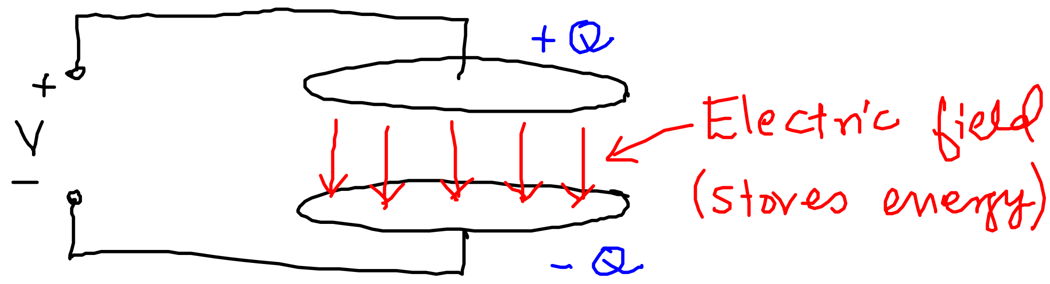
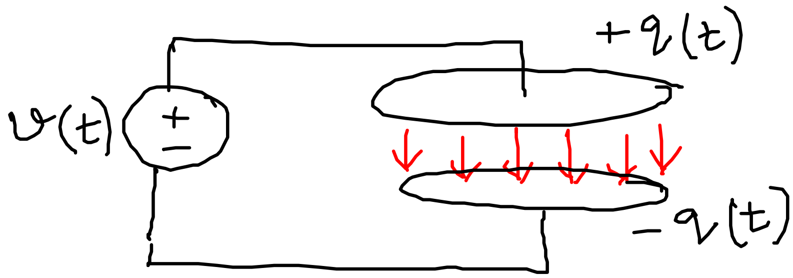
Capacitor



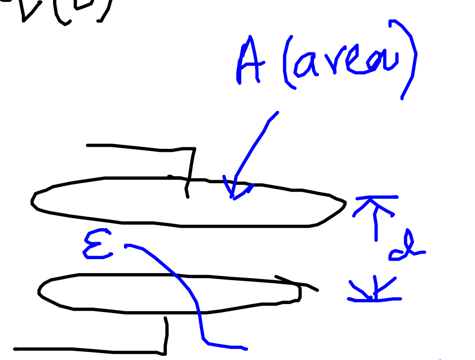
$$Q = VC$$

Q → coulombs, V → volt, C → Farad (F) — capacitance

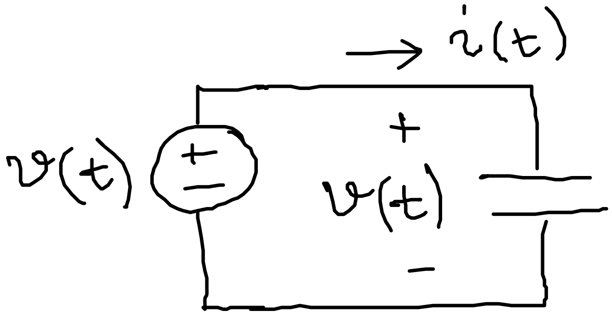


$$q(t) = C\,v(t)$$

$$C = \frac{\varepsilon A}{d}$$



$\varepsilon = 8.854 \times 10^{-12}$ F/m for air



$$i(t) = \frac{dq(t)}{dt} = C\frac{dv(t)}{dt}$$

$$i(t) = C\frac{dv(t)}{dt}$$

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(t')\,dt'$$

$$V_0 = v(t = t_0) = \frac{1}{C}\int_{-\infty}^{t_0} i(t')\,dt'$$

$t \geq t_0$

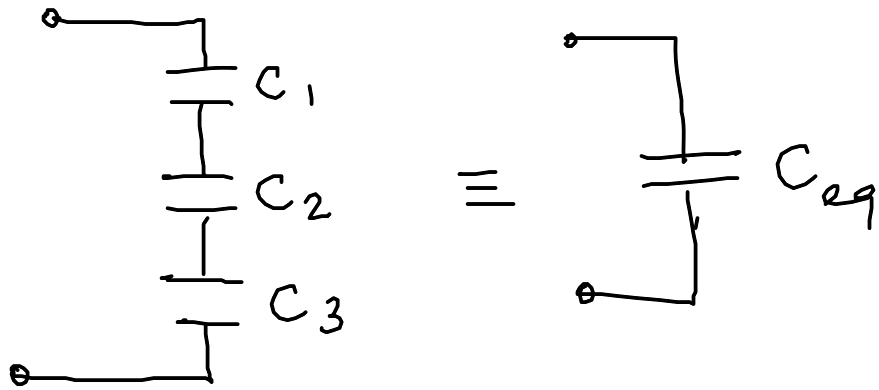
$$\hookrightarrow v(t) = V_0 + \frac{1}{C}\int_{t_0}^{t} i(t')\,dt'$$

typically $t = 0$

$$t \geq 0 \rightarrow v(t) = V_0 + \frac{1}{C}\int_{0}^{t} i(t')\,dt'$$

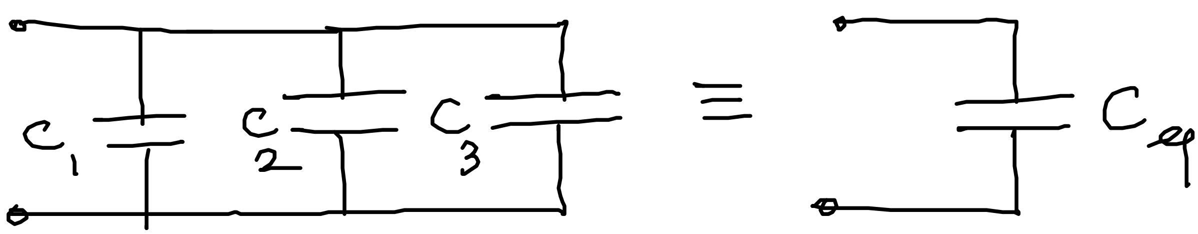
(V_0:) voltage across the capacitor at $t = 0$

series capacitors



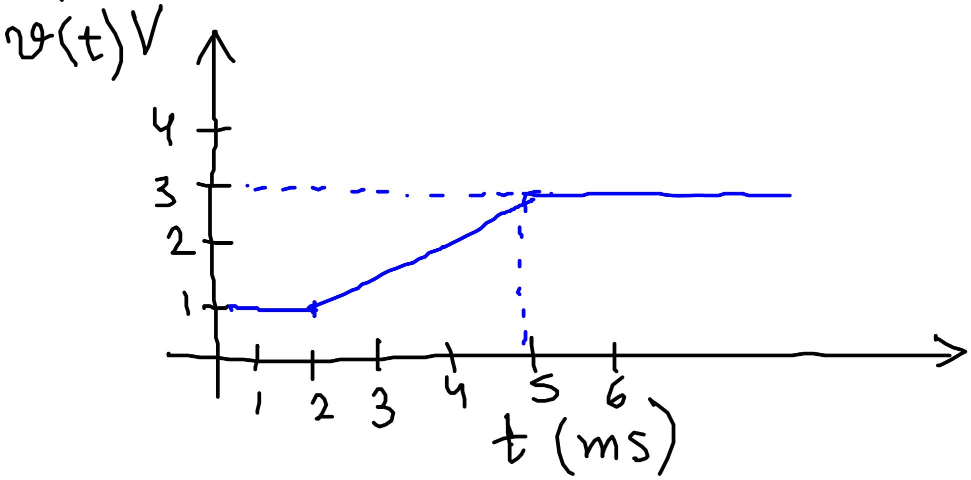
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

parallel capacitors



$$C_{eq} = C_1 + C_2 + C_3$$

Examples



$C = 1000\,\mu F$

$$i(t) = C\,\frac{dv}{dt}$$

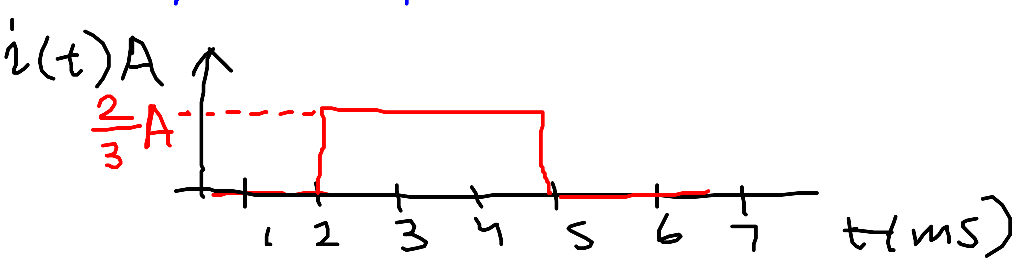
$i(t) = 0$ for $0 < t < 2\,ms$

$$i(t) = (1000\,\mu F)\left(\frac{2V}{3\,ms}\right)$$

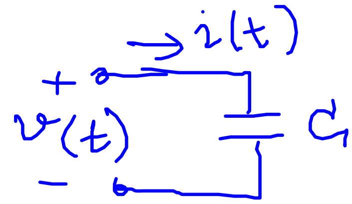
$$= 1000 \times 10^{-6}\,F\,\frac{2V}{3 \times 10^{-3}\,s}$$

$$= \frac{2}{3}\,A \quad \text{for } 2\,ms < t < 5\,ms$$

$i(t) = 0$ for $t > 5\,ms$



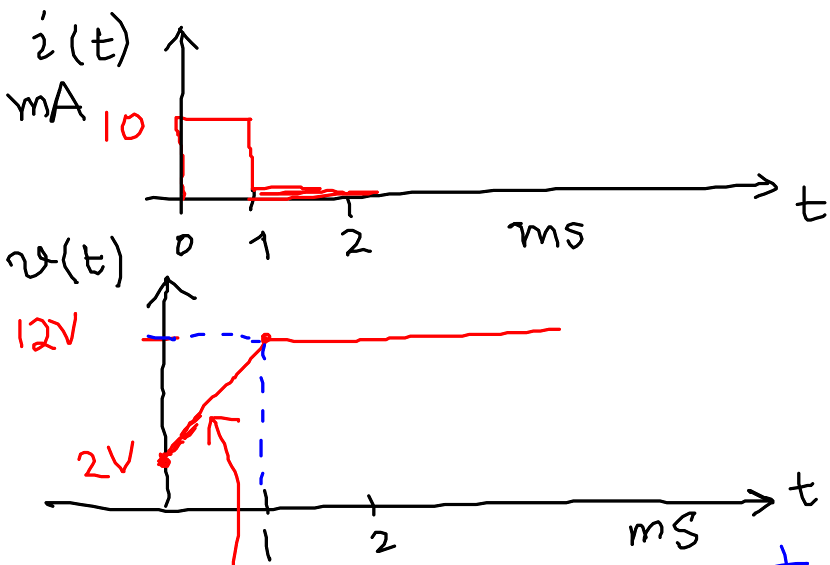
Example



$$i(t) = \begin{cases} 10\,mA & 0s \leq t \leq 1s \\ 0 & t > 1s \end{cases} \qquad C = 1000\,\mu F$$

$$V_0 = v(t=0) = 2V$$

$$v(t) = \overset{2V}{V_0} + \frac{1}{C}\int_0^t i(t')\,dt' \qquad t > 0$$



$0s < t < 1\ ms$

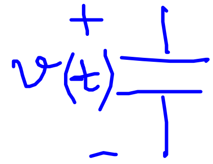
$$v(t) = 2V + \frac{1}{10^{-3}F}\int_0^t 10\,mA\,dt'$$

$$= 2V + 10t$$

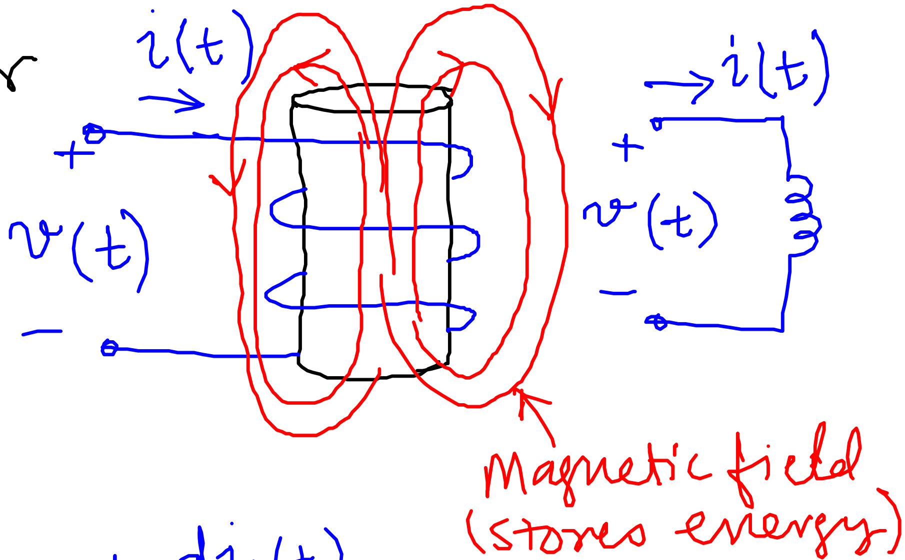
Energy stored in a capacitor

$$W_C(t) = \frac{1}{2} C v^2(t)$$

$\swarrow$ J



Inductor



$$v(t) = L \frac{di(t)}{dt}$$

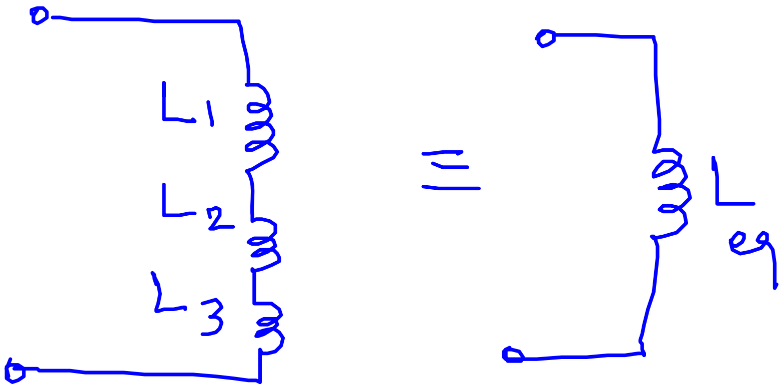
L → inductance (H) — henrys

$$i(t) = \frac{1}{L} \int_{-\infty}^{t} v(t')\,dt'$$

$$I_0 = i(t = t_0)$$

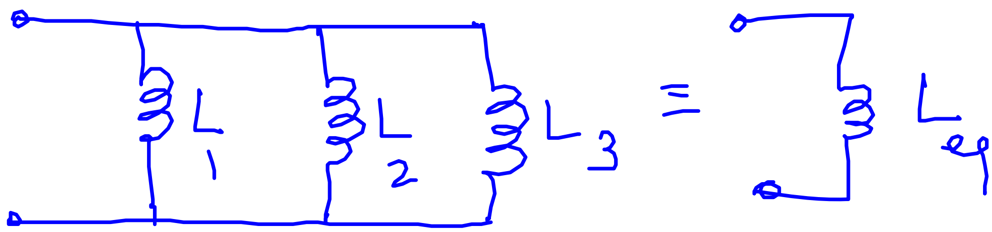
$$\underset{t \geq t_0}{i(t)} = I_0 + \frac{1}{L} \int_{t_0}^{t} v(t')\,dt'$$

Inductors in series



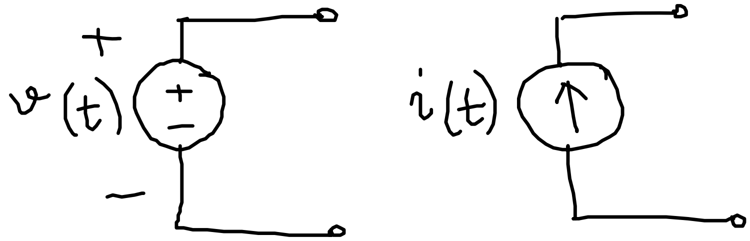
$$L_{eq} = L_1 + L_2 + L_3$$

Inductors in parallel



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

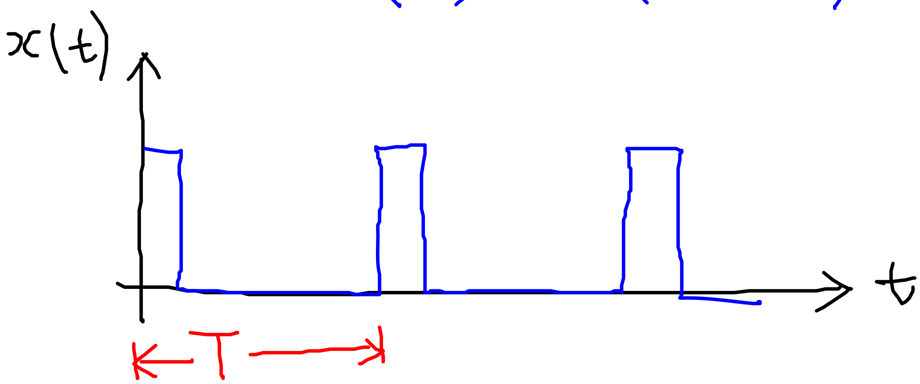
Time dependent signal sources

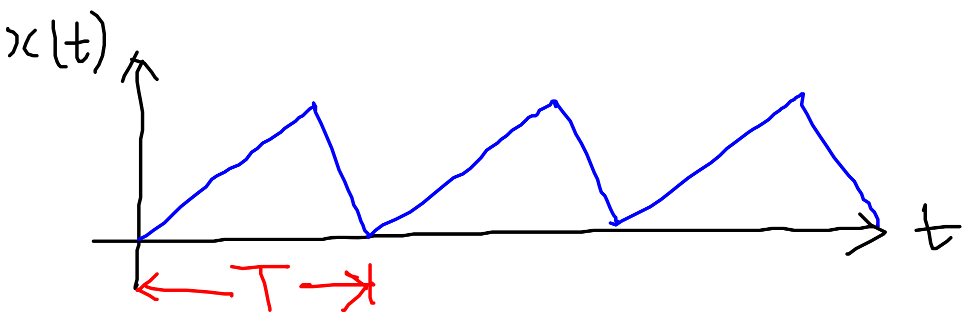


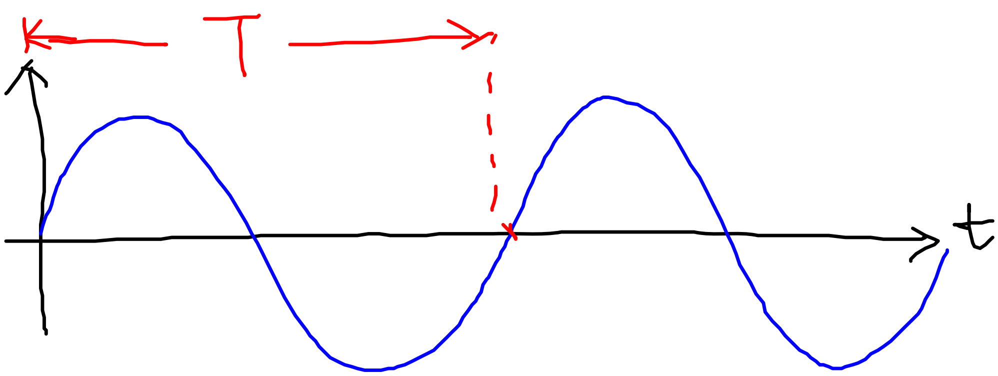
periodic signals

$$x(t) = x(t + nT) \quad n = 1, 2, 3 \ldots$$

(T → period)







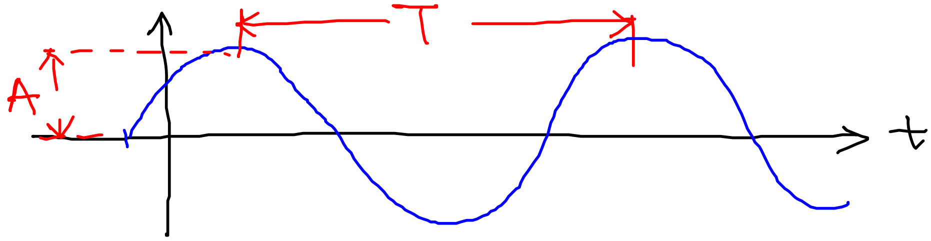
sinusoidal wave forms

$$x(t) = A\cos(\omega t + \phi)$$

- A → Amplitude (V or A)
- ω → angular (radian) frequency rad/sec
- ϕ → phase (radians or degrees)

Note $A\sin(\omega t) = A\cos(\omega t - \frac{\pi}{2})$

$A\cos(\omega t) = A\sin(\omega t + \frac{\pi}{2})$



frequency cycles/sec → hertz (Hz)

$\omega = 2\pi f$

ω ← radial frequency (rad/sec)

f ← natural frequency (cycles/sec, Hz)

$f = \frac{1}{T}$

$T = \frac{1}{f}$

$T = \frac{2\pi}{\omega}$