2.6. EBRT in $A_1,...,A_k,fA_1,...,fA_k,\subseteq$ on (MF,INF).

In this section, we use the tree methodology presented in section 2.1 to analyze EBRT in $A_1,...,A_k,fA_1,...,fA_k,\subseteq$ on (MF,INF). This turns out to be very easy, and we obtain the same classification if we replace MF by any subset of MF satisfying some weak conditions. In particular, we show that EBRT in $A_1,...,A_k,fA_1,...,fA_k,\subseteq$ on (MF,INF) is RCA_0 secure.

Note that in sections 2.4 and 2.5, we have stayed within EBRT in A,B,fA,fB,⊆. EBRT in A,B,fA,fB on (SD,INF),(ELG ∩ SD,INF), (ELG,INF),(EVSD,INF), is a major additional undertaking, and is beyond the scope of this book. The same can be said for various fragments of EBRT in A,B,C,fA,fB,fC,⊆ on (SD,INF),(ELG ∩ SD,INF),(ELG,INF),(EVSD,INF).

However, EBRT on (MF,INF) is considerably easier to analyze, due to the presence of constant functions and projection functions.

As usual, we start with the list of all $A_1,...,A_k,fA_1,...,fA_k,\subseteq$ elementary inclusions.

EBRT in $A_1,...,A_k,fA_1,...,fA_k$ on (MF,INF).

$A_i = \emptyset$. No.
$fA_i = \emptyset$. No. Set $f(x) = 0$.
$A_i \cap fA_j = \emptyset$. No. Set $f(x) = x$.
$A_i = N$.
$fA_i = N$. No. Set $f(x) = 0$.
$A_i \cup fA_j = N$.
$A_i \subseteq A_j$, $j < i$.
$A_i \subseteq fA_j$. No. Set $f(x) = 0$.
$A_i \subseteq A_j \cup fA_p$, $j < i$.
$fA_i \subseteq A_j$.
$fA_i \subseteq fA_j$, $j < i$.
$fA_i \subseteq A_j \cup fA_p$, $p < i$.
$A_i \cap fA_j \subseteq A_p$, $p < i$.
$A_i \cap fA_j \subseteq fA_p$, $p < j$.
$A_i \cap fA_j \subseteq A_p \cup fA_q$, $p < i$ and $q < j$.

EBRT in $A_1,...,A_k,fA_1,...,fA_k,\subseteq$ on (MF,INF).*
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$A_i = N$.
$A_i \cup fA_j = N$.
$A_i \subseteq A_j$, $j < i$.
$A_i \subseteq A_j \cup fA_k$, $j < i$.
$fA_i \subseteq A_j$.
$fA_i \subseteq fA_j$, $j < i$.
$fA_i \subseteq A_j \cup fA_p$, $p < i$.
$A_i \cap fA_j \subseteq A_p$, $p < i$.
$A_i \cap fA_j \subseteq fA_p$, $p < j$.
$A_i \cap fA_j \subseteq A_p \cup fA_q$, $p < i$ and $q < j$.

Entirely RCA_0 correct. Set $A_1 = ... = A_k = N$.

THEOREM 2.6.1. The following is provable in RCA_0. Let $V \subseteq MF$ contain at least one constant function of some arity, and at least one projection function of some arity. For all $k \geq 1$, EBRT in $A_1,...,A_k,fA_1,...,fA_k,\subseteq$ on (V,INF) and (MF,INF) have the same correct formats. For all $k \geq 1$, EBRT in $A_1,...,A_k,fA_1,...,fA_k,\subseteq$ on (MF,INF) is RCA_0 secure.

Proof: We can use any constant function and any projection function in place of the unary functions $f(x) = 0$ and $f(x) = x$ that were used above. RCA_0 suffices due to the obvious explicitness of the classification. QED

THEOREM 2.6.2. There is an algorithm for determining the truth value of any statement in EBRT in any $A_1,...,A_k,fA_1,...,A_k,\subseteq$ on (MF,INF). In fact, an algorithm can be given that can be proved to work in RCA_0.

Proof: The result follows from the explicitness of the classification, the algorithm presented in section 2.1, and Theorem 2.1.4. QED