of residence of an investor is irrelevant. Thus a representative investor can be looked at, and only real returns used. This investor chooses a portfolio that is mean-variance efficient—that is, a portfolio such that, for a given variance of real return, no other portfolio has a higher real expected return. Random variables are denoted by tildes. Let $\tilde{r}_i$ be the real return of security i and σ_{ij} be the covariance of the real return of security i with the real return of security j.

Given the assumptions about the investor's expected utility function, the solution of the investor's portfolio choice problem is a portfolio, that is, a set of weights w_i, i=1, . . . ,n, which maximizes his expected real wealth subject to the constraint that the variance of the real return of his portfolio equals some number K and that the w_i's sum to one. Consequently, to solve for the optimal w_i's, the investor maximizes the following Lagrangian function:

$$L = E\left(\sum_{i=1}^{n} w_i\tilde{r}_i\right) + \lambda\left(\sum_{i=1}^{n}\sum_{j=1}^{n} w_iw_j\sigma_{ij} - K\right) + \mu\left(\sum_{i=1}^{n} w_i + 1\right) \qquad (1)$$

where E(.) denotes the expectation operator. Let w_i^o be the optimal value of w_i, for all i's. By the fact that w_i^o and w_j^o are optimal, an infinitesimal change in w_i^o and w_j^o, so that $w_i^{o'} = w_i^o + \Delta$ and $w_j^{o'} = w_j^o - \Delta$, leaves the optimized value of L unchanged. It follows that

$$\frac{\partial L}{\partial w_i^o} - \frac{\partial L}{\partial w_j^o} = E(r_i) - E(r_j) + \lambda\left[\sum_{k=1}^{n} w_k^o\sigma_{ik} - \sum_{k=1}^{n} w_k^o\sigma_{jk}\right] = 0. \qquad (2)$$

The economy being looked at has many investors who are not all the same. Suppose, however, that an artificial economy could be constructed that would be exactly the same as the economy considered here, except for the fact that in that artificial economy all investors are the same. Our representative investor is equivalent to an investor in that artificial economy. Therefore, the representative investor must hold the market portfolio—otherwise the market portfolio would not be held in the artificial economy.[5] Let $\tilde{r}_m$ be the real return on the market portfolio and $\tilde{r}_z$ the real return on a portfolio uncorrelated with the return on the market portfolio. Equation (2) then implies that[6]

$$E(\tilde{r}_i) - E(\tilde{r}_z) = \beta_i\,[E(\tilde{r}_m) - E(\tilde{r}_z)], \qquad (3)$$

where

$$\beta_i = \frac{Cov(\tilde{r}_i, \tilde{r}_m)}{Var(\tilde{r}_m)}. \qquad (4)$$

Equation (3) shows that the real expected return on a security is a linear function of the beta of the security. Furthermore, similarly to the Black [1972] model, the expected real return on a security in excess of the expected real return on the zero beta portfolio is an increasing function of the expected real return on the market portfolio in excess of the expected real return on the zero beta portfolio. Notice, however, that the market portfolio includes all securities in the world and not only domestic securities. Whereas it seems reasonable to argue that investors cannot borrow or lend at a real risk-free rate, note also that when such borrowing and lending opportunities are available, the real risk-free rate of return replaces $E(\tilde{r}_z)$ in equation (3).

SECTION 3: SOME IMPLICATIONS OF DIFFERENCES IN PURCHASING POWER RISKS ACROSS CURRENCIES

As the model presented in Section 2 is similar to the Black model of asset pricing without risk-free borrowing and lending, it is not surprising that its implications for holdings of risky assets by investors are the same as those of the Black model. In particular, in this model, investors hold portfolios that are mean-variance efficient in real terms. The representative investor holds the market portfolio, which is mean-variance efficient. An investor more (less) risk-tolerant than the average investor holds a portfolio more (less) risky than the market portfolio. As there is no good reason to expect the risk tolerance of investors to differ much across countries, the model implies that investors tend to hold similar portfolios everywhere in the world. As it is generally believed that investors hold very different portfolios across countries and that the portfolio held by an investor tends to be heavily weighted toward investments in his home country, the model developed so far offers an incomplete view of portfolio choice and, consequently, of asset pricing in open economies. It is, however, worthwhile to pursue the implications of this model for equilibrium expected returns, as this allows us to understand how differences in purchasing power risks across currencies affect equilibrium expected real returns. The section concludes with a discussion of the concept of exchange rate risk.

Equilibrium Expected Returns of Common Stocks

The real return on security i is equal to the percentage change in the real price of security i. The real price of security i is its price in a currency divided by the price in the same currency of the commodity consumed by investors. As the law of one price holds, one can use domestic currency prices without loss of generality. Consequently, the real return on security i is

$$\tilde{r}_i = \frac{\tilde{P}_{i1}}{\tilde{P}_1}\left(\frac{P_{i0}}{P_0}\right)^{-1} - 1 = \frac{\tilde{R}_i - \tilde{I}}{1 + \tilde{I}} \tag{5}$$

where $\tilde{I}$ is the domestic rate of inflation, that is, $\tilde{P}_1/P_0 - 1$, and $\tilde{R}_i$ is the nominal rate of return on asset i, that is, $\tilde{P}_{i1}/P_{i0} - 1$. Substituting for $\tilde{r}_i$ in equation (3) and using equation (5),

$$E\left(\frac{\tilde{R}_i - \tilde{I}}{1 + \tilde{I}}\right) - E\left(\frac{\tilde{R}_z - \tilde{I}}{1 + \tilde{I}}\right) = \beta_i\left[E\left(\frac{\tilde{R}_m - \tilde{I}}{1 + \tilde{I}}\right) - E\left(\frac{\tilde{R}_z - \tilde{I}}{1 + \tilde{I}}\right)\right] \tag{6}$$

where

$$\beta_i = \frac{\mathrm{Cov}\left(\frac{\tilde{R}_i - \tilde{I}}{1 + \tilde{I}}, \frac{\tilde{R}_m - \tilde{I}}{1 + \tilde{I}}\right)}{\mathrm{Var}\left(\frac{\tilde{R}_m - \tilde{I}}{1 + \tilde{I}}\right)} \tag{7}$$

Equation (6) shows that, in general, the model developed in this section yields different expected returns for securities than converting returns into domestic currency returns and using the Sharpe-Lintner capital asset pricing model.[7] With the model developed in this section, the expected real return of individual assets in excess of the expected real return on the zero beta portfolio is a linear function of the beta of these assets computed using real returns for the assets and for the market portfolio. In contrast, the Sharpe-Lintner model is generally interpreted as stating that the expected nominal return of individual assets in excess of the return of the riskless nominal asset is a linear function of the beta of these assets computed using nominal returns for these assets and for the market portfolio. To gain some insights into how the expected returns given in equation (6) differ from those given by the Sharpe-Lintner model, it is useful to pause briefly and look at the relation between nominal returns and inflation. The traditional view is that common stocks are good hedges against unanticipated inflation because common stocks are claims to real resources. In general, however, common stock returns can be positively or negatively correlated with inflation depending on the activities of the corporation considered. For instance, one would expect that the higher the face value of the nominal debt of a firm, ceteris paribus, the higher the gain of that firm from unanticipated inflation. Nevertheless, the empirical evidence is that, on average, nominal common stock returns are negatively correlated with anticipated inflation and uncorrelated or negatively correlated with unexpected inflation.[8]

To make the implications of equation (6) more explicit, discussion turns briefly to the special case in which the nominal rate of return on the market portfolio is uncorrelated with the rate of inflation. Consider a security whose nominal return is uncorrelated with the rate of inflation and with the nominal rate of return of the market portfolio. This security has a Sharpe-Lintner beta equal to zero. However, the real beta of this security, that is, the beta computed according to equation (7), is positive, as both the real return of this security and the real return of the market portfolio are negatively correlated with the rate of inflation. Furthermore, the Sharpe-Lintner beta will underestimate (overestimate) the risk of assets whose real rate of return is negatively (positively) correlated with the rate of inflation.[9] To understand this, notice that an asset whose Sharpe-Lintner beta is equal to zero but whose real rate of return is negatively (positively) correlated with the rate of inflation has a positive (negative) real beta.

The Relation between the Forward Exchange Rate and the Expected Future Spot Exchange Rate

The model derived in Section 2 can be used to show how the forward exchange rate is related to the expected future spot exchange rate. The payoff from holding a one-period foreign currency forward contract is $e_1 - F$, where e_1 is the spot exchange rate at date 1 and F is the forward exchange rate at date 0 for a contract that matures at date 1. This follows from the fact that the investor can sell at a price e_1 the currency he has to purchase at a price F. As a forward contract involves no initial outlay, the forward price F must be such that the present value of a payoff equal to $e_1 - F$ is equal to zero. Consequently, the forward exchange rate is a decreasing function of the systematic risk an investor must bear when he holds a forward contract. The difference between the expected future spot exchange rate and the forward exchange rate compensates investors for the systematic risk they bear while being long in the foreign currency. The extent to which the forward exchange rate underpredicts the future spot exchange rate is an increasing function of this systematic risk.

Using the model of Section 2,

$$E\left(\frac{\tilde{d}-f}{1+\tilde{I}}\right) = \beta_d \left[E\left(\frac{\tilde{R}_m - \tilde{I}}{1+\tilde{I}}\right) - E\left(\frac{\tilde{R}_z - \tilde{I}}{1+\tilde{I}}\right) \right], \tag{8}$$

where

$$\beta_d = \frac{\operatorname{Cov}\left(\frac{\tilde{d}-f}{1+\tilde{I}}, \frac{\tilde{R}_m - \tilde{I}}{1+\tilde{I}}\right)}{\operatorname{Var}\left(\frac{\tilde{R}_m - \tilde{I}}{1+\tilde{I}}\right)}. \tag{9}$$

In equation (8), $\tilde{d}$ is the percentage rate of change of the exchange rate, that is, $\tilde{e}_1/e_o-1$, while f is the forward premium. Equation (8) gives a formal content to the discussion, as it shows that, for a given expected rate of change of the exchange rate, the forward exchange rate is a decreasing function of β_d, which is the beta of the payoff of a forward contract expressed as a fraction of the current spot exchange rate. Notice that β_d is almost equal to the beta of the rate of change of the exchange rate. If β_d is equal to zero, the forward exchange rate is roughly an unbiased predictor of the future spot exchange rate. This result is not surprising in the light of this discussion, as in this case a long position in the foreign currency has no systematic risk.[10]

Defining Exchange Rate Risk

The model derived in Section 2 is useful to characterize exchange rate risk in terms of its impact on the expected return of risky securities. One way to define exchange rate risk is by the standard deviation of the exchange rate. This definition is not very useful when investors can hold a portfolio of risky assets, as it is possible that holding an asset whose return is perfectly correlated with the exchange rate does not add risk to a portfolio. It is, therefore, more useful to use the concept of systematic exchange rate risk, that is, that part of exchange rate risk which gives rise to a risk premium, and the concept of diversifiable exchange rate risk, that is, that part of exchange rate risk that does not add to the risk of a well-diversified portfolio. It is important to notice that exchange rate risk is not necessarily a risk intrinsic to the exchange rate. In fact, one would not expect it to be. Under a regime of fixed exchange rates, exchange rates would not be risky, but this does not imply that, somehow, there would be less risk in the world. Furthermore, in the model developed in Section 2, the exchange rate risk is a function of each country's purchasing power risks only. Some authors, therefore, would argue that in this simplified model, with more than one currency but no departures from purchasing power parity, exchange rate risk does not exist, since in the absence of purchasing power risks there would be no exchange rate uncertainty.[11]

Some authors associate exchange rate risk with unanticipated deviations from purchasing power parity, that is, with unanticipated deviations of P/eP^* from one, where P and P^* are price indices.[12] The rather subtle argument made by those who favor this definition of exchange rate risk is that an asset whose return is correlated with deviations from purchasing power parity may have an expected rate of return that differs from the expected rate of return predicted by the model of Section 2. The next section shows that an asset whose return is correlated with deviations from purchasing power parity may earn a risk premium in addition to the risk premium explained by the covariance of the real return of the asset with the real return of the market portfolio. This additional risk premium is not explained, however, by the fact that investors bear the risk of deviations from purchasing power parity that, somehow, are an exogenous factor in the economy. Instead, this additional risk premium is explained by the fact that deviations from purchasing power parity are correlated with changes in other variables that affect the investors' expected lifetime utility.

SECTION 4: DIFFERENCES IN CONSUMPTION ACROSS COUNTRIES

Section 2 assumed that all investors are alike except for their risk tolerance. The fact that investors use different numeraires was of no importance once it was recognized that investors cared only about real consumption. This section assumes that investors consume different baskets of goods across countries. As a result, the model of this section differs from the one developed in Section 2 because purchasing power risks differ across countries, rather than merely across currencies. This crucial difference between Section 2 and this section implies that in general an investor's portfolio depends on his country of residence and that the expected real return on a risky asset is a function of other characteristics of the real return of this asset than its covariance with the real return on the market portfolio only.

Differences in Consumption Baskets and Asset Holdings

At each point in time, investors in different countries are likely to consume different consumption baskets.[13] These differences in consumption baskets arise for essentially 2 reasons. First, investors do not have the same tastes across countries. Second, relative prices of commodities differ across countries because of tariffs, transportation costs, taxes, and so on. Furthermore, changes in relative prices differ across countries over time. A commodity can become relatively more expensive in one country while it becomes relatively less expensive in another country.

Assume that there exists a price index P which can be used to deflate the nominal expenditures of a domestic investor to obtain his real expenditures.[14] In other words, P is equivalent to the investor's CPI, and if there is only one commodity, as in Section 2, P is equal to the price of that commodity. If P^* is the price index for a foreign investor, there is no reason for purchasing power parity to hold when consumption baskets differ across countries. Consequently, P need not be equal to eP^*. For simplicity, it is assumed that all investors in a country use the same price index.

If P is not equal to eP^*, and all the other assumptions of Section 2 hold, it is still true that the investors care only about the distribution of real returns. (In particular, the optimization problem of investors is still the same as in Section 2.) Now, however, the real return of a security is not the same for individuals in different countries because the price index used to compute real returns differs among investors. All investors of a given country choose their portfolio of risky assets among the same set of efficient portfolios as, by assumption, all investors of a given country consume the same basket of commodities. In contrast, investors in different countries choose their portfolio among different sets of efficient portfolios.

Consider now the portfolio choice problem of an individual investor. If this investor is highly risk-averse, he is willing to sacrifice substantial return to reduce the variance of his real wealth. In the absence of purchasing power risks, this investor would invest a substantial proportion of his wealth in a bond that promises a safe nominal return as it would yield a safe consumption stream. In the presence of purchasing power risks, however, such a bond is not a safe asset in real terms. To reduce the purchasing power risks of his investment in the nominal bond, the investor hedges all or part of his holdings of the nominal bond against purchasing power risks. A perfect hedge portfolio, that is, a portfolio that can be used to eliminate all purchasing power risks, is a portfolio whose real return is perfectly correlated with changes in the purchasing power of a currency.

Assume, for simplicity, that such a perfect hedge portfolio exists and call it a domestic purchasing power risks hedge portfolio, as it can be used to eliminate the purchasing power risks of the domestic riskless nominal asset as well as of risky assets whose real return is negatively correlated with inflation. In the absence of purchasing power risks, the domestic investor invests his wealth in the domestic risk-free bond and in the tangency portfolio of risky assets, that is, the portfolio that, for the domestic investor, is at the point of tangency of the capital market line and the mean-variance frontier. With purchasing power risks, the investor also invests in the purchasing power risks hedge portfolio.[15] By investing in the hedge portfolio, the investor can further decrease the variance of the real return of his wealth for a fixed expected real return.

To understand the costs and benefits of investing in the hedge portfolio for the domestic investor, it is useful to focus first on the effect of such an investment on the variance of the return of his real wealth. If, prior to investing in the hedge portfolio, the real return of the investor's wealth is negatively correlated with the rate of inflation, an investment in the hedge portfolio decreases the variance of the investor's end-of-period real wealth. This follows from the fact that the real payoff of the hedge portfolio is, in this case, negatively correlated with the real payoff of the other investments of the investor, and, therefore, an unexpectedly high real payoff of the hedge portfolio is expected to offset an unexpectedly low real payoff of the other investments of the investor. Consequently, if the investor's real wealth is exposed to unanticipated inflation before the increase in the investor's holdings of the hedge portfolio, it is less exposed to unanticipated inflation afterwards. As the nominally risk-free bond, on theoretical grounds, and the tangency portfolio, on empirical grounds, are exposed to unanticipated inflation, in the sense that their real return is negatively correlated with the rate of inflation, one expects the change in the investor's portfolio to decrease the variance of the real return of that portfolio.

The effect of an increase in the investor's holdings of the hedge portfolio on the expected real rate of return of his wealth is more difficult to understand. For expositional purposes, it is useful to start by looking at the special case in which the expected nominal return of the domestic purchasing power risks hedge portfolio is equal to the nominal return of the domestic safe bond. Without loss of generality, one can choose the expected nominal payoff of both bonds to be one dollar. In this case, the real payoff of the safe asset must be $1/E(\tilde{P}_1)$, as the nominal payoff is then equal to $\tilde{P}_1/E(\tilde{P}_1)$ and its expected value is equal to $E(\tilde{P}_1/E(\tilde{P}_1)) = 1$. The expected real payoff of the nominal bond is, however, equal to $E(1/\tilde{P}_1)$. By Jensen's Inequality, $E(1/\tilde{P}_1) > 1/E(\tilde{P}_1)$ and, consequently, the expected real payoff of the nominal bond exceeds the real payoff of the safe real asset.

The previous analysis shows that if it is costless to hedge a nominal bond against purchasing power risks in nominal terms, the expected real return of a portfolio hedged against purchasing power risks is lower than the expected real return of the same portfolio not hedged against purchasing power risks. It turns out that, in this setting, if the risk tolerance of the investor is large enough, he is even willing to go short in the hedge portfolio.[16] In a more general setting, the expected nominal return of the hedge portfolio in general differs from the nominal return on the safe nominal bond. As the expected real return of the hedge portfolio increases, ceteris paribus, one would in general expect investors to be more likely to have a long position in that portfolio. Furthermore, one would, in general, expect the value of the position in the hedge portfolio to increase as the expected real return of that portfolio increases.

Expected Returns with Differences in Consumption Baskets across Countries

A large body of empirical research has shown that changes in P are rarely equal to changes in eP^*.[17] In other words, the real exchange rate, eP^*/P, changes over time. Furthermore, the variation in eP^*/P is often large compared to the variation in P or P^*. Although this does not mean that there exist arbitrage opportunities on commodity markets, it does imply that asset holdings may differ substantially across countries because investors in different countries face substantially different purchasing power risks and hence hold different hedge portfolios. For hedge portfolios to matter in the pricing of assets, however, it is required that, across countries, investors take positions in such portfolios. As shown in the earlier part of this section, this is not necessarily the case. Furthermore, for the investors' holdings of hedge portfolios to matter, the positions taken in these portfolios must not offset each other across countries.

To see how holdings of hedge portfolios affect expected returns, suppose that asset i has a nominal return that is positively correlated with changes in P. If the risk tolerance of domestic investors is low, they want to hedge against purchasing power risks. Suppose, however, that asset i is priced in line with the Sharpe-Lintner model. In this case, domestic investors want to take a long position in asset i in excess of their holdings of asset i in the tangency portfolio, as they want to hedge against purchasing power risks. For a given supply of asset i, the usefulness of asset i in hedging against domestic purchasing power risks decreases its expected return. Whether asset i has a higher expected return than predicted by the Sharpe-Lintner model, however, depends also on whether asset i is useful to hedge purchasing power risks for investors in other countries. If investors in some other country have to go short in asset i to hedge against unanticipated changes in their price index, it may turn out that the expected return of asset i is higher than predicted by the Sharpe-Lintner model.

With this analysis, the expected return of a security depends on how useful it is for investors to hedge against purchasing power risks and on how much they are willing to pay to do it. As the purchasing power risks differ across countries, it would seem that one has to find how useful a security is to hedge the purchasing power risks in each country and how much investors are willing to pay to hedge purchasing power risks in each country. Whereas some authors have constructed models along these lines,[18] it is doubtful that such models are very useful as they make the expected return of a security depend on the sum of a large number of risk premia whose value is difficult to compute in a reliable way.

It turns out, however, that it is possible to obtain a simple model to describe expected returns.[19] Notice first that the model of Section 2 still holds, except that the market portfolio in the equation for expected returns is replaced by the portfolio held by a domestic investor and that real returns are computed by using the price index of domestic investors. To understand this, notice that domestic investors still care only about the distribution of real returns and still choose to hold an efficient portfolio of risky assets. There is no reason for them to hold the market portfolio, however, since they face purchasing power risks that differ from those faced by other investors and hence may hold a hedge portfolio in addition to their holdings of the tangency portfolio and of the nominal risk-free asset. The portfolio of risky assets held by a representative domestic investor is not directly observable, which makes the model of Section 2 not very useful in this context. As in Section 2, however, the fact that at the optimum a change in the investor's asset holdings cannot increase his expected utility can be used to derive a useful result on the pricing of risky assets when consumption baskets differ across countries.

Assume that the representative domestic investor's utility at date 1 is $U(c_1)$, where c_1 is his real consumption at date 1. The investor is assumed to be risk-averse and c_1 is a random variable as of date 0. Suppose that the investor is holding an optimal portfolio. If this investor, at the optimum, decreases his holdings of security i and increases his holdings of security j marginally, his expected lifetime utility should be unchanged. Clearly, the change in the investor's portfolio has no effect on his consumption at date 0, as the purchase of security j is financed by the sale of security i. Consequently, the change in the investor's portfolio should not affect his expected utility for date 1, as otherwise his current portfolio would not be optimal. This implies that

$$E(\tilde{U}_{c_1}(\tilde{r}_i - \tilde{r}_j)) = 0, \qquad (10)$$

where U_{c_1} is the first derivative of the investor's utility of real consumption at date 1. Using the definition of a covariance, (10) can be rewritten as:

$$-\text{Cov}(\tilde{U}_{c_1}, \tilde{r}_i) + \text{Cov}(\tilde{U}_{c_1}, \tilde{r}_j) = E(\tilde{U}_{c_1})E(\tilde{r}_i) - E(\tilde{U}_{c_1})E(\tilde{r}_j). \tag{11}$$

Let r_{zu} be the return on a portfolio whose return is uncorrelated with changes in marginal utility. One can choose asset j to be such that $r_{zu} = r_j$ and obtain from equation (11) that

$$-\frac{\text{Cov}(\tilde{U}_{c_1}, \tilde{r}_i)}{E(\tilde{U}_{c_1})} = E(\tilde{r}_i) - E(\tilde{r}_{zu}). \tag{12}$$

It follows from equation (12) that the expected return on security i is a decreasing function of the covariance of the security's return with the marginal utility of consumption. Furthermore, this function is linear with a slope equal to $-(1/E(\tilde{U}_{c_1}))$ and an intercept equal to $E(\tilde{r}_{zu})$. To understand the relation between a security's expected return and its covariance with the marginal utility of real consumption, consider an investor who can choose between receiving a dollar if his marginal utility is high or if his marginal utility is low at date 1. The investor will choose to receive the dollar when his marginal utility is high, as this is when he values a dollar the most. By the same token, the individual prefers an asset that pays off when his marginal utility is high and hence requires a smaller risk premium to hold such an asset.

It is useful to notice that marginal utility is negatively correlated with consumption. This implies that an asset whose return is negatively correlated with c_1 has a lower expected return than an asset whose return is positively correlated with c_1. The relation between consumption and marginal utility is not linear, and, consequently, the relation between the expected returns of risky assets and the covariance of their returns with consumption is not linear in general. Under certain assumptions, however, equation (12) can be rewritten so that expected returns are a function of the covariance of asset returns with changes in consumption rather than with changes in marginal utility. The most innocuous of these assumptions is that trading takes place continuously, so that one can take the limit in continuous time of the expression for expected returns. Alternatively, one can assume that the joint distribution of marginal utility and asset returns is multivariate normal.[20] Notice finally that if the utility function is quadratic (like $U = c_1 - \frac{1}{2}bc_1$), then the marginal utility is linear in consumption ($U_{c_1} = a - bc_1$) and the expected return of an asset is a linear function of the covariance of the real return of that asset with consumption.

When one rewrites expected returns as a function of $\text{Cov}(\tilde{c}_1, \tilde{r}_i)$, one obtains a relation in terms of observables that makes it possible to compute expected real returns for all assets on markets that are perfect for domestic investors. Notice that a first-order Taylor-series expansion of marginal utility around $E(\tilde{U}_{c_1})$ makes it possible to write $\text{Cov}(\tilde{U}_{c_1}, \tilde{r}_i)$ as $E(\tilde{U}_{c_1c_1})\,\text{Cov}(\tilde{c}_1, \tilde{r}_i)$ plus a remainder term that, in the limit of continuous time, is negligible. Let w_i^p, $i=1, \ldots, n$, be known portfolio weights. One can use equation (12), after replacing $\text{Cov}(\tilde{U}_{c_1}, \tilde{r}_i)$ with $E(\tilde{U}_{c_1c_1})\,\text{Cov}(\tilde{c}_1, \tilde{r}_i)$, to compute the real expected return $E(\tilde{r}_p)$ on the portfolio with weights w_i^p, $i=1, \ldots, n$:

$$E(\tilde{r}_p) - E(\tilde{r}_{zc}) = \left[\frac{-E(\tilde{U}_{c_1c_1})}{E(\tilde{U}_{c_1})}\right] \text{Cov}(\tilde{c}_1, \tilde{r}_p), \tag{13}$$

where $E(\tilde{r}_{zc})$ is the expected real return on a portfolio whose real return is uncorrelated with real consumption. Using equation (12) and (13), it follows that

$$E(\tilde{r}_i) - E(\tilde{r}_{zc}) = \beta_{ic}[E(\tilde{r}_p) - E(\tilde{r}_{zc})], \tag{14}$$

where

$$\beta_{ic} = \frac{\mathrm{Cov}(\tilde{r}_i, \tilde{c}_1)}{\mathrm{Cov}(\tilde{r}_p, \tilde{c}_1)}. \tag{15}$$

Equation (14) implies that there is a linear relation between expected real returns and consumption betas. As c_1 is the real consumption of a representative domestic investor, one can use domestic real consumption per capita in equations (14) and (15). Notice that equation (14) can be used to compute the risk premium incorporated in the forward exchange rate in the same way as in Section 3.

A More General Interpretation

Equations (14) and (15), which make it possible to compute real expected returns for all assets traded on perfect markets for domestic investors, hold under assumptions that are much less restrictive than those made so far in this section. Indeed, equations (14) and (15) hold in a multiperiod world in which the distribution of the returns changes over time and in which the distribution of the rates of change of goods prices changes over time. Because the model of asset pricing developed in this section holds under very general assumptions about exchange rate dynamics, it is particularly well-suited to price assets in an international setting. In particular, this model holds even when the risk premium incorporated in the forward exchange rate changes over time.

To understand why the model developed in this section holds even when investment and consumption opportunity sets change over time, it is useful to notice that to obtain the equation for expected returns (that is, equation (14)), use is made of the fact that at the optimum the investor cannot be made better off by rearranging his portfolio. Thus the following relation is obtained:

$$E(\tilde{U}_c(\tilde{r}_i - \tilde{r}_j)) = 0. \tag{16}$$

This relation still holds in a multiperiod model in which investment and consumption opportunity sets change over time. To understand this, suppose that in a multiperiod world the investor has chosen an optimal investment policy that prescribes how he will invest his wealth over time. Assume that the investor has a von Neuman-Morgenstern time-additive expected utility function of lifetime real consumption. Consider now a hypothetical experiment in which the investor deviates from his optimal policy at date 0 by increasing his investment in asset i and decreasing his investment in asset j marginally for one period, that is, at date 1 the investor goes back to his optimal policy. This change in the investor's portfolio does not affect his consumption at date 0. If the change in his investment policy makes it possible for the investor to increase his expected utility for date 1 and maintain the same expected utility for all other dates, the investor has not chosen an optimal investment policy. This implies that relation (11) must hold. It has not been assumed, however, that consumption and investment opportunity sets are constant through time.

Notice that if the investment opportunity set changes over time, investors in general want to hedge against unanticipated changes in the investment opportunity set, as these changes affect the investors' lifetime expected utility. Consequently, investors take positions in a variety of hedge portfolios in addition to taking a position in a purchasing power risks hedge portfolio. In this setting, when one wishes to compute the proper discount rate for a risky project, one can still compute the risk premium by looking at the covariance of the real cash flows of the project with changes in real consumption. As the investment opportunity set changes over time, however, the covariance of changes of the real cash flows with changes in real consumption is a function of the state of the world. Although the consumption-based capital asset pricing model offers a simple and intuitive explanation for the cross-sectional variation of the expected returns of risky assets, its empirical implementation is made difficult by the fact that consumption rates are endogenous variables that are solved for by investors when they maximize their lifetime expected utility.[21]

SECTION 5: THE IMPLICATIONS OF BARRIERS TO INTERNATIONAL INVESTMENT

The analysis of Section 4 makes it possible to understand why investors in a given country do not hold the world market portfolio and why there are some similarities in asset holdings among investors of a given country. It would not be reasonable to believe that the results of Section 4 alone can explain the asset holdings of investors. There is a wide variety of barriers to international investment that play a significant role in how investors allocate their wealth among risky assets. Barriers to international investment can take the form, for instance, of exchange rate restrictions, explicit limits on capital flows, differential taxes, holding costs, risks of expropriation, and so on. Because barriers to international investment are so numerous and diverse, only some general types of barriers to international investment can be discussed. First, the model developed in Section 4 is shown to hold in the presence of barriers to international investment. It can be used to compute the expected real return of risky assets that are traded in perfect markets (that is, not subject to taxes, transactions costs, holding costs and political risks) for investors of at least one country. Barriers to international investment that take the form of differential taxes and transactions costs are then explored. Finally, after brief discussion of political risks, analysis turns to how barriers to international investment affect the financing policies of firms.

Consumption Betas and Barriers to International Investment

Section 4 showed how the equilibrium expected real rate of return on a risky asset can be computed for all assets that domestic investors can trade in perfect markets. Consequently, one can apply the model of Section 4 even when foreign or domestic investors cannot trade some assets in perfect markets. If foreign investors face restrictions in holding domestic assets but domestic investors face no restrictions on holding any assets, all assets are traded on perfect markets for domestic investors. Consequently, domestic consumption betas can be used to compute the real expected returns of all assets. If domestic investors face restrictions in holding some foreign securities but foreign investors face no restrictions on holding these assets, one can compute the expected real returns on these assets from the perspective of foreign investors, as foreign investors trade these assets on perfect markets, and use foreign consumption betas.

Taxes, Transactions Costs, and Barriers to International Investment

The model of Section 4 can be applied under a wide variety of circumstances when barriers to international investment exist. Now it will be shown how the model can be used to compute the expected return abroad of foreign assets which are subject to barriers to international investment for domestic investors.[22] To make this comparison, it is assumed for simplicity that there exists a safe real asset in the domestic country and that the reference portfolio p is chosen to be a portfolio of domestic assets only.

Consider a situation where the barriers to international investment take a form that is equivalent to a tax. This way of characterizing barriers to international investment is fairly general, as the tax rate can be a surrogate for the certaintyequivalent cost of expropriation, cross-border tax adjustments, holding costs, transactions costs, or for the shadow price of quantity constraints on foreign investment (like an upper limit imposed by the host country on the fraction of a firm that can be owned by foreign investors). For simplicity, it is assumed that the tax rate is the same for all foreign assets held long by domestic investors and that it applies to the end-of-period price of a share. To model barriers to international investment, it is more reasonable to apply the tax to the end-of-period value of an investor's position abroad than to tax positive returns on that position only, because many costs associated with investing abroad must be paid even if returns are negative. If the tax rate on a long position is equal to τ, the after-tax real return for domestic investors on some foreign asset held long, say security i, is equal to

$$r_i^A = \left(\frac{e_1 P_{i1}}{e_o P_{io}^*}\right)\left(\frac{P_o}{P_1}\right)(1-\tau) - 1 = \frac{(1+d)(1+R_i)(1-\tau)}{(1+I)}, \tag{17}$$

where d is the rate of change of the exchange rate and I is the rate of inflation in the domestic country. If foreign assets held long by domestic investors are subject to a tax, the model of Section 4 still applies, but now it applies to after-tax returns:

$$E(\tilde{r}_i^A) - r = \beta_{ic}[E(\tilde{r}_p) - r], \tag{18}$$

where

$$\beta_{ic}^A = \frac{\mathrm{Cov}(\tilde{r}_i^A, \tilde{c}_1)}{\mathrm{Cov}(\tilde{r}_p, \tilde{c}_1)}. \tag{19}$$

It follows that assets that have the same before-tax beta can have different after-tax returns. As foreign assets held long have higher after-tax foreign currency expected returns for foreign investors than for domestic investors, one would expect that, in general, domestic investors have a smaller long position in foreign assets than foreign investors.

Domestic investors may hold some foreign assets short. Consequently, the effects of a tax on short holdings of foreign assets should be investigated. Let γ be the tax rate on short holdings of foreign assets by domestic investors. If γ is positive, short sales benefit from a subsidy. For instance, in a situation in which only net holdings of foreign assets are subject to a tax, $\tau = \gamma$, and the Black [1972] model holds. As after-tax expected real returns on foreign assets held short by domestic investors are still given by the model of Section 4, it follows that

$$E(\tilde{r}_i) - \gamma - r = \beta_{ic}[E(\tilde{r}_p) - r]. \tag{20}$$

If $\gamma \neq \tau$, some foreign assets may not be held at all by domestic investors. In other words, some assets may not be traded internationally. To understand this, suppose that $\gamma < 0$. Consider 2 risky assets, one domestic and the other foreign, whose returns in domestic currency are perfectly correlated. If foreign investors face no barriers to international investment, they can arbitrage between 2 assets so that their expected pretax returns in domestic currency are the same. In such a situation, domestic investors do not hold the foreign asset at all, as the domestic asset is a perfect substitute for the foreign asset and is not subject to a tax. If foreign asset j is not held by domestic investors, its expected real return is given by

$$E(\tilde{r}_j) - h - r = \beta_{jc} [E(\tilde{r}_p) - r], \tag{21}$$

where $\tau > h > \gamma$. Not surprisingly, in this setting, a domestic investor takes a position in a foreign asset only if the benefits from doing so are large enough to offset the tax he has to pay when he holds that asset. Not all foreign assets have expected benefits large enough to insure that domestic investors take a position in these assets.

Political Risks

One barrier to foreign investment often mentioned is the possibility that investments made abroad will be expropriated by the government of the country in which these investments are made.[23] The model developed in this paper makes it possible to compute the risk premium that investors of one country, say country A, require to bear political risks when they invest in another country, say country B. Note that expropriation can be complete or partial. Governments often impose regulations that discriminate against foreign investors. The imposition of such a regulation in country B will decrease the value of investments made in that country by investors in country A and is called here a partial expropriation. Let x, $0 \leq x \leq 1$, be the fraction of the holdings of assets of country B by investors of country A that is expropriated at the end of the period. At the beginning of the period, x is a random variable and is written $\tilde{x}$. If $\tilde{x}$ is independent of the level of real consumption, the expected return on an asset subject to the risk of expropriation is the same as the expected return on an equivalent asset that is not subject to this risk. This implies, however, that the expected real return on an asset of country B is $E[(\tilde{r}_j + \tilde{x})/(1-\tilde{x})]$ where r_j is the real return on an asset of country A with the same consumption beta as the asset of country B. It follows that the expected real return on an asset of country B exceeds the expected real return on an asset of country A if one does not take into account political risks. This discussion is relevant, however, only if investors of country A hold assets of country B, as otherwise a consumption beta from country A cannot be used to compute the expected real return of an asset in country B.

It is possible for x to be correlated with changes in real consumption in country A. To the extent that investors can share risks internationally, it is likely that real consumption is positively correlated across countries. If country B is a net borrower from country A and is more likely to declare unilaterally a moratorium on its interest payments to investors of country A, x is negatively correlated with changes in real consumption in country A, assuming that changes in real consumption are positively correlated across countries. As the real return for investors of country A on an investment in country B is negatively correlated with x, the lower the covariance of x and changes in the real consumption of investors of country A, the higher the expected rate of return required by investors of country A on investments in country B. This discussion shows that if the probability of expropriation increases when the rate of real consumption unexpectedly decreases, the investors of country A require a (positive) risk premium to bear the risk of expropriation when they invest in country B.

Barriers to International Investment and Financial Policies of Firms

In the earlier sections of this paper, the expected return on an asset in a given numeraire is the same for all investors, irrespective of where they are located. In such a situation, all the results of the theory of finance about capital budgeting and financial policies hold. That a firm's shareholders are located in different countries is of no relevance to the firm's managers. The usefulness of the results developed in the earlier sections of this paper for corporate policies is that they make it possible to assess more carefully the risk of the projects whose present value has to be computed. In particular, they make clear how to compute the present value of a project undertaken in country B by a firm of country A when the capital markets of countries A and B are fully integrated. The capital markets of 2 countries are fully integrated if investors in each country face no barriers to international investment when they invest in the other country.

When there are barriers to international investment, it matters where an asset is traded. In this case, investors in different countries are willing to pay different prices for the rights to the same stream of cash flows, which has wide-ranging implications for the financial and investment policies of firms. However, these implications can be discussed only briefly here.

In the presence of barriers to international investment, the same project may not have the same value for different firms. Such a situation may arise for a variety of reasons. For instance, investors in country B may not have access to capital markets from other countries and investors from other countries may not have access to the capital market of country B. In such a situation, investors from country B, in general, use a different discount rate to evaluate projects than investors from other countries. In terms of the model of Section 2, risks that are diversifiable for most investors in the world may not be diversifiable for investors of country B. Also, different firms may value the same project differently because these firms are exposed to different types of political risks or different tax structures. As a result, in the presence of barriers to international investment, a firm may undertake a project that some other firm would not undertake.[24]

With barriers to international investment, the domestic currency price of securities sold by a firm is likely to depend on the country in which securities are sold. In efficient markets, one would not expect to make profits by issuing securities in some countries and investing the proceeds in other countries. It is perfectly possible, however, that selling securities in one country yields more income for the firm than selling securities in another country. Taxes which will be paid by investors who hold a security may depend on where the security is issued and traded. For instance, many countries impose withholding taxes on interest income paid by domestic corporations to nonresident investors when securities are issued in the domestic country. If a corporation wants to sell securities to nonresident investors, it will generally be able to sell securities at a higher price if it issues them on offshore markets in such a way as to eliminate the requirement for nonresident investors to pay withholding taxes in the domestic country.

Firms can take advantage of barriers to international investment in numerous ways through financial transactions. For instance, mergers with firms of foreign countries can provide diversification benefits for shareholders in the domestic country which they would not be able to obtain on their own when the firms face less stringent barriers to international investment than individual investors.[25] As another example, joint ventures can enable domestic firms to make it possible for investors in foreign countries to share some risks internationally.

SECTION 6: CONCLUSION

This paper discusses some problems that arise when one tries to use capital asset pricing models in an international setting. In such a setting, barriers to international investment and dynamics of commodity prices play a crucial role in the determination of expected returns on risky assets.

Not all assets are equally good hedges against unanticipated changes in the price of a consumption good. Therefore, one would not expect the expected return of risky assets to be a function of their Sharpe-Lintner beta, the expected rate of return of the market portfolio, and the risk-free interest rate only. One would also believe that how good an asset is to hedge various risks (for instance, commodity price risks) that differ from stock market risk would affect the expected return of this asset.

In general, models that allow the expected return of a risky asset to be a function of the covariance of the return of that risky asset with variables other than the return of the market portfolio can be formulated in 2 different ways. First, one can obtain an equation for the expected return of risky assets, which makes each expected return equal to the expected return of a risk-free portfolio plus a sum of risk premia. Second, one can get an equation that yields expected returns as the sum of the expected return of a risk-free portfolio plus a positive constant times the covariance of the return of the risky assets with changes in consumption. Both approaches can be derived from the same assumptions and both approaches are difficult to implement empirically.

The consumption-based approach to asset pricing has, however, some distinct advantages. First, from a theoretical standpoint, it seems to provide a more intuitive understanding of the difference between the expected return of a risky asset and of the riskless portfolio. This difference is proportional to the covariance of the return of the risky asset with changes in consumption, rather than being a function of a large number of covariances of the return of the risky asset with changes in state variables. Second, the consumption-based approach is a general equilibrium approach, while models that impose artificial constraints on price dynamics or on the number of state variables of interest cannot claim this distinction. Third, the consumption-based approach makes it easy to take into account barriers to international investment, which are believed to play a significant role in the pricing of assets from some countries. Finally, while the consumption-based approach is more empirically tractable than other approaches—especially in the presence of barriers to international investment—there is currently no consensus on this issue among financial economists. Given the current state of research in international asset pricing, however, it is safe to expect that the most exciting discoveries will come from empirical work.

FOOTNOTES

1. See Jensen [1972] for a review of these models.

2. This paper is not meant to be a review of the literature; see Adler and Dumas [1983] for additional references to the literature. This paper does not deal with issues which are primarily of interest to international economists. This explains why there is no discussion of the role of money or extensions of asset pricing models which derive exchange rates as asset prices. See Lucas [1982], Stulz [1984], and Svensson [1983].

3. See especially Grauer, Litzenberger, and Stehle [1976], Kouri [1977], and Fama and Farber [1979]. These models are often contrasted with the model of Solnik [1974], which assumes that exchange rate changes are perfectly correlated with changes in the terms of trade. Solnik's [1974] model is a special case of the model developed in Section 4. Throughout the paper, a discrete-time framework is used. Furthermore, the model developed in this section is a mean-variance model. While the assumptions one has to make for a mean-variance model to result from the solution of the investor's portfolio choice problem [see Dybvig and Ingersoll 1982 and the references therein] are unpalatable, it turns out that the results of this section can be obtained in a continuous-time model that requires less stringent assumptions.

4. The commodity may be viewed as a basket of goods. With this interpretation, all investors must have identical and constant expenditure shares on the various goods. See Adler and Dumas [1983] for references to the law of one price and to purchasing power parity.

5. An alternative and more general derivation would aggregate asset demands across investors. Such a derivation would not require the non-trivial assumption that a representative investor exists, but it would still yield equations (3) and (4). See Constantinides [1982] and Rubinstein [1974] for conditions under which a representative investor exists.

6. To obtain (3), note that $\Sigma w_k \sigma_{ik}$ is the covariance of $\tilde{r}_i$ with $\tilde{r}_m$ if the w_k's are the weights of the market portfolio. Let $Cov(\tilde{r}_j, \tilde{r}_m) = 0$, so that j has the same expected return as the portfolio with return $\tilde{r}_z$. Finally, eliminate λ by noticing that $E(\tilde{r}_m) - E(\tilde{r}_z) = \lambda Var(\tilde{r}_m)$.

7. See Sharpe [1964] and Lintner [1965]. If one is willing to assume that $E(\tilde{I}\tilde{r}_i)$ is negligible for all securities, (6) and (7) can be rewritten as

$$E(\tilde{R}_i) - E(\tilde{R}_f) = \beta_i [E(\tilde{R}_m) - E(\tilde{R}_f)] \quad (6')$$

and

$$\beta_i = \frac{Cov(\tilde{R}_i - \tilde{I}_1, \tilde{R}_m - \tilde{I})}{Var(\tilde{R}_m - \tilde{I})}. \quad (7')$$

8. See Fama and Schwert [1977] and the references therein. See Solnik [1983] and Gultekin [1983] for international evidence.

9. See Friend, Landskroner, and Losq [1976] for a similar result.

10. For papers which obtain similar results, see the papers referenced in footnote 3 and Frankel [1979].

11. See Grauer, Litzenberger, and Stehle [1976].

12. See, for instance, Solnik [1974].

13. For models which use this assumption in one form or another, see Solnik [1974], Sercu [1980], Stulz [1981b], and Adler and Dumas [1983]. Stulz [1981b] allows explicitly for nontraded goods and changes in price levels as well as changes in relative prices.

14. A utility function which satisfies the assumption of constant expenditure shares satisfies this assumption. See Samuelson and Swamy [1974]. The analysis in Stulz [1981b] does not require this assumption. Stulz [1983] discusses some implications of utility functions which do not have constant expenditure shares.

15. See Breeden [1984] for an analysis of hedging and references to the literature.

16. See Stulz [1983] for a proof of this statement in a continuous-time model and for references to the literature.

17. See Levich [1983] for references.

18. Adler and Dumas [1983] assume that price indices differ across countries but have a constant investment opportunity set, and assume that the dynamics for price indices are constant. Hodrick [1981] assumes that purchasing power parity holds but lets the investment opportunity set change over time.

19. The following presents a discrete-time version of Stulz [1981a] assuming constant expenditure shares. See also Grossman and Shiller [1982] and, especially, Wheatley [1983].

20. For the continuous-time approach, see Breeden [1979]. For an approach which assumes that the joint distribution of price relatives and consumption is lognormal, see Rubinstein [1976].

21. For a critical view of consumption-based asset pricing models, see Cornell [1981]. For a discussion of some practical problems one encounters when using such models, see Hansen and Singleton [1982] and Wheatley [1983]. Various measures of consumption expenditures are published regularly in the U.S. It must be noted that the measure of consumption used in the model includes expenditures on nondurable goods plus the services consumed from the stock of durable goods, while published estimates of total consumption expenditures include purchases of durable goods.

22. This is especially useful in tests of whether asset markets are segmented internationally. For recent work which derives asset pricing equations when markets are segmented and then proceeds to test them, see Bodurtha [1983], Wheatley [1983], and Errunza and Losq [1983]. The present discussion builds on Stulz [1981a] and Wheatley [1983], who derives the Stulz [1981a] model in a consumption-based capital asset pricing model.

23. For a review of the forms political risk can take and of the literature on political risk, see Lessard, Bollier, Eckaus, and Kahn [1983]. See also Shapiro [1982].

24. See, for instance, Adler [1974].

25. See, for instance, Adler and Dumas [1975].

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