CONCEPT CALCULUS

APA PANEL on LOGIC IN PHILOSOPHY

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1. INTRODUCTION.

We focus on an unexpectedly close connection between the logic of mathematical concepts and the logic of informal concepts from common sense thinking.

This connection is new and there is the promise of establishing similar connections involving a very wide range of informal concepts.

We call this development the Concept Calculus. It is in a very early stage of development.

The results in Concept Calculus are rather specific. For the initial result, we identify the "logic of mathematical concepts" to be the usual axioms of set theory that provide the usual foundations of mathematics - the ZFC axioms (Zermelo Frankel with the axiom of choice).

And for the initial result, we focus on a two particular informal concepts from common sense thinking. These are the binary relations

BETTER THAN.
MUCH BETTER THAN.

In the initial result, we present some axioms involving only "better than", "much better than", and identity between objects. These axioms are of a simple basic character, and range from obvious to plausible.

The initial result asserts the following. Let T be the system of axioms involving better than and much better than, presented below.

THEOREM. ZFC and T are mutually interpretable. I.e., there is an interpretation of ZFC in T, and there is an interpretation of T in ZFC.

COROLLARY. ZFC is consistent if and only if T is consistent.

It should be noted that both of these results are proved in an extremely weak fragment of ordinary mathematics. If we use Gödel numberings throughout, then these results are proved in a very weak fragment of PRA = primitive recursive arithmetic, which is itself a very weak fragment of PA = Peano Arithmetic. In particular, EFA = exponential function arithmetic = $I\sum_0(\exp)$, suffices. With some care, PFA = polynomial function arithmetic = bounded arithmetic = $I\sum_0$, suffices. There are corresponding weak theories of finite strings that suffice if we treat formal systems more directly, without Gödel numberings.

We also show that certain extensions of ZFC via so called "large cardinal hypotheses" correspond in the same way to certain very natural extensions of T, still using only better than and much better than.

2. INTERPRETATION POWER.

The notion of interpretation plays a crucial role in Concept Calculus.

Interpretability between formal systems was first precisely defined by Tarski. We work in the usual framework of first order predicate calculus with equality. See [4].

An interpretation of S in T consists of

i. A one place relation defined in T which is meant to carve out the domain of objects that S is referring to, from the point of view of T.

ii. A definition of the constants, relations, and functions in the language of S by formulas in the language of T, whose free variables are restricted to the domain of objects that S is referring to (in the sense of i).

iii. It is required that every axiom of S, when translated into the language of T by means of i,ii, becomes a theorem of T.

In ii, we usually allow that the equality relation in S need not be interpreted as equality - but rather as an equivalence relation.

We give two illustrative examples.

S consists of the axioms for linear order, together with "there is a least element".

i. $\neg(x < x)$.
ii. $(x < y \wedge y < z) \rightarrow x < z$.
iii. $x < y \vee y < x \vee x = y$.
iv. $(\exists x)(\forall y)(x < y \vee x = y)$.

T consists of the axioms for linear order, together with "there is a greatest element".

i. $\neg(x < x)$.
ii. $(x < y \wedge y < z) \rightarrow x < z$.
iii. $x < y \vee y < x \vee x = y$.
iv. $(\exists x)(\forall y)(y < x \vee x = y)$.

Note that S,T are theories in first order predicate calculus with equality, in the same language: just the binary relation symbol <.

CLAIM: S is interpretable in T and T is interpretable in S. They are mutually interpretable.

Here is the obvious interpretation of S in T. In T, take the objects of S to be everything (according to T).

Define $x < y$ of S to be $y < x$ in T.

Interpretation of the axioms of S formally yields

i'. $\neg(x < x)$.
ii'. $(y < x \wedge z < y) \rightarrow z < x$.
iii'. $y < x \vee x < y \vee x = y$.
iv'. $(\exists x)(\forall y)(y < x \vee x = y)$.

These are obviously theorems of T.

Here is a more sophisticated example. PA = Peano Arithmetic is the first order theory with equality, using 0,S,+,•. The axioms: successor axioms, defining equations for +,•, and the scheme of induction for all formulas in this language.

Now consider "finite set theory". This is a bit ambiguous: could mean either

ZFC without the axiom of infinity; i.e., or ZFC\I; or

ZFC with the axiom of infinity replaced by its negation; i.e., ZFC\I + ¬I.

THEOREM (well known). PA, ZFC\I, ZFC\I + ¬I are mutually interpretable.

PA in ZFC\I: nonnegative integers become finite von Neumann ordinals. Induction in PA gets translated to a consequence of foundation and separation.

ZFC\I + ¬I in PA: Sets of ZFC\I + ¬I, are coded by the natural numbers in PA – in an admittedly ad hoc manner.

The various axioms of ZFC\I + ¬I get translated into theorems of PA.

In many such examples of mutual interpretability, the considerably stronger relation of synonymy holds. We will not delve into this further here.

3. BASIC FACTS ABOUT INTERPRETATION POWER.

We begin with the observation that for any two S,T, if T is inconsistent (proves a sentence and its negation) then S is interpretable in T. I.e., the most powerful level of interpretation power is inconsistency. Of course, this level is to be avoided.

A very fundamental fact about interpretation power is that there is no greatest interpretation power - short of inconsistency.

THEOREM 3.1. (In ordinary predicate calculus with equality). Let S be a consistent recursively axiomatized theory. There exists a consistent finitely axiomatized extension T of S which is not interpretable in S.

The above is most easily proved using Gödel's second incompleteness theorem. This tells us that, roughly speaking, S + Con(S) is never interpretable in S (assuming S is consistent). There are some problems with using this, as S may prove its own inconsistency. These problems can be overcome.

COMPARABILITY(?). Let S,T be recursively axiomatized theories. Then S is interpretable in T or T is interpretable in S?

There are plenty of natural and interesting examples of incomparability for finitely axiomatized theories that are rather weak. To avoid trivialities, we give an example of incomparability where there are only infinite models.

S is the theory of discrete linear orderings without endpoints.

T is the theory of dense linear orderings without endpoints.

We get plenty of incomparability arbitrarily high up:

THEOREM 3.2. Let S be a consistent recursively axiomatized theory. There exist consistent finitely axiomatized theories T_1,T_2, both in a single binary relation symbol, such that

i) S is provable in T_1,T_2;
ii) T_1 is not interpretable in T_2;
iii) T_2 is not interpretable in T_1.

BUT, are there examples of incomparability between natural theories that are metamathematically strong? Say in which PA is interpretable?

STARTLING OBSERVATION. ***Any two natural theories S,T, known to interpret PA, are known (with small numbers of exceptions) to have: S is interpretable in T or T is interpretable in S. The exceptions are believed to also have comparability.***

Because of this observation, there has emerged a rather large linearly ordered table of "interpretation powers" represented by natural formal systems. Generally, several natural formal systems may occupy the same position.

We call this growing table, the ***Interpretation Hierarchy***.

See [4] for much more information about Tarski interpretations.

4. INITIAL DEVELOPMENT OF CONCEPT CALCULUS.

We begin with the notions: better than (>), and much better than (>>). These are binary relations. This is an example of what we call *concept amplification*.

One can also view > and >> mereologically, as

x > y iff y is a "proper part of x".

x >> y iff y is a "small proper part of x".

An important idea is that of minimality. We say that x is minimal if and only if x is not better than anything.

We say that x is minimally better than y if and only if x is better than y and the things y is better than, and nothing else.

BASIC. Nothing is better than itself. If x is better than y and y is better than z, then x is better than z. If x is much better than y, then x is better than y. If x is much better than y and y is better than z, then x is much better than z. If x is better than y and y is much better than z, then x is much better than z. There is something that is much better than any given x,y.

MINIMAL. If x is much better than y, then x is much better than some, but not all, things that are minimally better than y.

EXISTENCE. Let x be a thing better than a given range of things. There is something that is better than the given range of things and the things that they are better than, and nothing else. Here we use L(>,>>) to present the range of things.

AMPLIFICATION. Let x,y be given, as well as a true statement about x, using the two binary relations >,=, and the unary relation >> x. The corresponding statement about x, using the two binary relations >,=, and the unary relation >> x,y, is also true.

i. MIINIMAL says that if x is much better than y, then y can be slightly improved so that x remains much better. However, there is great diversity among these slight improvements, so that nothing is much better than all of them.
ii. From BASIC and MINIMAL, we get lots of incomparables.
iii. EXISTENCE says that there is something of any bounded level of goodness. This corresponds to the Separation Axiom in set theory.
iv. AMPLIFICATION is a particular way of saying this: “I cannot tell the difference (using >,=), collectively, between the things that are much better than me, and the things that are much better than both you and me”.
v. AMPLIFICATION embodies both the Power Set Axiom and the Replacement Axiom of set theory. It makes “much better than” correspond to “jumping up greatly in cardinality”.
vi. The last axiom of BASIC corresponds to the Infinity Axiom in set theory.

THEOREM 4.1. BASIC + MINIMAL + EXISTENCE + AMPLIFICATION is mutually interpretable with ZFC. This is provable in EFA.

COROLLARY 4.2. ZFC is consistent if and only if BASIC + MINIMAL + EXISTENCE + AMPLIFICATION is consistent. This is provable in EFA.

There are several tricky points involved in establishing these results. Careful investigation reveals that many of the above axioms can be strengthened, weakened, or dropped.

AMPLIFIED LIMIT. There is something that is better than something, and also much better than everything it is better than.

vii. AMPLIFIED LIMIT postulates the existence of a kind of STAR. Stars correspond to certain large cardinals.

THEOREM 4.3. BASIC + MINIMAL + EXISTENCE + AMPLIFICATION + AMPLIFIED LIMIT interprets ZFC + "there is an almost ineffable cardinal" and is interpretable in ZFC + "there exists an ineffable cardinal".

BINARY AMPLIFICATION. Let x,y be given, as well as a true statement about x, using the three binary relations >,=, and z >> w >> x. The corresponding statement about x, using the three binary relations >,=, and z >> w >> x,y, is also true.

viii. BINARY AMPLIFICATION is a powerful extension of AMPLIFICATION that throws us far beyond ZFC, even with "small large cardinals" such as ineffable cardinals.

ix. BINARY AMPLIFICATION is a particular way of saying this: "I cannot tell the difference (using >,=) between the much better relation among the things that are much better than me, and the much better relation among the things that are much better than both you and me".

THEOREM 4.4. BASIC + MINIMAL + EXISTENCE + BINARY AMPLIFICATION interprets ZFC + "there exists a Ramsey cardinal" and is interpretable in ZFC + "there exists a measurable cardinal".

THEOREM 4.5. BASIC + MINIMAL + EXISTENCE + AMPLIFIED LIMIT + BINARY AMPLIFICATION interprets ZFC + "there exists a measurable cardinal with arbitrarily large lesser measurable cardinals" and is interpretable in ZFC + "there exists a measurable cardinal with a normal measure 1 set of lesser measurable cardinals".

Can we reason confidently within this world of abstract "better than and much better than"?

We anticipate a logical analysis of all of the "simple" propositions (and schemes) involving "better than, much better than". There should emerge a handful of preferred, divergent, "complete views", which determine the truth values of all of these "simple" propositions (and schemes).

A major preliminary step is to analyze all of the “simple” statements that hold in a fundamental model of the kind that we discuss below.

What happens to Russell’s Paradox? In sets, we start with

THERE IS A SET WHOSE ELEMENTS ARE EXACTLY THE SETS WITH A GIVEN PROPERTY

and obtain a contradiction that Frege missed and Russell saw.

The corresponding principle here is

THERE IS SOMETHING WHICH IS BETTER THAN, EXACTLY, THE THINGS WITH A GIVEN PROPERTY AND THOSE THINGS THEY ARE BETTER THAN.

This immediately leads to a contradiction, even before "and". This is because there cannot be anything which is better than all things - by irreflexivity. I.e., nothing can be better than itself.

Thus Russell’s Paradox now becomes entirely transparent and never would have trapped anyone. In fact, it disappears as a Paradox. Clearly there is no residual feeling of mystery as there is in the context of sets and properties.

Sections 5,6,7 below discuss examples of might be thought of as “naïve physics”.

5. SINGLE VARYING QUANTITY.

We now consider a single varying quantity – where the time and quantity scales are the same, and are linearly ordered.

This is common in ordinary physical science, where the time scale and the quantity scale may both be modeled as nonnegative real numbers.

The language has >,>>,=,F, where >,>> are binary relations, and F is a one place function.

F(x) is the value of the varying quantity at time x.

When thinking of time, >,>> is later than and much later than. When thinking of quantity, >,>> is greater than and much greater than.

BASIC. Nothing is better than itself. If x is better than y and y is better than z, then x is better than z. If x is much better than y, then x is better than y. If x is much better than y and y is better than z, then x is much better than z. If x is better than y and y is much better than z, then x is much better than z. There is something that is much better than any given x,y. For any x ≠ y, x is better than y or y is better than x.

MINIMAL. If x is much better than y, then x is much better than something minimally better than y.

ARBITRARY BOUNDED RANGES. Every bounded range of values is the range of values over some bounded interval. Here we use L(>,>>,=,F) to present the bounded range of values.

AMPLIFICATION. Let x,y be given, as well as a true statement about x, using F, the two binary relations >,= and the unary relation >> x. The corresponding statement about x, using F, the two binary relations >,= and unary relation >> x,y, is also true.

THEOREM 5.1. Using the above versions, BASIC + MINIMAL + ARBITRARY BOUNDED RANGES + AMPLIFICATION is mutually interpretable with ZFC. This is provable in EFA.

We can strengthen as before:

AMPLIFIED LIMIT. There is something that is greater than something, and also much greater than everything it is greater than.

THEOREM 5.2. Using the above versions, BASIC + MINIMAL + ARBITRARY BOUNDED RANGES + AMPLIFICATION + AMPLIFIED LIMIT interprets ZFC + "there is an almost ineffable cardinal" and is interpretable in ZFC + "there exists an ineffable cardinal".

BINARY AMPLIFICATION. Let x,y be given, as well as a true statement about x, using F and the three binary relations >,=, and z >> w >> x. The corresponding statement about x, using F, the three binary relations >,=, and z >> w >> x,y, is also true.

THEOREM 5.3. Using the above versions, BASIC + MINIMAL + ARBITRARY BOUNDED RANGES + BINARY AMPLIFICATION interprets ZFC + “there exists a Ramsey cardinal” and is interpretable in ZFC + “there exists a measurable cardinal”.

THEOREM 5.4. Using the above versions, BASIC + MINIMAL + ARBITRARY BOUNDED RANGES + AMPLIFIED LIMIT + BINARY AMPLIFICATION interprets ZFC + “there exists a measurable cardinal with arbitrarily large lesser measurable cardinals” and is interpretable in ZFC + “there exists a measurable cardinal with a normal measure 1 set of lesser measurable cardinals”.

As before, these latter two principles push the interpretation power well into the large cardinal hierarchy.

There are versions where we do not assume that the time scale is the same as the quantity scale. Some of these versions use two varying quantities, and there are three separate scales (time, first quantity, second quantity).

6. SINGLE VARYING BIT.

We now use a bit varying over time. Physically, this is like a flashing light. Mathematically, it corresponds to having a time scale with a unary predicate.

In order to get logical power out of this particularly elemental situation, we need to use forward translations of time. We think of b+c so that the amount of time from b to b+c is the same as the amount of time before c.

We use >,>>,=,+,P, where P(t) means that the varying bit at time t is 1.

Instead of a time scale, we can think of one dimensional space with a direction. P(t) means that there is a pointmass at position t.

In the earlier contexts, we did not support continuity. Here we simultaneously support discreteness and continuity.

BASIC. Nothing is better than itself. If x is better than y and y is better than z, then x is better than z. If x is much better than y, then x is better than y. If x is much better than y and y is better than z, then x is much better than z. If x is better than y and y is much better than z, then x is much better than z. There is something that is much better than any given x,y. If x is much better than y, then x is much better than something better than y. For any $x \neq y$, x is better than y or y is better than x.

BOUNDED TIME TRANSLATION. For every given range of times before a given time b, there exists a translation time c such that a time before b lies in the range of times if and only the bit at time b+c is 1. Here we use L(>,>>,=,+,P) to present the range of times.

The idea is our usual one: the behavior of P over bounded intervals is arbitrary, up to translation.

AMPLIFICATION. Let x,y be given, as well as a true statement about x, using P, the binary function +, the two binary relations >,= and the unary relation >> x. The corresponding statement about x, using P, the binary function +, the two binary relations >,= and the unary relation >> x,y, is also true.

AMPLIFIED LIMIT. There is something that is greater than something, and also much greater than everything it is greater than.

BINARY AMPLIFICATION. Let x,y be given, as well as a true statement about x, using P, the binary function +, and the three binary relations >,=, and z >> w >> x. The corresponding statement about x, using P, the binary function +, and the three binary relations >,=, and z >> w >> x,y, is also true.

THEOREM 6.1. Using the above versions, BASIC + ARBITRARY BOUNDED RANGES + AMPLIFICATION is mutually interpretable with ZFC. This is provable in EFA.

AMPLIFIED LIMIT. There is something that is greater than something, and also much greater than everything it is greater than.

THEOREM 6.2. Using the above versions, BASIC + BOUNDED TIME TRANSLATION + AMPLIFICATION + AMPLIFIED LIMIT interprets ZFC + “there is an almost ineffable cardinal” and is interpretable in ZFC + “there exists an ineffable cardinal”.

BINARY AMPLIFICATION. Let x,y be given, as well as a true statement about x, using F and the three binary relations >,=, and z >> w >> x. The corresponding statement about x, using F, the three binary relations >,=, and z >> w >> x,y, is also true.

THEOREM 6.3. Using the above versions, BASIC + BOUNDED TIME TRANSLATION + BINARY AMPLIFICATION interprets ZFC + "there exists a Ramsey cardinal" and is interpretable in ZFC + "there exists a measurable cardinal".

THEOREM 6.4. Using the above versions, BASIC + BOUNDED TIME TRANSLATION + AMPLIFIED LIMIT + BINARY AMPLIFICATION interprets ZFC + "there exists a measurable cardinal with arbitrarily large lesser measurable cardinals" and is interpretable in ZFC + "there exists a measurable cardinal with a normal measure 1 set of lesser measurable cardinals".

7. PERSISTENTLY VARYING BIT.

The objection can be raised that a varying bit realistically has to have persistence. It cannot be varying "densely". Specifically, if the bit is 1 then it remains 1 for a while, and if the bit is 0 then it remains 0 for a while.

Define a persistent range of times in the obvious way.

PERSISTENT TIME TRANSLATION. For any time b and persistent range of times before b, there exists a translation time c such that any time before b lies in the range of times if and only if the bit at time b+c is 1. Here we use L(>,>>,=,P,+) to present the range of times.

We need to have two additional time principles.

ADDITION. $y < z \rightarrow x+y < x+z$.

ORDER COMPLETENESS. Every nonempty range of times with an upper bound has a least upper bound. Here we use L(>,>>,=,P,+) to present the nonempty range of times.

We also have BASIC, AMPLIFICATION, AMPLIFIED LIMIT, BINARY AMPLIFICATION as in section 6.

We get the analogous results.

There are also versions that treat the idea of an expanding universe, where expansion is internal, and not just at the end. This leads to the logical strength of elementary embedding hypotheses from large cardinal theory.

8. AN INTERPRETATION IN SET THEORY.

We present a set theoretic interpretation of BASIC + MINIMAL + EXISTENCE + AMPLIFICATION, for {BETTER THAN, MUCH BETTER THAN}. The other direction is too technical for this talk.

We first form an underlying structure $(D,>,=)$.

We define pairs $(D_\alpha,>_\alpha)$, for all ordinals α. Define $(D_0,>_0) = (\emptyset,\emptyset)$. Suppose $(D_\alpha,>_\alpha)$ has been defined, and is transitive and irreflexive. Define $(D_{\alpha+1},>_{\alpha+1})$ to extend $(D_\alpha,>_\alpha)$ by adding an exact strict upper bound for every subset of D_α - even if it already has an exact strict upper bound. For limit ordinals λ, define $D_\lambda = \cup_{\alpha<\lambda} D_\alpha$, $>_\lambda = \cup_{\alpha<\lambda} >_\alpha$. Let $D = \cup_\alpha D_\alpha$, $> = \cup_\alpha >_\alpha$.

The new elements introduced at each stage are incomparable, and never below ("worse than") previously introduced elements. Thus the (eventual) predecessors of x are introduced earlier than x. Each exact upper bound introduced remains valid later.

LEMMA 8.1. $(D,>)$ is irreflexive and transitive, satisfies Minimality, and also Existence in second order form. The same claims are true for $(D_\kappa,>_\kappa)$, where κ is a limit ordinal.

Now fix S to be a nonempty set of limit ordinals, with no greatest element, whose union is κ. We define M[S] to be $(D_\kappa,>_\kappa,>>_S)$, where $D_\kappa,>_\kappa$ is as above. We define $x >>_S y$ if and only if

$$x,y \in D_\kappa \wedge (\exists\alpha,\beta \in S)(\alpha < \beta \wedge y \in D_\alpha \wedge (\forall w \in D_\beta)(x > w)).$$

LEMMA 8.2. M[S] satisfies Basic.

LEMMA 8.3. MK proves that there exists a set S of limit ordinals, with no greatest element, with the following indiscernibility property. For all $\alpha < \beta < \gamma$ from S, β,γ

have the same first order properties over V, relative to any parameters from $V(\alpha)$.

LEMMA 8.4. Let $n < \omega$. ZF proves that there exists a set S of limit ordinals, with no greatest element, with the following indiscernibility property. For all $\alpha < \beta < \gamma$ from S, β,γ have the same first order properties over V, with at most n quantifiers, relative to any parameters from $V(\alpha)$.

LEMMA 8.5. Let S to be a nonempty set of limit ordinals, with no greatest element. In M[S], x >> y if and only if $(\forall w \in D_\beta)(x > w)$, where β is the least ordinal in S after α, and α is the least ordinal in S such that $y \in D_\alpha$.

LEMMA 8.6. Suppose S has the indiscernibility property in Lemma 8.3. Then M[S] satisfies BASIC + MINIMAL + EXISTENCE + AMPLIFICATION.

THEOREM 8.7. BASIC + MINIMAL + EXISTENCE + AMPLIFICATION is interpretable in ZF.

For AMPLIFIED LIMIT, we need S to contain a limit point of S. This can be obtained from an ineffable cardinal.

For BASIC + MINIMAL + EXISTENCE + BINARY AMPLIFICATION, we need S of type ω satisfying a more powerful form of indiscernibility.

LEMMA 8.8. Suppose there is a countable transitive model of ZC + "there exists a measurable cardinal". There is a countable transitive model A of ZF, and an unbounded $S \subseteq A$ consisting of limit ordinals, of order type ω, with the following indiscernibility property. Let $\alpha < \beta < \gamma$ be from S. Then $S\backslash\beta$ and $S\backslash\gamma$ have the same first order properties over A, relative to any parameters from the $V(\alpha)$ of A.

Let A be a transitive model of ZF, and S be an unbounded subset of A consisting of limit ordinals. Define M[A,S] as follows. The domain and the > is defined internally in M, as proper classes of M, as before. We write these as DM, $>_M$. The >>, which is a binary relation on D_M, is then defined as before. We write this as $>>_S$.

LEMMA 8.9. M[A,S] satisfies Basic + Minimality + Existence.

LEMMA 8.10. Let A,S be as in the conclusion of Lemma 8.8. Then M[A,S] satisfies BINARY AMPLIFICATION.

THEOREM 8.11. BASIC + MIIMALITY + EXISTENCE + BINARY AMPLIFCATION interprets ZFC + "there exists a Ramsey cardinal" and is interpretable in ZFC + "there exists a measurable cardinal".

There are three older manuscripts present on this website http://www.math.ohio-state.edu/~friedman/.

[1] H. Friedman, "Concept Calculus", October 25, 2006, 24 pages. Downloadable Manuscripts, #53, http://www.math.ohio-state.edu/~friedman/manuscripts.html.

[2] H. Friedman, "Interpreting Set Theory in Ordinary Thinking: Concept Calculus", May 24, 2007, 12 pages. Third Tarski Lecture, 2007. Downloadable Manuscripts, #60, http://www.math.ohio-state.edu/~friedman/manuscripts.html.

[3] H. Friedman, "Concept Calculus", lecture delivered at Mathematical Methods in Philosophy, Banff, Canada, February 17, 2007, 9 pages. Lecture Notes, #37, http://www.math.ohio-state.edu/~friedman/manuscripts.html.

[4] H. Friedman, "Interpretations, According to Tarski", May 24, 2007, 42 pages. First Tarski Lecture, 2007. Downloadable Manuscripts, #60, http://www.math.ohio-state.edu/~friedman/manuscripts.html.

PRINCIPLE OF PLENITUDE

From Wikipedia, Plenitude Principle.

The principle of plenitude asserts that everything that can happen will happen.

The historian of ideas Arthur Lovejoy was the first to discuss this philosophically important Principle explicitly, tracing it back to Aristotle, who said that no possibilities which remain eternally possible will go unrealized, then forward to Kant, via the following sequence of adherents:

Augustine of Hippo brought the Principle from Neo-Platonic thought into early Christian Theology.

St Anselm 's ontological arguments for God's existence used the Principle's implication that nature will become as complete as it possibly can be, to argue that existence is a 'perfection' in the sense of a completeness or fullness.

Thomas Aquinas's belief in God's plenitude conflicted with his belief that God had the power not to create everything that could be created. He chose to constrain and ultimately reject the Principle.

Giordano Bruno's insistence on an infinity of worlds was not based on the theories of Copernicus, or on observation, but on the Principle applied to God. His death may then be attributed to his conviction of its truth.

Leibniz believed that the best of all possible worlds would actualize every genuine possibility, and argued in *Théodicée* that this best of all possible worlds will contain all possibilities, with our finite experience of eternity giving no reason to dispute nature's perfection.

Kant believed in the Principle but not in its empirical verification, even in principle.

The Infinite monkey theorem and Kolmogorov's zero-one law of contemporary mathematics echo the Principle. It can also be seen as receiving belated support from certain radical directions in contemporary physics, specifically the many-worlds interpretation of quantum mechanics and the cornucopian speculations of Frank Tipler on the ultimate fate of the universe.