FOUNDATIONAL ADVENTURES FOR THE FUTURE

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FOUNDATIONAL ADVENTURES
CONFERENCE IN HONOR OF MY 60TH BIRTHDAY
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HONOREE LECTURE
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I'm now going to have to get serious

- I'm still relatively invisible
- Not too many decades left
- Need for increased rate of publication
- Have been mostly focusing on excruciatingly challenging odysseys
- Need to greatly expand the scope of activities

WHAT EXCRUCIATINGLY CHALLENGING ODYSSEYS?

- Wright Brothers made a two mile flight
- Wright Brothers made a 42 mile flight
- Want to ship goods
- Want to move lots of passengers
- Want reliability and safety
- Want low cost
- ... Modern aviation
- Each major advance spawns reasonable demands for more and more
- Excruciating difficulties overcome
- Armies of people over decades or more
- Same story for any practically any epoch breaking advance in anything

WHAT EXCRUCIATINGLY CHALLENGING ODYSSEYS?

- Gödel's Incompleteness Phenomena
- Always sentences neither provable nor refutable
- But undesirably abstract from the mathematical viewpoint
- Want concrete examples
- Want nice concrete examples
- Want very nice concrete examples
- Want extremely nice concrete examples
- Want famous concrete examples
- Want diverse famous iconic concrete examples
- After > 40 years, at very early stages
- Boolean Relation Theory monograph
- Abstracts on Π01 Incompleteness - a few days ago

WHAT EXCRUCIATINGLY CHALLENGING ODYSSEYS?

- More recently: consistency proof for mathematics
- Have simple, intriguing, arguably plausible, axioms for “better” and “much better” that prove the consistency of mathematics
- Want compelling axioms for “better”, “much better”
- Want evident axioms for “better”, “much better”
- Want compelling/evident axioms for “better”, “much better” compatible with only finitely many entities at any time
- Want to instead use fundamental physical notions - “naive physics”
- Concept Calculus - develop axioms for any informal/semiformal concepts
- Concept Calculus: much better than (available)

WHAT EXCRUCIATINGLY CHALLENGING ODYSSEYS?

- Has resulted in substantial satisfaction
- Has resulted in substantial dissatisfaction
- About 100 longish papers and about 650 page monograph in 42 years
- Possibly 500 longish papers and about 5 650 page monographs if excruciating challenges were avoided

FRIEDMAN

UNLEASHED

CATEGORIES OF ADVENTURES

- foundations of mathematics
- mathematical philosophy
- philosophy of mathematics
- programming languages and verification
- finitization
- computer assisted education
- foundations of probability/statistics
- foundations of physical theories
- foundations of law/politics/ethics/economics
- piano performance

FOUNDATIONS OF MATHEMATICS

- Boolean Relation Theory development - to appear
- Strict Reverse Mathematics - to appear - recovers EFA from strict math
- Finite to Infinite (through Comprehension) - in preparation, partial manuscript exists
- Completeness/Solvability - e.g., 3 quantifier sentences in $\in,=$
- Incompleteness/Unsolvability - e.g., 4 quantifier sentences in $\in,=$, more recently: in Euclidean Geometry
- Interpretability: mutual interpretability = equiconsistency, book with Visser in preparation
- New Axioms (especially large cardinals) large cardinals via simple general algebra, published
- Practical Proof Theory - in preparation - structural properties of actual proofs

MATHEMATICAL PHILOSOPHY
PHILOSOPHY OF MATHEMATICS

- Concept Calculus development - scope is everything except current science and engineering
- Foundational Exposition - most effective way to present subjects - principles and development - supports “universal education”, and completely new era in interdisciplinary communication
- Isms - detailed novel logical analysis of platonism, realism, empiricism, finitism, ultrafinitism, predicativism, conventionalism, nominalism, foundationalism, etcetera

PROGRAMMING LANGUAGES AND VERIFICATION

- Programming Language design - functional, but efficiency tightly controlled
- Software Specification - design of friendly specification languages, with friendly rigorous semantics - already an issue for computer arithmetic
- Software Verification - math behind ordinary software is ridiculously trivial - algorithms needed to recognize ridiculous trivialities
- Programming Language Design fundamentally integrated with Verification method

FINITIZATION

- Rework the whole of infinitary mathematics in clear and appropriately robust finite terms
- Pixelization - more radical - theory of mathematics present on your computer screen
- Hypothesis - all significant understandable mathematical phenomena can be adequately represented by a diagram on a computer screen with relatively small resolution
- Prove all celebrated facts about the first 8 levels of the cumulative hierarchy, or alternatively, the first 2^2^2^2^2^2^2^2 positive integers, using almost no axioms (largely done)
- Identity “first” place where finitization is impossible

COMPUTER ASSISTED EDUCATION

- Longstanding pioneer is Patrick Suppes
- Major problems with user friendliness as the practical math/sci/eng settings get even slightly sophisticated
- Detailed analysis of special proof techniques for fundamental contexts - new completeness theorems - micro completeness (well under way)
- Templating - investigations into the nature of random sentences meeting certain educationally motivated constraints

Foundations of Probability/Statistics

Physical Theories

Law/Politics/Ethics/ Economics

- Don't have time to discuss these - have some definite ideas that are not standard
- Great practical need for foundations of law/politics/ethics
- Contentious arguments commonly ensue between parties who disagree at a hidden fundamental level
- Tremendous need to "agree to disagree" at the fundamental level
- Desperate practical need for foundations
- Left unchecked, may evolve into violent breakdown of society

PIANO PERFORMANCE

- Digitally Sculptured Piano Performance
- Creation of classical piano recordings at a new level of beauty and technical brilliance
- Apparent use for extraordinary ordinary piano performance
- Major advances in artistic production, education, and aesthetics

ENHANCED INTERROGATION AND WATERBOARDING

- Current methodology in mathematical logic can be greatly improved
- Same for mainstream mathematics
- Same for mainstream philosophy
- Same for mainstream computer science
- Same for musicians, music theorists, and music educators
- Same for everything that I have seen in academia