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INTEREST RATES AND MONETARY POLICY UNCERTAINTY

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This paper analyzes a model in which optimizing households are uncertain about monetary policy and learn about it over time. In this model, the variance of changes in nominal and *ex post* real interest rates increases with the variance of the household's predictive distribution over money growth. A positive relation between unexpected changes in nominal interest rates and the money stock results from monetary policy uncertainty. Finally, if money growth covaries negatively with real wealth, an increase in monetary policy uncertainty increases the expected real rate of interest and decreases the nominal rate of interest.

1. Introduction

This paper analyzes the effect of uncertainty about monetary policy on interest rates. A model in which optimizing households are unsure about the distribution of money growth and learn about it over time is presented. Households can be uncertain about monetary policy for a variety of reasons. However, two reasons which have implications for current policy debates come to mind. First, the monetary authority's announcements may lack credibility, so that the time series of money growth conveys information to households.[1] Second, a change in monetary regime can leave households unsure about the true distribution of money growth.[2]

In the following analysis, monetary policy uncertainty implies that the households' predictive distribution over money growth has a higher variance than the variance of money growth. Households use the predictive distribution over money growth to determine their holdings of risky assets. As the households' holdings of risky assets are best studied in models in which households maximize their expected lifetime utility of consumption, this paper

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[1] See Brunner (1984), for instance.

[2] Many economists consider that the change in the operating procedures of the Federal Reserve Board in October 1979 constitutes a change in monetary regime. See, for instance, Meltzer and Mascaro (1983) and Roley and Walsch (1983).

uses a variant of the continuous-time stochastic model of Cox, Ingersoll and Ross (1985) popular in the finance literature.[3] A government *cum* monetary authority is introduced in that model. The analysis allows for money growth to take place either through transfers to households or through open-market operations in the only produced commodity to finance government expenditures.[4] The actions of the government are assumed to be exogenous.

In this paper, the real rate of return of nominal bonds is uncertain because the price level changes randomly over time. However, random changes in the price level do not *per se* imply the existence of a positive risk premium for nominal bonds. To use a terminology common in the finance literature, such a premium arises only if the risk associated with nominal bonds is not diversifiable.[5] If the representative household has a logarithmic utility function, nominal bonds are risky and their yield incorporates a risk premium whenever money growth covaries negatively with real wealth. When money growth covaries negatively with real wealth, nominal bonds have high real payoffs when real wealth is unexpectedly high and hence when marginal utility is unexpectedly low, i.e., when households put the least value on unexpectedly high real payoffs.

When money growth takes place through transfers fully capitalized by households, investment does not depend on expected money growth. In this paper, expected investment is a constant fraction of real wealth, which is the sum of the stock of capital, real balances and the capitalized value of future transfers. As the distribution of output is exogenously given, a change in the predictive distribution over money growth leaves expected investment unaffected only if it does not change the ratio of real wealth to the stock of capital. In this setting, the change in the stock of capital is equal to output minus the consumption rate of the commodity. This means that, over sufficiently short intervals of time, changes in wealth are perfectly correlated with output. Hence, the risk premium for nominal bonds has the opposite sign as the covariance of money growth with output when future transfers are capitalized by households.

In the absence of capitalized transfers, changes in real wealth are not perfectly correlated with output because a change in expected money growth induces a change in real balances which is not offset by a change in the real value of future transfers. With monetary policy uncertainty, unexpected mo-

[3] Cox, Ingersoll and Ross (1985) provide a continuous-time stochastic growth model in which one commodity can be produced through a variety of technologies. For earlier models which incorporate money in the Cox, Ingersoll and Ross (1985) model, see Jones (1980), Gertler and Grinols (1982), Stulz (1984) and Stulz and Wasserfallen (1985).

[4] See King and Plosser (1985) for empirical evidence on the use of money growth to finance government expenditures.

[5] The capital asset pricing model of Sharpe (1969) and Lintner (1965) implies that the non-diversifiable or systematic risks explain the cross-sectional variation of expected returns of risky assets.

ney growth conveys information about the true mean growth rate of the money stock. An unexpectedly large increase in the money stock induces households to revise upwards their expectation of money growth and, because of the associated increase in the nominal rate of interest, to decrease their holdings of real balances. As, in this case, real wealth is equal to the stock of capital plus real balances, unexpectedly high money growth is associated with unexpectedly low real wealth. Hence, in general, money growth covaries negatively with real wealth and nominal bonds have a positive risk premium.

The issue of whether the present value of future transfers belongs in the marketable wealth of households or not plays a crucial role in the pricing of nominal bonds. This is because in this paper unexpected money growth conveys information about future money growth and hence affects the stock of real balances. In a model with infinitely-lived agents, perfect markets and no agency costs, one would expect households to capitalize future transfers, as otherwise, with heterogeneous households, welfare could be increased by the opening of a market for claims on future transfers. However, it may well be that a model in which households do not capitalize future transfers better predicts the effects of changes in the degree of monetary policy uncertainty.[6] Contracting costs alone make one reluctant to dismiss such a model.

When the risk premium for nominal assets is positive, it increases with the variance of the predictive distribution over money growth. Consequently, in this case, an increase in monetary policy uncertainty, which means an increase in the variance of the predictive distribution over money growth, is associated with higher *ex post* real interest rates. It turns out surprisingly that, in general equilibrium, an increase in the variance of the predictive distribution over money growth decreases the nominal rate of interest when the representative household has a logarithmic utility function. An increase in the variance of the predictive distribution over money growth increases the expected real rate of return of a nominal bond for a given nominal interest rate because the real value of a nominal bond is a convex function of the money supply. This increase in the expected real rate of return of the nominal bond exceeds the increase in the expected real rate of return of the bond due to the increase in the risk premium. To maintain equilibrium, the nominal rate of return of the bond must fall, so that the increase in the expected real rate of return of the bond is equal to the increase in the risk premium for a given real rate of return on safe real assets.

While households act rationally in the model developed in this paper, they behave differently from the way they behave in standard rational expectations models. Here, households do not know the true dynamics for the money

[6]See Gertler and Grinols (1982) for a continuous-time model similar to the one used here which does not capitalize transfers. Jones (1980) and Stulz (1984) use a similar approach but capitalize transfers. The results of the analysis in the absence of capitalized transfers seem better able to account for the empirical evidence on the behavior of interest rates since 1979 as presented in Mascaro and Meltzer (1983).

supply. This feature offers a tractable yet useful way to introduce monetary policy uncertainty. No attempt is made to explain why monetary policy uncertainty exists. This uncertainty is taken to be a fact of life rather than modeled as the product of actions which maximize the objective function of a monetary authority. Households are viewed as Bayesian dynamic programmers who learn the true dynamics of the money supply as they acquire more data about changes in the money supply. This learning effect explains why, in this model, monetary policy uncertainty implies that unexpected money growth increases the yield for bonds of all maturities and that the variance of bond yields is higher than otherwise.[7]

The plan of the paper is as follows. Sections 2 through 4 assume that all money growth takes place through transfers which are fully capitalized by households. The model is presented in section 2. Section 3 derives the price of money and the present value of future transfers. In section 4, equations for the real and nominal rates of interest are derived. Section 5 considers the implications for our results of assuming that money growth takes place through purchases of the commodity and non-capitalized transfers. Finally, section 6 offers some concluding remarks.

2. The model

For simplicity, this paper considers an economy in which there is a representative household. Households consume the services of real balances and a commodity. It is assumed that all markets are perfect, i.e., there are no transactions costs, all households have the same information set and each household acts as a price taker. Furthermore, it is assumed that trading takes place continuously. The commodity is produced by a constant stochastic returns to scale technology. The only input required to produce a random amount of the commodity is the commodity itself. k is the *per capita* stock of the commodity available to households. The instantaneous output from investing k in production is given by

$$\mathrm{d}y = \mu_y k\,\mathrm{d}t + \sigma_y k\,\mathrm{d}z_y, \tag{1}$$

where $\mathrm{d}z_y$ is the increment of a standard Wiener process.[8] In the analysis of this paper, it is assumed that the technology available to produce the commodity does not change over time.

While households choose their holdings of real balances, the dynamics of the money stock are determined by the government *cum* monetary authority.

[7] Cornell (1983) documents that, after the change in monetary policy in 1979, the correlation between unexpected changes in nominal yields and the money stock increased substantially.

[8] For references on the methodology used throughout this paper, see Cox, Ingersoll and Ross (1985).

It is assumed that there is no government debt and that the money stock per capita M follows a lognormal diffusion process such that

$$\mathrm{d}M = \mu_M M\,\mathrm{d}t + \sigma_M M\,\mathrm{d}z_M, \tag{2}$$

where μ_M and σ_M are constant and positive. Until section 5, money growth takes place only through transfers which are fully capitalized by households and identical for each household.

In this economy, changes in the money stock are random. Monetary policy is chosen to be constant to keep the households' optimization problem tractable and to allow the economy to converge to an equilibrium in which interest rates are constant. Eq. (2) describes a monetary policy such that the monetary authority pursues a target growth rate for the money stock, but is not able or willing to choose a deterministic path for the growth rate of the money stock.

To model the effect on interest rates of the fact that households are uncertain about monetary policy, it is assumed that households do not know the true expected rate of growth of the money stock. Households can compute their predictive distribution over the rate of growth of the money supply by using the information available to them. This predictive distribution can be derived explicitly for the case in which households have a diffuse prior before sampling and their only relevant information is the time series of changes in the money supply.[9] In this case, the predictive distribution is normal with mean

$$\mu_M^{\mathrm{e}}(t) = \tfrac{1}{2}\sigma_M^2 + \frac{1}{t}\log\frac{M(t)}{M(0)}, \tag{3}$$

per unit of time and variance $((t+1)/t)\sigma_M^2 = \Omega^2\sigma_M^2$ per unit of time. t is the time elapsed since the monetary policy was introduced. For households to be uncertain about the mean growth rate of the money stock, it is required that $t < \infty$. In the following, Ω is used to measure the degree of monetary policy uncertainty. When $\Omega = 1$, there is no monetary policy uncertainty and households know the true dynamics of the money stock. Finally, μ_M^{e} follows:

$$\mathrm{d}\mu_M^{\mathrm{e}} = \frac{1}{t}\frac{\mathrm{d}M}{M} - \frac{1}{t}\mu_M^{\mathrm{e}}\,\mathrm{d}t. \tag{4}$$

In this model, by construction, there is no uncertainty about the instantaneous

[9]See Williams (1977) for a derivation of the predictive distribution for a random variable which follows the same dynamics as the money stock here.

variance of the growth rate of the money supply because households sample continuously.[10]

In general, households have other information available than the time series of changes of the money stock. For instance, households have some model of how the monetary authority functions and are able to assess the usefulness of statements made by the monetary authority. Furthermore, the new monetary regime might have some characteristics which make it similar to an earlier monetary regime or to some foreign monetary regime. Finally, as time evolves, other variables besides the history of the money stock can be useful to households. If households have more information available than is assumed in this paper, the variance of their predictive distribution is smaller. However, a more realistic model would also recognize the fact that the mean growth rate of the money stock changes over time, so that the filtering problem faced by households is more complex than assumed here. When the mean growth rate of the money stock follows itself a stochastic process, the variance of the predictive distribution over money growth is larger than in this paper if households have no other relevant information than the time series of money growth.[11]

The representative household is assumed to be infinitely-lived and to maximize the following expected utility function of lifetime consumption:

$$\mathrm{E}_t \int_t^{\infty} \mathrm{e}^{-\rho\gamma}[(1-\alpha)\ln c(\gamma) + \alpha \ln m(\gamma)]\,\mathrm{d}\gamma,$$

where E_t is the expectation operator conditional on information available at time t, $c(\gamma)$ is the instantaneous rate of consumption of the commodity, and $m(\gamma)$ corresponds to the holdings of real balances.

Households can invest in N risky assets and one bond with a safe instantaneous real rate of return $r(t)$ per unit of time, called here a purchasing power bond. The risky assets are the investments in the production of the commodity, a bond with a safe instantaneous nominal rate of return $R(t)$ per unit of time, called here a safe nominal bond, an asset which represents a claim to the household's future transfers, and undefined contingent claims which are in zero net supply. The assumptions of constant returns to scale and perfect markets imply that the return of an investment in the production of the commodity is given by eq. (1).

Let $\boldsymbol{n}$ be the $1 \times N$ vector of investment proportions in risky assets, $\mathrm{d}I_i/I_i$ the instantaneous real rate of return of the ith risky asset, and w the real wealth of the representative household. By construction, w is equal to the

[10] See Williams (1977).

[11] Dothan and Feldman (1984) and Gennotte (1984) describe the filtering technique which applies if the mean growth rate of the money stock follows itself a stochastic process.

household's stock of capital k plus its holdings of real balances m plus the present value of future transfers v, as no other asset has a positive net supply. Hence, in equilibrium, the value of the household's holdings of claims to future transfers must be equal to the present value of the transfers which it will receive. The flow budget constraint of the representative household can be written as

$$\mathrm{d}w = \sum_{i=1}^{N} n_i \left(\frac{\mathrm{d}I_i}{I_i} - r\,\mathrm{d}t \right) w + rw\,\mathrm{d}t - Rm\,\mathrm{d}t - c\,\mathrm{d}t, \tag{5}$$

where transfer income, which is the sum of the current transfer plus the change in the present value of future transfers, is included in the return of the household's portfolio for simplicity, as the return of the asset which represents a claim to the household's future transfers is perfectly correlated with the household's transfer income. This procedure is equivalent to assuming that the household has sold its claim to future transfers and then holds assets which represent claims to future transfers. To solve for its investments in risky assets, a household maximizes its expected lifetime utility subject to eq. (5). The solution of the household's optimization problem is

$$\boldsymbol{n} = \boldsymbol{V}^{-1}(\boldsymbol{\mu} - r \cdot \mathbf{1}), \tag{6}$$

where $\boldsymbol{n}$ is the $1 \times N$ vector of investment proportions, $\boldsymbol{\mu}$ is the $1 \times N$ vector of expected real returns of risky assets, $\mathbf{1}$ is a $1 \times N$ vector of ones, and $\boldsymbol{V}^{-1}$ is the inverse of the $N \times N$ variance–covariance matrix of risky asset returns. The investment proportion n_1 is the fraction of the representative household's real wealth invested in safe nominal assets, i.e., cash balances and nominal bonds. To solve the representative household's optimization problem, it is necessary to aggregate its holdings of cash balances and of the nominal safe bond. Otherwise, the variance–covariance matrix cannot be inverted as two assets have perfectly correlated real returns. The expected real rate of return of the first risky asset is $R + \mu_q^{\mathrm{e}}$ per unit of time, where μ_q^{e} is the expected rate of change of the price of money q conditional on the information set of the representative household.

Eq. (6) makes it possible to derive an equation which must be satisfied by the expected real rate of return of all assets in equilibrium. To see this, pre-multiply eq. (6) by $\boldsymbol{V}$ to get

$$\boldsymbol{V}\boldsymbol{n} = \boldsymbol{\mu} - r \cdot \mathbf{1}. \tag{7}$$

The ith row of eq. (7) can be written as

$$\sigma_{i,w} = \mu_i - r, \tag{8}$$

where $\sigma_{i,w}$ is the instantaneous covariance between the real rate of return of risky asset i and the rate of growth of the household's real wealth, and μ_i is the ith element of the vector $\boldsymbol{\mu}$. Eq. (8) is closely related to the equation which yields expected returns in the capital asset pricing model of Sharpe (1964) and Lintner (1965), and it has the same interpretation. It states that the expected real return of a risky asset in excess of the real rate of interest depends only on the covariance of the real rate of return of the risky asset with the real rate of change of invested wealth. However, in this model, contrary to the capital asset pricing model, the distribution of the real rate of return of invested wealth is endogenous.

The expected real rate of return of the nominal bond must satisfy eq. (8). However, to solve for R as a function of exogenous variables, one must first solve for the dynamics of the price of money q and for the value of future transfers.

3. The value of future transfers and money

The representative household's real wealth is equal to $m+v+k$. In the following, $m+v$ is called monetary wealth and denoted by u. The model described in section 3 has only three state variables, which are the stock of capital, the expected growth rate of the money stock and time. In this model, time plays a non-trivial role because the variance of the predictive distribution over money growth falls over time. Consequently, one can write u as a function of k, μ_M^e and t, i.e., $u(k, \mu_M^e, t)$. To simplify the notation, x denotes μ_M^e in the following.

To compute the value of monetary wealth, note first that the real return to holding real balances is the sum of their pecuniary return, i.e., the change in their real value due to inflation, and of their non-pecuniary return, i.e., the value of the services rendered, which corresponds to the opportunity cost $Rm\,\mathrm{d}t$ of holding real balances:

$$M\,\mathrm{d}q + Rm\,\mathrm{d}t. \tag{9}$$

The real transfer income $\mathrm{d}T$ is equal to the real value of the transfer plus the change in the present value of future transfers:

$$q\,\mathrm{d}M + \mathrm{cov}(\mathrm{d}q, \mathrm{d}M) + \mathrm{d}v. \tag{10}$$

As mentioned in section 2, the nominal transfer is equal to $\mathrm{d}M$. As the transfer is a flow, its real value depends on its covariance with the change in the price of money.

The real return of monetary wealth is equal to the sum of expressions (9) and (10):

$$\mathrm{d}m + \mathrm{d}v + Rm\,\mathrm{d}t = \mathrm{d}u + Rm\,\mathrm{d}t, \tag{11}$$

as $\mathrm{d}m = q\,\mathrm{d}M + \mathrm{cov}(\mathrm{d}q, \mathrm{d}M) + M\,\mathrm{d}q$. Eq. (8) must be satisfied by the expected real rate of return of any risky asset. Consequently, the real rate of return of monetary wealth must satisfy

$$\mu_u u + Rm - ru = \sigma_{u,k} u + \sigma_u^2 u^2, \tag{12}$$

where $\sigma_{u,k}$ is the covariance between the rates of change of u and k. $\mathrm{d}k$ is equal to $\mathrm{d}y - c\,\mathrm{d}t$ and hence is perfectly correlated with $\mathrm{d}y$, so that $\sigma_y = \sigma_k$. Furthermore, μ_u, $\sigma_{u,k}$ and σ_u^2 are obtained by applying Ito's Lemma to the function $u(k, x, t)$:

$$\mu_u u = u_k k \mu_k + u_x \mu_x x + \tfrac{1}{2}\left(u_{kk} k^2 \sigma_k^2 + u_{xx} x^2 \sigma_x^2 + 2u_{xk}\sigma_{x,k} xk\right) + u_t, \tag{12a}$$

$$\sigma_{u,k} u = u_k \sigma_k^2 k + u_x \sigma_{x,k} x, \tag{12b}$$

$$\sigma_u^2 u^2 = u_k^2 k^2 \sigma_k^2 + u_x^2 x^2 \sigma_x^2 + 2u_k u_x x k \sigma_{x,k}. \tag{12c}$$

Inspection of eq. (12), using eqs. (12a)–(12c), shows that all terms are functions of u and its partial derivatives except for Rm. The inhomogeneous term Rm has the interpretation of a dividend.[12] It corresponds to the expenditures on the services of real balances which are a fraction $\alpha/(1-\alpha)$ of the household's purchases of the commodity for consumption purposes, c. From the national accounts identity for the economy, c must be equal to the total *per capita* dividend paid by firms. The present value of the dividend paid by firms is equal to the capital stock k, while the present value of the dividend paid by monetary wealth must equal the value of monetary wealth u. As $rm = (\alpha/(1-\alpha))c$, the value of monetary wealth is therefore proportional to the capital stock:

$$u = \frac{\alpha}{1-\alpha} k. \tag{13}$$

Eq. (13) implies that in this economy, monetary wealth is a constant fraction of the capital stock. Total wealth is therefore $u + k = [1/(1-\alpha)]k$. To simplify the notation, let $1/(1-\alpha) = \zeta$. Hence, real balances are given by

$$m = \frac{1}{R}\alpha\rho\zeta k. \tag{14}$$

As real balances are equal to qM, eq. (14) can be used to derive the dynamics for the price of money. Substituting qM in eq. (14), dividing by M

[12] See Jones (1980) for this interpretation.

on both sides and applying Ito's Lemma yields

$$\frac{\mathrm{d}q}{q} = -\frac{\mathrm{d}R}{R} - \frac{\mathrm{d}M}{M} + \frac{\mathrm{d}k}{k} + \sigma_R^2\,\mathrm{d}t + \Omega^2\sigma_M^2\,\mathrm{d}t + \Omega\sigma_{M,R}\,\mathrm{d}t$$
$$- \sigma_{k,R}\,\mathrm{d}t - \Omega\sigma_{k,M}\,\mathrm{d}t. \tag{15}$$

Note that $\mathrm{d}k$ is the change in the capital stock net of the dividends paid by firms. Because an unexpected increase in the money stock leads households to believe that the mean growth rate of the money stock is higher than previously thought, it brings about a decrease in the expected growth rate of the price of money. Hence, in this model, the expected rate of inflation depends on *past* changes in the money stock.

4. Interest rates with capitalized transfers

In equilibrium, all the stock of capital must be invested in production. Using the result that real wealth is equal to ζk and applying eq. (8) to investments in production yields

$$r = \mu_y - \sigma_y^2. \tag{16}$$

As changes in the expected growth rate of the money stock do not affect real wealth, they leave the real rate of interest unchanged. Note, however, that the real rate of interest r is not the expected real rate of return of nominal bonds but the real rate of return on default-free indexed bonds. As shown in Cox, Ingersoll and Ross (1985), the real rate of interest also satisfies eq. (16) in an economy without money which is otherwise identical to the economy studied in this paper.[13]

Applying eq. (8) to nominal bonds and using eq. (16) yields

$$R = r - \mu_q + \zeta\sigma_{q,k}. \tag{17}$$

Using the expression for the rate of change of q given in eq. (15) results in the following equation for the nominal rate of interest:

$$R = r + \mu_M^e + \mu_R^e - \mu_k - \sigma_R^2 - \Omega^2\sigma_M^2$$
$$- \Omega\sigma_{M,k}(\zeta - 1) - \sigma_{k,R}(\zeta - 1) - \Omega\sigma_{M,R} + \zeta\sigma_k^2. \tag{18}$$

At this point, the first result of this paper about the nominal rate of interest

[13] See also Breeden (1986).

can be stated:

Proposition 1. With fully capitalized transfers, uncertainty about the mean growth rate of the money stock leads to:

(*a*) *a higher variance of the nominal rate of interest*;

(*b*) *a lower nominal rate of interest provided that the covariance of money growth with output is not too low.*

To verify part (a) of this result, note that without uncertainty about the mean growth rate of the money stock, the expected rate of change of the price of money is constant, $\Omega = 1$, and hence the nominal rate of interest is constant. With uncertainty about the mean growth rate of the money stock, an unanticipated increase in the money stock leads households to revise upwards their expectation of inflation. The equation for the nominal rate of interest implies that expected inflation variability induces variability in the nominal rate of interest. When output is uncorrelated with money growth, all terms which depend on Ω or uncertainty in R are negative in eq. (18), which motivates part (b) of the result. Notice, however, that if $\Omega\sigma_{M,k}(\zeta - 1)$ is large and negative, it could be possible for that term to overwhelm $\Omega^2\sigma_M^2$ (remember that $\zeta - 1$ is positive). Hence, for some values of Ω, the interest rate R increases with Ω, but this is not possible when Ω is very large and when σ_M exceeds σ_k. If households are more risk-averse than assumed here, the risk premium term would be multiplied by the households' coefficient of relative risk aversion, which implies that it is then more likely that the risk premium term overwhelms $\Omega^2\sigma_M^2$, when $\sigma_{M,k}$ is negative. Part (b) of Proposition 1 corresponds to a well-known result, which is that the real end-of-period value of a nominal safe asset is a convex function of the end-of-period price level. Hence, for a lognormal diffusion process for the price of money the higher the variance of inflation, the higher the expected real rate of return on a nominal bond for a given yield to maturity.[14] As the required real rate of return on a bond is a function of its non-diversifiable risk, i.e.,

$$\zeta\sigma_{q,k} = \zeta\left(\sigma_k^2 - \Omega\sigma_{k,M} - \sigma_{k,R}\right),$$

it does not depend directly on the variance of the predictive distribution of money growth. In particular, if money growth is uncorrelated with output, an increase in the variance of the predictive distribution over money growth leaves the non-diversifiable risk of nominal bonds unaffected. In this case, the nominal rate of interest must fall when the variance of the predictive distribution over money growth increases to maintain constant the expected real

[14] See, for instance, Fischer (1975) and Gertler and Grinols (1982).

rate of return of nominal bonds. If, as some empirical evidence suggests,[15] the covariance of money growth with output is negative, an increase in the variance of the predictive distribution of money growth increases the risk of nominal bonds as it increases $\zeta\sigma_{q,k}$. Hence, in this case, the expected real rate of return of nominal bonds increases with an increase in the variance of the predictive distribution over money growth, which motivates the next result:

Proposition 2. With fully capitalized transfers, an increase in the degree of uncertainty about the mean growth rate of the money stock decreases (increases) the expected real rate of return on nominal bonds if money growth covaries positively (negatively) with output.

If money growth covaries negatively with output, inflation is unexpectedly low when output is unexpectedly high. Hence, the real return of nominal bonds is high when output is high and marginal utility is low. Households would be willing to settle for a lower expected rate of return on an asset which has high payoffs when marginal utility is high. Nominal bonds are such an asset when money growth covaries positively with output. Note finally that *ex post* real interest rates on nominal bonds are more variable with monetary policy uncertainty because the households' expected money growth differs from the true mean growth rate of the money stock.

5. Monetary policy uncertainty and wealth effects

So far, money growth took place through transfers which were fully capitalized by households. It was shown that, in this case, monetary policy uncertainty has no effect on the real wealth of households. In this section, the analysis is extended to include the possibility that monetary policy uncertainty affects the real wealth of households. In the first part of this section, the effect of monetary policy uncertainty on real and nominal interest rates is studied when all changes in the money stock take place through purchases of the commodity by the government, which ties monetary policy to fiscal policy. It is therefore assumed that the government holds a stock of the commodity large enough to make this policy possible and uses this stock in a way which does not affect the marginal utility of consumption expenditures of households. If government expenditures decrease the marginal utility of consumption ex-

[15] The empirical evidence implies that $\sigma_{k,M}$ is negative if it is measured as the covariance of money growth with real stock returns [see, for instance, Cornell (1983)] and is not significantly different from zero but positive if measured as the covariance of output growth with the growth rate of high-powered money [see King and Plosser (1984)]. Bhagat and Wakeman (1984) provide indirect evidence for a negative estimate of $\sigma_{k,M}$ as they show that Treasury bill returns exhibit a positive risk premium, i.e., have some non-diversifiable risk when the capital asset pricing model is formulated in real terms.

penditures, the results are more similar to those obtained in the second part of this section. There, the case in which money growth takes place through transfers which are not capitalized by households is analyzed. This case implicitly assumes that financial markets are incomplete.

5.1. The real rate of interest and wealth effects

In the absence of transfers, the change in wealth is equal to the return of real balances plus the return of capital minus purchases of the commodity for consumption. The rate of change of real wealth is therefore

$$\frac{\mathrm{d}w}{w}=\left(\frac{m}{w}\right)\frac{\mathrm{d}q}{q}+\left(\frac{k}{w}\right)\frac{\mathrm{d}y}{k}-\frac{c}{w}\,\mathrm{d}t. \tag{19}$$

Substituting eq. (19) in the asset pricing equation of section 2 yields a formula for the real rate of interest:

$$r=\mu_y-\sigma_{q,y}\left(\frac{m}{w}\right)-\sigma_y^2\left(\frac{k}{w}\right). \tag{20}$$

This formula reduces to the formula for the real rate of interest obtained in section 4 when output is uncorrelated with money growth, as in this case $\sigma_{q,y}=\sigma_y^2$. When output is correlated with money growth, as $\sigma_{q,y}$ is a decreasing function of $\sigma_{M,y}$, an increase in the covariance of output with money growth increases the real rate of interest. This result is explained by the fact that the variance of the rate of change of real wealth is a decreasing function of the covariance of output with money growth, as a high value of $\Omega\sigma_{M,y}$ means that real balances have low payoffs when output is unexpectedly high. As the risk-free real asset becomes more attractive when the variance of real wealth increases, the demand for risk-free real bonds falls as $\Omega\sigma_{M,y}$ increases, so that the real rate of interest must increase. It immediately follows that:

Proposition 3. An increase in monetary policy uncertainty increases (decreases) the real rate of interest when money growth covaries positively (negatively) with output. Furthermore, unless $\sigma_{M,y}=0$, monetary policy uncertainty increases the variance of changes in the real rate of interest.

The real rate of interest changes over time in the presence of monetary policy uncertainty because changes in expected money growth cause changes in the distribution of the real rate of return of invested wealth. In the absence of monetary policy uncertainty, the joint distribution of inflation and output is constant, so that the distribution of the rate of change of real wealth and the real rate of interest are constant.

5.2. The nominal rate of interest and wealth effects

Using eq. (8), which yields equilibrium expected returns, and eq. (19), which defines the rate of change of real wealth, yields the following equation for the nominal rate of interest:

$$R = r - \mu_q^e + \sigma_q^2\left(\frac{m}{w}\right) + \sigma_{q,y}\left(\frac{k}{w}\right). \tag{21}$$

The sum of the last two terms on the right-hand side of eq. (21) is the risk premium incorporated in the expected real rate of return of nominal bonds. Here, an increase in the money stock is accompanied by a purchase of capital by the government so that the capital stock held by households follows:

$$\mathrm{d}k = \mu_y k\,\mathrm{d}t + \sigma_y k\,\mathrm{d}z_y - (q\,\mathrm{d}M + \mathrm{cov}(\mathrm{d}q, \mathrm{d}M)) - c\,\mathrm{d}t, \tag{22}$$

where $q\,\mathrm{d}M + \mathrm{cov}(\mathrm{d}q, \mathrm{d}M)$ is equal to the value of the transfer that households received in the model with capitalized transfers.

Substituting eq. (22) in the equation for the rate of change of the price of money of section 3 and using the equation for the real rate of interest developed earlier in this section yields an equation for the nominal rate of interest:

$$\begin{aligned} R = {} & \mu_y - \mu_q^e + \sigma_y^2 + \Omega^2\sigma_M^2\beta^2 + \sigma_R^2\beta^2 \\ & + 2\Omega\sigma_{M,R}\beta^2\left(\frac{m}{w}\right) - \Omega\sigma_{M,y}\beta - \sigma_{R,y}\beta, \end{aligned} \tag{23}$$

where

$$\beta = R/(R - \rho\alpha).^{16}$$

Note that if there is no monetary policy uncertainty, eq. (23) implies that the nominal rate of interest is constant as both μ_y and μ_q^e become constants. As discussed in section 3, μ_q^e increases with the variance of the predictive distribution over money growth for a given expected growth rate of the capital stock. Like in section 4, if households have a logarithmic utility function and $\sigma_{M,y} = 0$, then the expected growth rate of the price of money increases more

[16] In the following, it is assumed that $\beta < 2$. This assumption implies that $(m/k) < 1$.

than the risk premium when the variance of the predictive distribution over money growth increases. Consequently, Proposition 1 holds here also, which means that, in general, monetary policy uncertainty lowers the nominal rate of interest and increases its variance.

While the absence of transfers has little qualitative impact on the nominal rate of interest, it has a significant impact on the expected real rate of return of nominal bonds. Without transfers, when $\sigma_{M,y}$ is not too high, the risk premium is an increasing function of the variance of the predictive distribution of money growth. This follows from the fact that, while real balances are part of real wealth and their value falls when money growth is unexpectedly high, there is no longer an offsetting effect arising from a change in the present value of future transfers. As explained in section 4, the risk premium falls when $\sigma_{M,y}$ increases. Hence, if $\sigma_{M,y}$ is very large, it is possible for the expected real rate of return on a nominal bond to fall when the degree of monetary policy uncertainty increases. Note finally that, in section 4, the expected real rate of return on nominal bonds could change only because Ω fell over time. In this section, unexpectedly high money growth changes the ratio of real balances to real wealth and hence changes the risk premium for nominal bonds.

Proposition 4. *When changes in the money stock take place through purchases of the commodity by the government, monetary policy uncertainty*:

(*a*) *increases the expected real rate of return on nominal bonds unless the covariance between output and money growth is high*;

(*b*) *increases the variance of the expected real rate of return of nominal bonds.*

The effects of monetary policy uncertainty discussed in this section depend on the level of expected money growth. When expected money growth is large, the ratio of real balances to real wealth is small, which implies that the covariance of money growth with the return of real wealth is small in absolute value. As expected money growth falls, households hold more real balances and the absolute value of the risk premium of nominal bonds increases. The effect of an increase in the variance of the predictive distribution of money growth on the risk premium of nominal bonds is an increasing function of that premium.

5.3. *Non-capitalized transfers*

The following analysis considers the case in which money growth takes place through transfers when households cannot trade claims to future transfers, so that their tradeable wealth does not include the present value of these transfers. In this case, the representative household's tradeable wealth is equal to $m + k$, which is also the representative household's wealth in the absence of transfers. The rate of change of real wealth includes transfers, as unexpected money growth does not affect the stock of capital held by households, and is given by

$$\frac{\mathrm{d}w}{w} = \left(\frac{m}{w}\right)\frac{\mathrm{d}q}{q} + \left(\frac{m}{w}\right)\frac{\mathrm{d}M}{M} + \left(\frac{m}{w}\right)\mathrm{cov}\left(\frac{\mathrm{d}q}{q}, \frac{\mathrm{d}M}{M}\right) + \frac{k}{w}\frac{\mathrm{d}y}{k} - c\,\mathrm{d}t. \tag{24}$$

It follows from eq. (24) that the real rate of return of a nominal bond will be less correlated with the rate of return of real wealth than previously in this section, as now the real rate of return of real wealth is augmented by the transfer which is negatively correlated with the real rate of return of the nominal bond. As an unexpected increase in the money stock increases the expected growth rate of the money stock and hence decreases real balances, it has a wealth effect even with transfers, as long as these transfers are not capitalized. This means that all three previous results of this section hold qualitatively in the presence of non-capitalized transfers.

6. Concluding remarks

The results of this paper are derived in a model which relies on the assumption that markets are perfect and focuses on the households' optimization problem. Therefore, the model is not encumbered by the *ad hoc* assumptions about the households' consumption and investment policies encountered in many models used in macroeconomics. This paper shows that substantial insights into the effects of uncertainty about monetary policy can be gained by using general equilibrium models which solve for the households' consumption and investment policies. While the present model relies on many restrictive assumptions which limit its ability to explain the behavior of interest rates over the recent past, further research could relax some of these assumptions. In particular, it would be interesting to see how different assumptions about the dynamics of the money stock affect our results.

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